Value of Component Commonality in Managing Uncertainty in a Supply Chain

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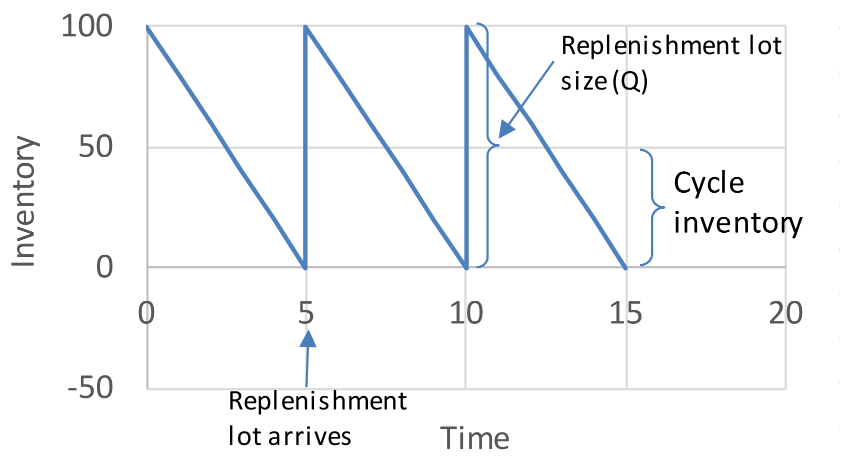
Role of Safety Inventory in a Supply Chain

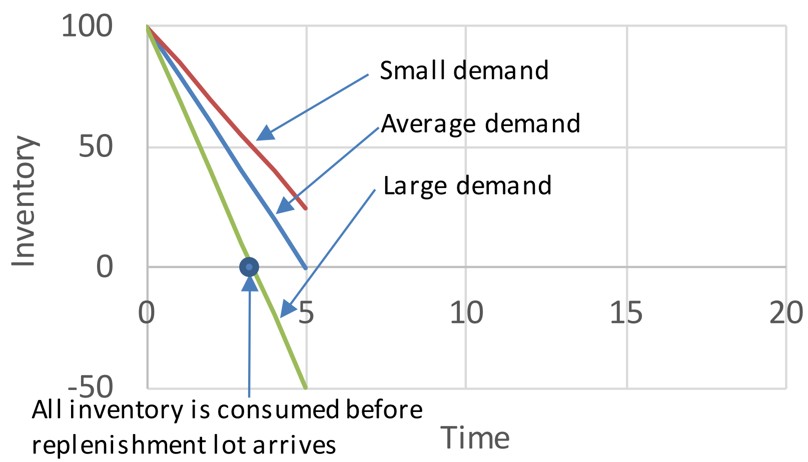
Role of Safety Inventory in a Supply Chain

- If demand is certain:
  - Daily demand is constant
  - We know how long it will take to consume all inventory to satisfy the demand
- Illustration
  - D=20 (Daily demand)
  - Q=100 (Replenishment lot size)
  - L=5 (Lead time for inventory replenishment)



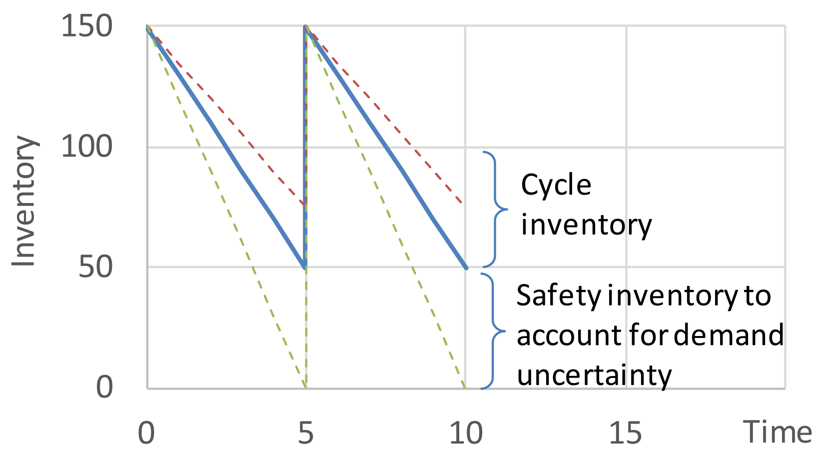
Role of Safety Inventory in a Supply Chain

- If demand is uncertain:
  - Demand may be larger or smaller than the average
  - If the demand is large than the average, all inventory may be consumed before next replenishment lot arrives
- Illustration
  - D=20 (Average daily demand)
  - Q=100 (Replenishment lot size)
  - L=5 (Lead time for inventory replenishment)



Role of Safety Inventory in a Supply Chain

- Safety inventory is used to account for uncertainty that:
  - Demand may be larger than the average (or forecasted)
- Illustration
  - D=20 (Average daily demand)
  - Q=100 (Replenishment lot size)
  - L=5 (Lead time for inventory replenishment)
  - ss=50 (Safety inventory)



Determining Appropriate Level of Safety Inventory

Trade-Off in Safety Inventory Decisions

- Large amount of safety inventory
  - Increases product availability (reduces product shortage); thus increases profit
  - Increases inventory holding costs; thus reduces profit
- Key questions in safety inventory decisions are:
  1. What is the appropriate level of safety inventory?
  2. What actions to take to reduce safety inventory while maintaining product availability?

(Reference) Supply Chain Management: Strategy, Planning, and Operation. Sunil Chopra and Peter Meindl. (2013) 5th Edition. Boston, MA: Pearson Prentice-Hall.

Determining Appropriate Level of Safety Inventory

- Safety inventory is determined by:
  - Demand and supply uncertainties
  - Product availability
- If uncertainty of demand or supply increases:
  - Safety inventory increases
- If product availability increases:
  - Safety inventory increases

(Reference) Supply Chain Management: Strategy, Planning, and Operation. Sunil Chopra and Peter Meindl. (2013) 5th Edition. Boston, MA: Pearson Prentice-Hall.

Demand Uncertainty

Measuring Demand Uncertainty

- Demand in each period (e.g., daily or weekly demand) is modeled by:
  - D_i: average demand per period
  - σ_i: standard deviation of demand per period (e.g., forecast error)
  - i = 1, 2, 3, …, L
- Notation is simplified if uncertainty of demand in each period is identical
  - D: average demand per period
  - σ_D: standard deviation of demand per period
- Lead time is the time between when an order is placed and when the order is received
  - L: lead time

(Reference) Supply Chain Management: Strategy, Planning, and Operation. Sunil Chopra and Peter Meindl. (2013) 5th Edition. Boston, MA: Pearson Prentice-Hall.

Measuring Demand Uncertainty

- Suppose demand in each period is:
  - Independent with one another
  - Normally distributed with:
  - Mean D_i and
  - Standard deviation σ_i
  - i = 1, 2, 3, … , L
- Total demand during the lead time of L periods is normally distributed with mean D_L and standard deviation σ_L

$$D_L = D_1 + D_2 + D_3 + \cdots + D_L = \sum_{i=1}^{L} D_i$$

$$\sigma_L = \sqrt{(\sigma_1)^2 + (\sigma_2)^2 + (\sigma_3)^2 + \cdots + (\sigma_L)^2} \quad (1)$$

when demand is independent

(Reference) Supply Chain Management: Strategy, Planning, and Operation. Sunil Chopra and Peter Meindl. (2013) 5th Edition. Boston, MA: Pearson Prentice-Hall.

Measuring Demand Uncertainty

- If mean and standard deviation of demand during each of L periods are the same:
  - Mean $D_i = D$
  - Standard deviation $\sigma_i = \sigma_D$
  - $i = 1, 2, 3, \ldots, L$
- Then, the total demand during the lead time of L periods is:
  - Normally distributed with:
  - Mean D_L
  - Standard deviation σ_L

$$D_L = \overbrace{D + D + D + \cdots + D}^{\text{L terms}} = LD$$

$$\sigma_L = \sqrt{\underbrace{(\sigma_D)^2 + (\sigma_D)^2 + (\sigma_D)^2 + \cdots + (\sigma_D)^2}_{\text{L terms}}} = \sqrt{L(\sigma_D)^2} = \sqrt{L}\sigma_D \quad (2)$$

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Product Availability and Replenishment Policy

Product Availability and Replenishment Policy

- Product availability
  - Ability to satisfy customer order from inventory
- A stockout occurs when:
  - Customer order is not satisfied due to no inventory
- Example: cycle service level (CSL)
  - Proportion of replenishment cycles that end without stockout
    - Replenishment cycle is the interval between the arrival of two successive replenishment lots
  - Equivalent to probability of replenishment cycle without stockout

- Replenishment policy consists of:
  - Timing of reorder
  - Quantity of reorder
- Example: continuous review policy
  - Inventory is continuously monitored
  - Lot size Q is ordered when the inventory reaches the reorder point (ROP)
  - Lot size is same
  - Time between order may change

(Reference) Supply Chain Management: Strategy, Planning, and Operation. Sunil Chopra and Peter Meindl. (2013) 5th Edition. Boston, MA: Pearson Prentice-Hall.

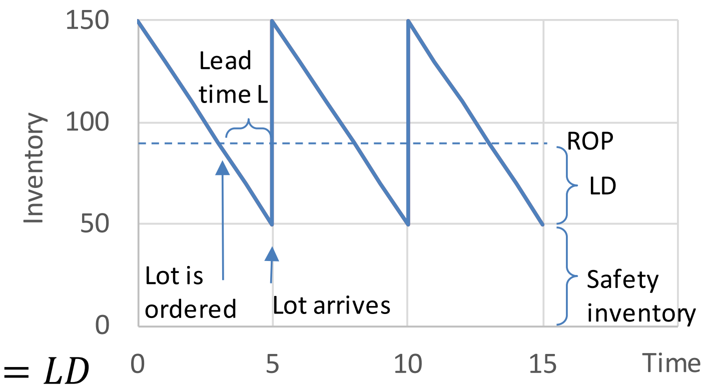
Evaluating CSL Given a Replenishment Policy

- Continuous review replenishment policy is assumed
  - Q: reorder lot size
  - L: replenishment lead time (number of weeks)
- Weekly demand
  - Normally distributed with:
  - Mean D and
  - Standard deviation σ_D
- Two cases are discussed next
  - Safety inventory given a replenishment policy
  - CSL given a replenishment policy

(Reference) Supply Chain Management: Strategy, Planning, and Operation. Sunil Chopra and Peter Meindl. (2013) 5th Edition. Boston, MA: Pearson Prentice-Hall.

Safety Inventory Given a Replenishment Policy

- Continuous review replenishment policy
  - When the amount of inventory reaches ROP, a new replenishment order is placed



- From Equation (2)

$$Expected\ demand\ during\ lead\ time = LD$$

- On average, LD products are sold between when the inventory reaches ROP and when the new replenishment lot arrives
  - Thus, the inventory when the replenishment lot arrives is:

$$ROP - LD$$

- Thus, safety inventory ss is:

$$Safety\ inventory\ ss = ROP - LD \qquad (4)$$

(Reference) Supply Chain Management: Strategy, Planning, and Operation. Sunil Chopra and Peter Meindl. (2013) 5th Edition. Boston, MA: Pearson Prentice-Hall.

CSL Given a Replenishment Policy

- Recall that CSL is equivalent to the probability of replenishment cycle without stockout
- Stockout does not occur as far as the demand during replenishment lead time L is smaller than ROP; Thus

$$CSL = Prob(demand\ during\ lead\ time\ of\ L\ periods\ or\ L\ weeks \leq ROP)$$

- If the total demand during the L periods is:
  - Normally distributed with mean D_L and standard deviation σ_L

CSL can be written as:

$$CSL = F(ROP, D_L, \sigma_L) = \underbrace{NORMDIST(ROP, D_L, \sigma_L, 1)} \qquad (5)$$

Excel function ("NORM.DIST" instead of "NORMDIST" is used in Excel 2010)

- If weekly demand is independent and identically distributed, then from Equation (2), D_L and σ_L in Equation (5) are:

$$D_L = LD$$
$$\sigma_L = \sqrt{L}\sigma_D$$

(Reference) Supply Chain Management: Strategy, Planning, and Operation. Sunil Chopra and Peter Meindl. (2013) 5th Edition. Boston, MA: Pearson Prentice-Hall.

Safety Inventory Given Desired CSL

- Companies often choose replenishment policies to achieve desired levels of product availability
- Thus, appropriate level of safety inventory needs to be calculated to achieve the desired level of product availability (e.g., in terms of CSL)
- Next
  - Safety inventory given desired CSL is discussed

Safety Inventory Given Desired CSL

- Goal
  - For continuous review replenishment policy,
  - Calculate appropriate safety inventory that achieves the desired CSL
- Assumption
  - L: lead time
  - CSL: desired cycle service level
  - D_L: mean demand during lead time
  - σ_L: standard deviation of demand during lead time

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Safety Inventory Given Desired CSL

- From Equation (4), ROP=D_L+ss
- Thus, safety inventory ss needs to satisfy the following condition

$$Prob(demand\ during\ lead\ time\ of\ L\ periods \leq D_L + ss) = CSL$$

- Given that demand is normally distributed, the above condition can be written as below using cumulative distribution function F()

$$F(D_L + ss, D_L, \sigma_L) = CSL$$

- Using inverse normal distribution F^{-1}(),

$$D_L + ss = F^{-1}(CSL, D_L, \sigma_L) \quad \rightarrow \quad ss = F^{-1}(CSL, D_L, \sigma_L) - D_L$$

- Using inverse standard normal distribution F_S^{-1}(),

$$ss = F_S^{-1}(CSL) \times \sigma_L = NORMSINV(CSL) \times \sigma_L = NORMSINV(CSL) \times \sqrt{L}\sigma_D \qquad (6)$$

Excel function ("NORM.S.INV" instead of "NORMSINV" is used in Excel 2010)

(Reference) Supply Chain Management: Strategy, Planning, and Operation. Sunil Chopra and Peter Meindl. (2013) 5th Edition. Boston, MA: Pearson Prentice-Hall.

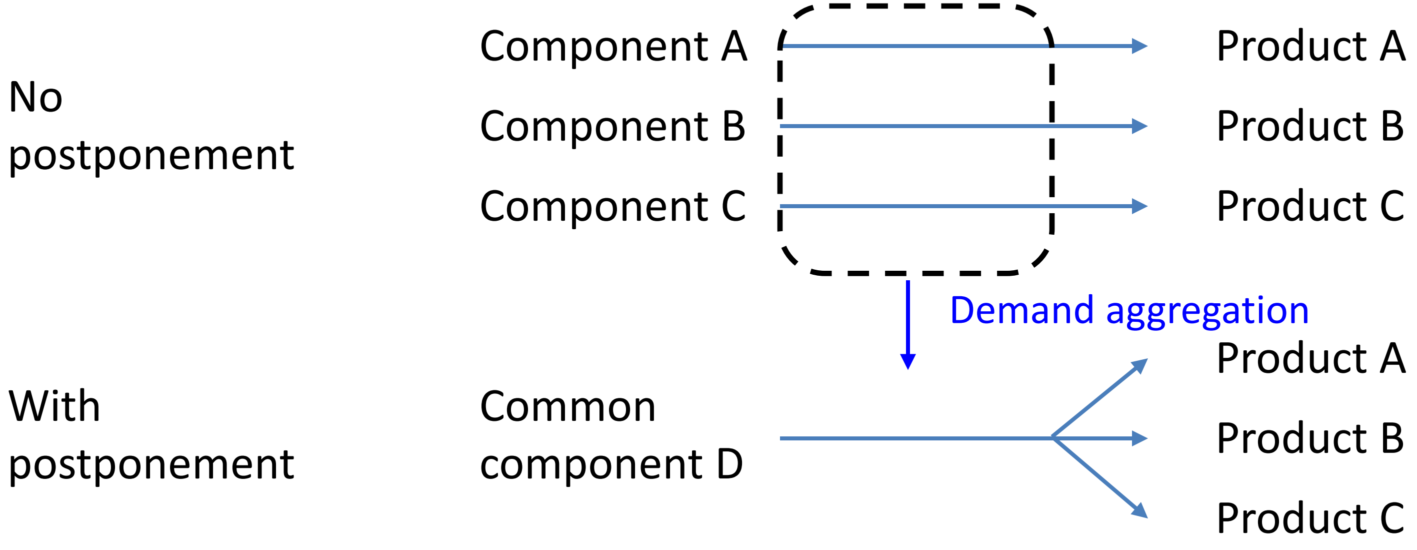
Impact of Product Design on Safety Inventory – Value of Component Commonality

Component Commonality

- Component commonality is a key to increasing product variety without reducing product availability or without increasing component inventory too much
- If product variety in a supply chain is large, the variety and amount of components in the supply chain can be significantly large
- Use of common components in multiple products enables a company to:
  - Aggregate component demand and
  - Reduce component inventory
- If a distinct component is used in each product:
  - Demand for the component is the same as demand for the finished product in which the component is used
- If the same component is used in multiple products:
  - Demand for the component is the same as the aggregate demand of all the finished products in which the component is used

Postponement

- Postponement is to delay product differentiation or customization until near the time of product sales
- Component commonality is the key enabler of product postponement
- Supply chains with and without postponement are graphically compared below



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Value of Component Commonality

- Value of component commonality and postponement comes from:
  - Reduction of safety inventory by demand aggregation
- Consider the following conditions
  - k: number of products
  - D_i: mean weekly demand of product i
    - i = 1, …, k
  - σ_i: standard deviation of weekly demand of product i
    - i = 1, …, k
  - L: replenishment lead time
  - CSL: cycle service level
  - D_L: mean demand during lead time
  - σ_L: standard deviation of demand during lead time
  - H: unit inventory holding cost $H = hC$
  - C: product cost per unit
  - h: inventory holding cost per year as a fraction of product cost

(Reference) Supply Chain Management: Strategy, Planning, and Operation. Sunil Chopra and Peter Meindl. (2013) 5th Edition. Boston, MA: Pearson Prentice-Hall.

Safety Inventory without Aggregation

- Step 1: Calculate safety inventory (ss) for each product (and component)
  - ss_1, ss_2, …, ss_k
- Step 2: Add safety inventory (ss) for all products
  - $ss_{Total} = ss_1 + ss_2 + \dots + ss_k$

- From Equation (6), safety inventory for each product is:
  - Product 1 (i=1): $ss_1 = F_S^{-1}(CSL) \times \sqrt{L}\sigma_1$
  - Product 2 (i=2): $ss_2 = F_S^{-1}(CSL) \times \sqrt{L}\sigma_2$

    ⋮
  - Product k (i=k): $ss_k = F_S^{-1}(CSL) \times \sqrt{L}\sigma_k$
- Total safety inventory without aggregation

$$\begin{aligned} ss_{Total} &= ss_1 + ss_2 + \cdots + ss_k \\ &= F_S^{-1}(CSL) \times \sqrt{L}\sigma_1 + F_S^{-1}(CSL) \times \sqrt{L}\sigma_2 + \cdots + F_S^{-1}(CSL) \times \sqrt{L}\sigma_k \end{aligned}$$

  - Using summation, the above formula can be written as:
    $ss_{Total} = \sum_{i=1}^{k} F_S^{-1}(CSL) \times \sqrt{L}\sigma_i$

  - If $F_S^{-1}(CSL) \times \sqrt{L}$ is same for all products, this term can be moved out of the summation
    $ss_{Total} = F_S^{-1}(CSL) \times \sqrt{L} \times \sum_{i=1}^{k} \sigma_i$ (7)

(Source) Supply Chain Management: Strategy, Planning, and Operation. Sunil Chopra and Peter Meindl. (2013) 5th Edition. Boston, MA: Pearson Prentice-Hall.

Safety Inventory with Aggregation

- Step 1: Calculate standard deviation of aggregated demand σ^C_D
  - Calculate variance of aggregate demand $var(D^C)$ by adding variance of demand for each product (variance is square of σ_i)
  - Calculate standard deviation of aggregated demand σ^C_D by taking square root of variance $var(D^C)$
- Step 2: Calculate safety inventory of product (and component) with aggregation

- Standard deviation of aggregated demand
  - Variance of aggregated demand is:

$$var(D^C) = \sum_{i=1}^{k} \sigma_i^2 = \sigma_1^2 + \sigma_2^2 + \cdots + \sigma_k^2$$

← This is when demands are independent

  - Then, standard deviation of aggregated demand σ^C_D is:

$$\sigma_D^C = \sqrt{var(D^C)} = \sqrt{\sum_{i=1}^{k} \sigma_i^2} = \sqrt{\sigma_1^2 + \sigma_2^2 + \cdots + \sigma_k^2} \quad (8)$$

- Total safety inventory with aggregation

$$Safety\ inventory\ with\ aggregation$$
$$= F_S^{-1}(CSL) \times \sqrt{L} \times \sigma_D^C = F_S^{-1}(CSL) \times \sqrt{L} \times \sqrt{\sigma_1^2 + \sigma_2^2 + \cdots + \sigma_k^2} \quad (9)$$

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Reduction in Inventory Holding Cost

- Reduction of inventory holding cost due to aggregation is:

$$Reduction\ of\ inventory\ holding\ cost$$
$$= Holding\ cost\ without\ aggregation\ - holding\ cost\ with\ aggregation$$

$$= (Safety\ inventory\ without\ aggregation - safety\ inventory\ with\ aggregation) \times unit\ inventory\ holding\ cost\ (if\ inventory\ holding\ cost\ is\ same\ for\ all\ components)$$

$$= \{Equation\ (7) - Equation\ (9)\} \times H$$

$$= \left(F_S^{-1}(CSL) \times \sqrt{L} \times \sum_{i=1}^{k} \sigma_i - F_S^{-1}(CSL) \times \sqrt{L} \times \sigma_D^C \right) \times H$$

$$= F_S^{-1}(CSL) \times \sqrt{L} \times \left(\sum_{i=1}^{k} \sigma_i - \sigma_D^C \right) \times H$$

Because $F_S^{-1}(CSL) \times \sqrt{L}$ is the common term, this term can be moved out of the parenthesis

$$= F_S^{-1}(CSL) \times \sqrt{L} \times hC \times \left(\sum_{i=1}^{k} \sigma_i - \sqrt{\sigma_1^2 + \sigma_2^2 + \cdots + \sigma_k^2} \right)$$

(Source) Supply Chain Management: Strategy, Planning, and Operation. Sunil Chopra and Peter Meindl. (2013) 5th Edition. Boston, MA: Pearson Prentice-Hall.

Problem

Question

- An electric bike company e-Motion designs, makes, and sells three models, EM-100, EM-200, and EM-300, with the following weekly demand
  - EM-100: average of 10, standard deviation of 4
  - EM-200: average of 20, standard deviation of 8
  - EM-300: average of 5, standard deviation of 1
- Currently, motor with different nominal output is used in each model
  - EM-100: 250 watt motor
  - EM-200: 500 watt motor
  - EM-300: 350 watt motor
  - Lead time of all these motors are 4 weeks (no standard deviation of lead time)
- e-Motion operates with continuous review replenishment policy
- Questions

  If e-Motion wishes to achieve 99% cycle service level (CSL),
  - (1-1) What is the safety inventory of each motor?

  If e-Motion decides to use 500 watt motor for all models while achieving 99% CSL,
  - (1-2) What is the safety inventory of the motor?

  If inventory holding cost of each motor is $60 per unit per year,
  - (1-3) How much inventory holding cost will e-Motion save by using common motor?

Answer

- (1-1)
  - EM-100: $ss = NORMSINV(0.99) \times \sqrt{4} \times 4 = 18.6$
  - EM-200: $ss = NORMSINV(0.99) \times \sqrt{4} \times 8 = 37.2$
  - EM-300: $ss = NORMSINV(0.99) \times \sqrt{4} \times 1 = 4.7$

- (1-2)
  - Standard deviation of the aggregate demand
    $\sigma_D^C = \sqrt{4^2 + 8^2 + 1^2} = \sqrt{81} = 9$
  - Aggregate: $ss = NORMSINV(0.99) \times \sqrt{4} \times 9 = 41.9$

- (1-3)

$$\{(18.6 + 37.2 + 4.7) - 41.9\} \times 60 = (60.5 - 41.9) \times 60 = \$1116.6$$