Impact of Product Modularity in Managing Uncertainty in Project Duration

Northern Illinois University
Department of Technology

Shun Takai

PERT (Program Evaluation and Review Technique)

PERT

- PERT (Project Evaluation and Review Technique)
  - Method to estimate probability of a project completion before target date
- History
  - Originated in US Navy's Polaris Missile System program
  - Developed by an operations research team that was formed in 1958 with members from:
    - Navy's Special Project Office
    - Booz, Allen, and Hamilton (consulting firm)
    - Lockheed Missile Systems (prime contractor)

(Reference) Project Management for Engineering, Business, and Technology, by John M. Nicholas and Herman Steyn, Routledge, 2012, 4th edition

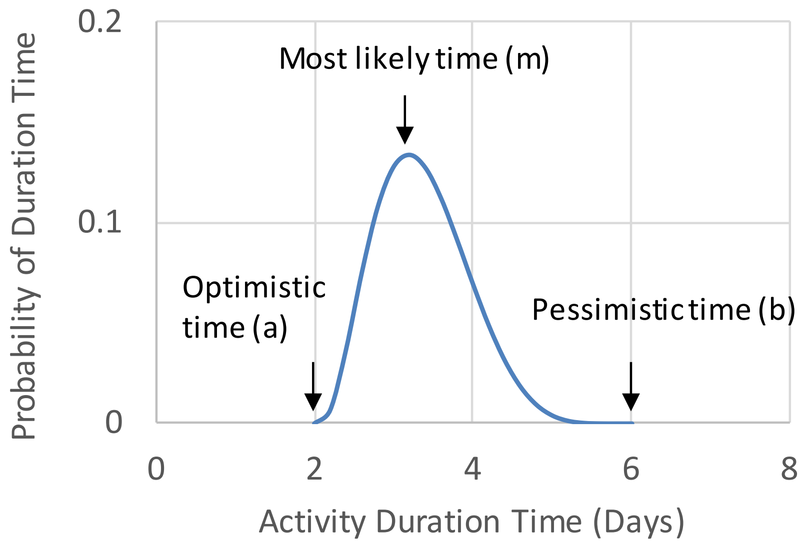
Activity Duration

- In PERT, uncertainty of each activity is modeled by beta distribution using three time estimates
  - a: Optimistic time (shortest)
  - m: Most likely time
  - b: Pessimistic time (longest)
- Mean t_e, variance V, and standard deviation σ of an activity duration are calculated from three time estimates

Expected time : $t_e = \frac{a+4m+b}{6}$

Variance : $V = \left(\frac{b-a}{6}\right)^2$

Standard deviation: $\sigma = \frac{b-a}{6}$



(Reference) Project Management for Engineering, Business, and Technology, by John M. Nicholas and Herman Steyn, Routledge, 2012, 4th edition

Project Duration

- Mean T_e, variance V_p, and standard deviation σ_p of each path in the project are calculated from mean t_e and variance V of activities in the path
  - Activities are assumed to be independent

Expected time: $T_e = \sum t_e$ for each path

Path

Activities in the path

Variance: $V_P = \sum V$ for each path

Standard deviation: $\sigma_P = \sqrt{V_P}$

Probability of Project Completion

- Assumption
  - Time of each path is distributed according to a normal distribution
- Probability that a project finishes before T_s (target time) is calculated from z value and z table

$$Z = \frac{T_s - T_e}{\sqrt{V_P}}$$

- Example: What is the probability that a project ends in 27 days?
  - $T_s = 27$
  - $T_e = 29$
  - $V_p = 6$

$$Z = \frac{T_s - T_e}{\sqrt{V_P}} = \frac{27 - 29}{\sqrt{6}} = -0.82$$

  - From Z-table, the probability that the project completes before 27 days is 0.21
  - Using Excel:

$$NORMSDIST(-0.82) = 0.206$$

(Reference) Project Management for Engineering, Business, and Technology, by John M. Nicholas and Herman Steyn, Routledge, 2012, 4th edition

Table IV

Standard Normal Distribution

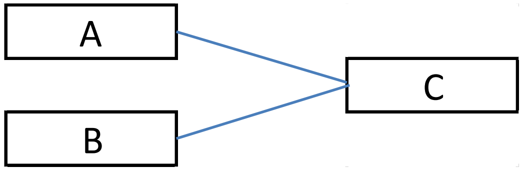
z = -0.82 →

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
−3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
−3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
−3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
−3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
−2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
−2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
−2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
−2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
−2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
−2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
−2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
−2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
−2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
−2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
−1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
−1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
−1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
−1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
−1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
−1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
−1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
−1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
−1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
−1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
−0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
−0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
−0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
−0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
−0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
−0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
−0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
−0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
−0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
−0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

(Reference) https://faculty.elgin.edu/dkernler/statistics/ch07/7-2.html

Critical Path

- Critical path
  - Path (sequence of activities) that gives the longest project duration
- Probability of project completion in PERT is based on the critical path
- This gives optimistic estimates of probability if there are near-critical paths
  - Near-critical path is the path with duration that is close to the duration of the critical path
- Example:
  - Suppose that a project consists of three activities A, B, and C
  - Means and standard deviations of the durations are:
    - Activity A: Mean 5, standard deviation 3 days
    - Activity B: Mean 6, standard deviation 1 days
    - Activity C: Mean 4, standard deviation 2 days



  - Critical path is B-C with 10 days
  - Probability that the path B-C completes within 11 days is $NORMSDIST\left(\frac{11-(6+4)}{\sqrt{1^2+2^2}}\right) = 0.673$
  - Probability that the path A-C completes within 11 days is $NORMSDIST\left(\frac{11-(5+4)}{\sqrt{3^2+2^2}}\right) = 0.710$
- Overall probability that the project completes within 11 days
  - If paths A-C and B-C are assumed to be independent (only for illustration purpose) → 0.673 x 0.71 = 0.478
  - If simulation is used, more accurate estimate of this probability → 0.545
  - These probabilities are smaller than 0.673 (the probability of project completion calculated based on the critical path)

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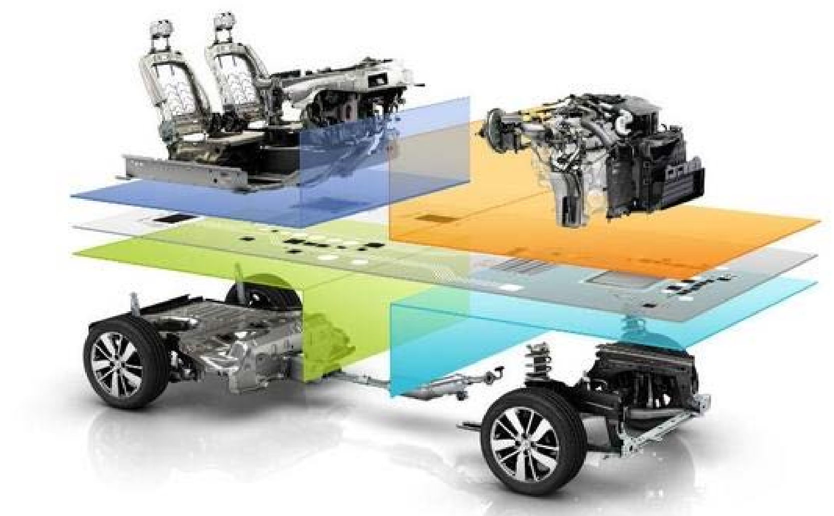
Benefit of Product Modularization on Project Duration and Uncertainty

Product Modularization

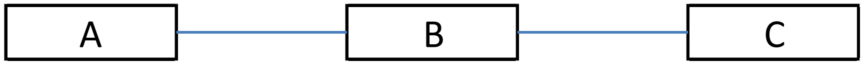
- Modularization is product design approach that divides product architecture into modules that can be independently designed, developed, and tested
- Example:
  - A Common Module Family (CMF) engineering architecture in Renault/Nissan Alliance vehicles
  - Vehicle is divided into engine bay, cockpit, front underbody, rear underbody and electrical/electronic architecture



(Reference) http://europe.autonews.com/article/20130619/ANE/306199957/renault-starts-modular-production-with-nissan-in-cost-cutting-drive

Benefit of Product Modularization

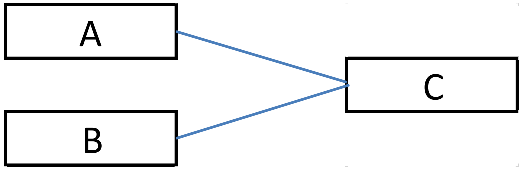
- Illustration
  - Suppose that a product is divided into two subsystems A and B
    - Activity A: Develop subsystem A
    - Activity B: Develop subsystem B
    - Activity C: Integrate subsystems A and B
  - Suppose that mean and standard deviation of the activity durations are:
    - Activity A: Mean 5, standard deviation 3 months
    - Activity B: Mean 6, standard deviation 1 months
    - Activity C: Mean 4, standard deviation 2 months
- Without modularization



  - Project activities are processed in series
  - Expected project duration: $T_e = \sum t_e = 5 + 6 + 4 = 15$ months
  - Variance of project duration: $V_P = \sum V = 3^2 + 1^2 + 2^2 = 14$
  - Standard deviation of project duration: $\sigma_P = \sqrt{14} = 3.74$ months
  - Probability that project complete within 15 months

$$NORMSDIST\left(\frac{15 - 15}{3.74}\right) = 0.5$$

Benefit of Product Modularization



- With modularization
  - Each module is individually designed, developed, and tested
  - Then the modules are integrated
    - Subsystem A is module A
    - Subsystem B is module B
  - Activities A and B are performed in parallel
  - Activity C is performed when both activities A and B are completed
- Impact of product modularization
  - Critical path is B-C with 10 months
  - Using simulation
    - Expected project duration: 10.8 months
    - Standard deviation of project duration: 2.55 months
    - Overall probability that the project completes within 15 months: 0.94