

Reliability-Based Multidisciplinary Optimization for Aircraft Wing Design

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Abstract

Aircraft wing design typically involves multiple disciplines such as aerodynamics and structure. Multidisciplinary design optimization (MDO) has been recently used to deal with the multidisciplinary efforts in wing design. When reliability is considered, MDO for the wing design becomes much more computationally intensive. To improve the efficiency, the strategy of using Sequential Optimization and Reliability Assessment (SORA) in MDO is proposed with the application of a simplified wing design problem. The overall reliability-based MDO is decomposed into sequential cycles of deterministic MDO and reliability analysis. The reliability analysis is therefore decoupled from the MDO loop, and the number of reliability analyses is then reduced significantly. It is shown that the use of the SORA method is efficient through the demonstration of a light aircraft wing design problem.

Keywords: Reliability; Multidisciplinary design optimization; Wing; Structure

1 Introduction

Aircraft wing design involves multiple disciplines such as aerodynamics and structure. To fully and efficiently make use of the synergism between the aerodynamics and wing structure, an integrated analysis and design approach is needed. Multidisciplinary Design Optimization (MDO) is such an approach that meets the need. MDO is a systematic approach to optimization of complex, coupled engineering systems. “Multidisciplinary” refers to the different aspects that must be included in designing a system that involves multiple interacting disciplines, such as those found in aerospace systems and other complex systems. Numerous MDO methods have been developed (Sobieski and Haftka 1997, Giesing and Barthelemy 1998, Alexandrov and Kodiyalam 1998, Sobieski et al. 2000). Typical MDO approaches include the Multidisciplinary Feasible method (MDF), Collaborative Optimization approach (CO), Individual Discipline Feasible method (IDF), Fully Integrated

Optimization (FIO), Simultaneous Analysis and Design (SAND), Concurrent Subspace Optimization (CSSO), and Bilevel Integrated System Synthesis (BLISS).

MDO has also been used in integrated aerodynamic/structural wing design. Grossman et al. (1988) applied the Fully Integrated Optimization method to the integrated aerodynamic/structural design of a sailplane wing. In their work, the lifting line theory was used to predicate the aerodynamic characteristics, and the beam theory was used to calculate the structure stresses and deformations. Their investigation indicated that the integrated design was superior to the sequential design in terms of performance or weight. In the work of Barthelemy et al. (1994), the generalized sensitivity formulation was used for determining the minimum wing weight with the consideration of aerodynamic and structural coupling. Gumbert et al. (2001) used SAND method to formulate and implement simultaneous aerodynamic and structural design optimization. Their work aimed to reduce the computational expense incurred in performing shape and sizing optimization using Computational Fluid Dynamics (CFD) flow analysis and Finite Element structure analysis tools.

The integrated aerodynamic/structural wing design, however, is usually modelled as a deterministic optimization problem. In reality, uncertainty is ubiquitous in any product development process (design, manufacturing, and operations) (Du and Chen, 2000). For example, for a wing design, uncertainties include:

- Uncertainties in operations: aerodynamic loads, payloads, flight speed, and altitude;
- Uncertainties in material properties: the strength and modulus;
- Uncertainties in manufacturing processes: component dimensions and shapes;
- Uncertainties in model structures and parameters.

To accommodate the uncertainties and ensure higher reliability and lower risk, reliability-based design (RBD) is widely applied in aircraft structure design (Zang et al. 2002). Pettit et al. (2002) modelled stochasticity in the wing joint and the wing roots and introduced randomness into deterministic finite element analysis. Allen and Maute (2004) used the first-order reliability method to approximate the system reliability of aeroelastic structures. A methodology of RBD for aircraft structures with multidisciplinary set of requirements was discussed in Xie and Rais-Rohani (2003). A wing spar design example was demonstrated in their work.

The benefit of RBD is the optimal trade-off between reliability and cost. On the other hand, RBD is much more computationally expensive than deterministic optimization design. Traditional RBD needs a double-loop procedure where the reliability analysis inner loop is nested with the optimization outer loop (Tu, et al. 1999). The optimizer calls reliability analysis not only at each iteration, but also for computing derivatives, if finite-difference derivatives are used. To reduce computational cost, strategies of decoupling reliability analysis from optimization have been sought. The common strategy is the use of sequential single loops proposed in (Chen and Hasselman 1997, Wu and Wang 1998, Wu et al. 2001). The entire RBD process is decomposed into a sequence of cycles. In each cycle, the deterministic optimization is performed first. Then the reliability analysis is conducted at the optimal point obtained from the preceding optimization. If the process does not converge, in the next cycle, a new deterministic optimization is formulated based on reliability analysis results in the previous cycle. Then, the optimization is performed again followed by a new reliability analysis. The process continues until the solution is found. Wu et al. (2001) developed a safety factor approach (SFA). They integrated the safety-factor concept and the MPP (Most Probable Point) concept and replaced reliability constraints with equivalent deterministic constraints. Du and Chen (2004) extended the SFA approach and referred to the type of single-loop methods as the Sequential Optimization and Reliability Assessment

(SORA) method. A comprehensive review and investigation of different single-loop methods has been reported recently in (Yang and Gu 2004).

RBD has also been integrated with MDO, and this integration is termed reliability-based multidisciplinary design optimization (RBMDO). Sues and Cesare (2000) proposed a RBMDO framework, under which the reliability analysis is decoupled from the optimization. Reliabilities are computed initially before the first execution of the optimization loop and then are updated after the optimization loop is executed. To alleviate the computational burden, in the optimization loop, approximate forms of reliability constraints are used. To integrate the existing reliability analysis techniques into the MDO framework more tightly, a multi-stage, parallel implementation strategy of probabilistic design optimization was proposed by Koch et al. (2000). Padmanabhan et al. (2003) demonstrated the use of Monte Carlo Simulation in MDO environment.

Reliability analysis, the task to calculate the reliability associated with probabilistic constraints, is an important component for RBMDO. Efficient reliability analysis methods under MDO environment have been reported. For example, the Concurrent Subsystem Optimization techniques are used for the Most Probable Point (MPP) search (Padmanabhan and Batill 2002). Du and Chen (2005) developed the collaborative reliability analysis where the MPP and multidisciplinary analysis are conducted concurrently. Ahn et al. (2004) proposed a new strategy named Sequential Approach on Reliability Analysis under Multidisciplinary Analysis Systems. In this approach, the reliability analysis and the multidisciplinary analysis are decomposed and arranged in a sequential manner, making a recursive loop. The integration of these efficient reliability analysis methods into the MDO framework can potentially improve the overall performance of RBMDO.

As for aircraft wing design, Xiao et al. (1999) applied a framework of RBD for an aircraft wing design with the maximum cruise range, involving coupled aero-structural analysis. Other example is the work done by Pettit and Grandhi (2000). Their results show that the RBMDO solution provides an optimum design that is improved over the deterministic design in terms of robustness and reliability.

Performing RBMDO, however, is a computationally expensive exercise, especially when the single-disciplinary RBD is directly combined with MDO. As mentioned above, a traditional single-disciplinary RBD involves a double-loop procedure where the reliability analysis inner loop is nested within the optimization outer loop. With the combination of MDO, an additional multidisciplinary analysis (MDA) loop is needed under the reliability loop, if the MDA is conducted iteratively. To reduce the computational cost, we propose to use the SORA method for RBMDO problems. In this work, our objective is to investigate the feasibility of the SORA method in RBMDO through a simple wing design problem for a light aircraft. Typical wing design models are much more complicated and computationally expensive; nevertheless, this simple example will sufficiently demonstrate the effectiveness of the integration of the SORA method with RBMDO.

This paper is organized as follows. A wing design problem for a light aircraft is formulated as a RBMDO problem in section 2, and the procedure of solving a RBMDO problem is discussed in section 3. The optimal results of the wing design are given in section 4 followed by the conclusions in section 5.

2 The Wing Design Problem

2.1 The wing design

A wing design problem involves aerodynamic design and structural design. The aerodynamic design aims to obtain good aerodynamic performances, such as the minimum drag or maximum lift, by selecting the external shape of the wing. The structural design aims to reduce structural weight by selecting the proper size of structural components to meet the requirements on allowable stresses, deformation limitations and, others. The overall performance of aircraft, such as the flight range and maximum flight speed, is largely determined by both the aerodynamic characteristics and structural weight. To obtain the overall optimal design of the wing, it is essential to consider the aerodynamic characteristics, structural weight, and the coupling between them. That is where MDO can play an important role for the integrated aerodynamic/structural wing design.

In this work, the wing design problem of a light aircraft is used for the demonstration of RBMDO. The basic design requirements for the aircraft are that the takeoff weight of the aircraft is around 700 kg, the flight attitude is about 3000m, the cruise speed is around 200km/h, and the external shape of the wing is rectangular. The other fixed parameters are the nominal wing area of 10m^2 and the wing section (airfoil) with 12% maximum thickness to chord ratio.

The structure model of the wing is shown in figure 1 schematically. The wing structure is subject to the aerodynamic loads. All the bending loads are carried by the spar caps; the torsion loads are carried by the wing box consisting of the skin, the front, and rear shear webs; and the shear loads are carried by the front and rear webs. In this structure model, the areas of spar caps at each section can be different while the thickness of the skin at each section remains the same.

The design task is to determine the parameters that define the aerodynamic external shape and the size of wing structure for the maximum flight range under uncertainty. The design variables for the external shape include the aspect ratio, incidence angle, and twist angle. The design variables for the size of the wing structure include the area of spar cap at each section and the thickness of the skin, and front and rear webs. The uncertainties in this problem are associated with (1) operations such as the variations of the flight altitude, flight speed, take-off weight, and gust load factor; (2) material properties such as the variations of bending strength of spar, shear strength and shear modulus of the skin, and the webs; and (3) manufacturing process such as the variations in wing area. The distributions of all the random variables will be provided in subsection 2.3.

We will first discuss the general MDO model and notations used in this paper and then formulate the wing design problem as an MDO problem.

2.2 RBMDO problems

A three-discipline system is used as an example for discussion. The extension of the discussion to a general multidisciplinary system will be obvious. A system with three coupled disciplines (subsystems) is shown in figure 2. The notations in the figure are explained in table 1.

As demonstrated in figure 2, discipline (subsystem) i has its own deterministic input variables \mathbf{d}_i and random variables \mathbf{x}_i , as well as the sharing input deterministic variables \mathbf{d}_s and sharing input random variables \mathbf{x}_s . \mathbf{d}_s and \mathbf{x}_s are also common inputs to all the disciplines. \mathbf{d}_i ($i = 1, 2, 3$) and \mathbf{d}_s are considered as design variables for a RBMDO problem.

All the disciplines are coupled through linking variables \mathbf{y}_{ij} . The complete set of linking variables which are output from discipline i and input to other disciplines is expressed by

$$\mathbf{y}_{i.} = (\mathbf{y}_{ij}, j = 1, 2, 3, j \neq i) = \mathbf{y}_{i.}(\mathbf{d}_s, \mathbf{d}_i, \mathbf{x}_s, \mathbf{x}_i, \mathbf{y}_{.i}) \quad (1)$$

where $\mathbf{y}_{i.}$ represents dependent variables as on the left-hand side and also the functional relationships between the dependent variables and the independent variables. We will use the same notation for other dependent variables in the rest of the paper. $\mathbf{y}_{.i}$ in equation (1) is the vector of linking variables which are the input to discipline i and the output from other disciplines, i.e.

$$\mathbf{y}_{.i} = (\mathbf{y}_{ji}, j = 1, 2, 3, j \neq i) \quad (2)$$

Expanding equation (1) over all disciplines yields the following simultaneous equations that determine the consistency (compatibility) over the interface between the disciplines.

$$\begin{cases} \mathbf{y}_{12} = \mathbf{y}_{12}(\mathbf{d}_s, \mathbf{d}_1, \mathbf{x}_s, \mathbf{x}_1, \mathbf{y}_{.1}) \\ \mathbf{y}_{13} = \mathbf{y}_{13}(\mathbf{d}_s, \mathbf{d}_1, \mathbf{x}_s, \mathbf{x}_1, \mathbf{y}_{.1}) \\ \mathbf{y}_{21} = \mathbf{y}_{21}(\mathbf{d}_s, \mathbf{d}_2, \mathbf{x}_s, \mathbf{x}_2, \mathbf{y}_{.2}) \\ \mathbf{y}_{23} = \mathbf{y}_{23}(\mathbf{d}_s, \mathbf{d}_2, \mathbf{x}_s, \mathbf{x}_2, \mathbf{y}_{.2}) \\ \mathbf{y}_{31} = \mathbf{y}_{31}(\mathbf{d}_s, \mathbf{d}_3, \mathbf{x}_s, \mathbf{x}_3, \mathbf{y}_{.3}) \\ \mathbf{y}_{32} = \mathbf{y}_{32}(\mathbf{d}_s, \mathbf{d}_3, \mathbf{x}_s, \mathbf{x}_3, \mathbf{y}_{.3}) \end{cases} \quad (3)$$

\mathbf{z}_i is the output of discipline i . Since an element of \mathbf{z}_i could be an objective or a constraint in RBMDO, \mathbf{z}_i is therefore partitioned into three components as follows

$$\mathbf{z}_i = (\mathbf{v}_i, \mathbf{g}_i, \mathbf{G}_i) \quad (4)$$

where \mathbf{v}_i is the vector of the system level objectives, \mathbf{g}_i is the vector of local deterministic constraints, and \mathbf{G}_i is the vector of local constraints, which are subject to reliability requirements. The components of \mathbf{z}_i are defined by

$$\mathbf{v}_i = \mathbf{v}_i(\mathbf{d}_s, \mathbf{d}_i, \mathbf{x}_s, \mathbf{x}_i, \mathbf{y}_{.i}) \quad (5)$$

$$\mathbf{g}_i = \mathbf{g}_i(\mathbf{d}_s, \mathbf{d}_i, \bar{\mathbf{x}}_s, \bar{\mathbf{x}}_i, \bar{\mathbf{y}}_{.i}) \quad (6)$$

$$\mathbf{G}_i = \mathbf{G}_i(\mathbf{d}_s, \mathbf{d}_i, \mathbf{x}_s, \mathbf{x}_i, \mathbf{y}_{.i}) \quad (7)$$

Deterministic constraint functions \mathbf{g}_i represent the design requirements that need not to be modeled probabilistically. They are therefore evaluated at the means of random variables ($\bar{\mathbf{x}}_s, \bar{\mathbf{x}}_i$) and the nominal value $\bar{\mathbf{y}}_i$ of \mathbf{y}_i as shown in equation (6). In this paper, the value of a function evaluated at the means of random input variables is called a nominal value. $\bar{\cdot}$ is used to represent the mean value of a random variables as well as the nominal value of a function.

Given a set of inputs ($\mathbf{d}_s, \mathbf{d}_1, \mathbf{d}_2, \mathbf{d}_3, \mathbf{x}_s, \mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3$), obtaining the output \mathbf{z}_i needs to solve the nonlinear simultaneous equations defined in equation (3). This is the task of deterministic multidisciplinary analysis (MDA). MDA typically requires numerical methods and an iterative process with a number of disciplinary analyses. The MDA can be, in principle, solved by any method for solving nonlinear equations, such as the Newton's method, or even optimization algorithms. For demonstrative purpose, we will use Sequential Quadratic Programming (SQP) to solve the MDA problems for the wing design. The optimization model for an MDA will be presented in section 3. It should be noted that the use of an optimizer herein is only for the analysis (MDA).

The general model of RBMDO is given by

$$\begin{cases} \min_{(\mathbf{d}_s, \mathbf{d})} v(\bar{\mathbf{v}}_1, \bar{\mathbf{v}}_2, \bar{\mathbf{v}}_3) \\ s.t. \quad \Pr\{\mathbf{G}_i(\mathbf{d}_s, \mathbf{d}_i, \mathbf{x}_s, \mathbf{x}_i, \mathbf{y}_i(\mathbf{d}_s, \mathbf{d}_i, \mathbf{x}_s, \mathbf{x}_i)) \leq 0\} \geq \mathbf{R}_i \quad i = 1, 2, 3 \\ \mathbf{g}_j(\mathbf{d}_s, \mathbf{d}_i, \bar{\mathbf{x}}_s, \bar{\mathbf{x}}_i, \bar{\mathbf{y}}_i(\mathbf{d}_s, \mathbf{d}_i, \bar{\mathbf{x}}_s, \bar{\mathbf{x}}_i)) \leq 0, j = 1, 2, 3 \end{cases} \quad (8)$$

where $\mathbf{d} = (\mathbf{d}_1, \mathbf{d}_2, \mathbf{d}_3)$ is the vector of local (disciplinary) design variables, the objective v is the function of the nominal values, $(\bar{\mathbf{v}}_1, \bar{\mathbf{v}}_2, \bar{\mathbf{v}}_3)$, of the individual disciplinary outputs. $\bar{\mathbf{v}}_i$ is computed by

$$\bar{\mathbf{v}}_i = \mathbf{v}_i(\mathbf{d}_s, \mathbf{d}_i, \bar{\mathbf{x}}_s, \bar{\mathbf{x}}_i, \bar{\mathbf{y}}_i) \quad (9)$$

The nominal values of linking variables $\bar{\mathbf{y}}_i$ are determined by

$$\bar{\mathbf{y}}_i = \mathbf{y}_i(\mathbf{d}_s, \mathbf{d}_i, \bar{\mathbf{x}}_s, \bar{\mathbf{x}}_i, \bar{\mathbf{y}}_i) \quad (10)$$

$\Pr\{\mathbf{G}_i(\mathbf{d}_s, \mathbf{d}_i, \mathbf{x}_s, \mathbf{x}_i, \mathbf{y}_i(\mathbf{d}_s, \mathbf{d}_i, \mathbf{x}_s, \mathbf{x}_i)) \leq 0\} \geq \mathbf{R}_i \quad i = 1, 2, 3$ are the reliabilities associated with the constraint functions \mathbf{G}_i , and \mathbf{R}_i are corresponding required reliabilities.

Solving the above optimization problem requires to call reliability analysis and MDA repeatedly. The MDA usually needs to solve the nonlinear simultaneous equations defined in equation (3) iteratively. Therefore, the overall computational effort is intensive. As shown in figure 2, the major difficulty in dealing with uncertainty comes from the fact that the uncertainties in one discipline propagate to other disciplines due to the data flow among disciplines (Batill et al. 2000).

2.3 Formulate the wing design problem as an RBMDO problem

After the introduction of the RBMDO, we will discuss how to model the wing design as a RBMDO problem. The external shape of the wing and the structural size of the wing are designed by two design groups, namely, the aerodynamic group and structural group. The whole system, therefore, consists of two subsystems (disciplines), which are coupled with each other as shown in figure 3. The input of the structural analysis subsystem requires the output from the aerodynamic analysis subsystem, i.e., the distribution of aerodynamic loads. Meanwhile, the distribution of the aerodynamic load in subsystem 1 is affected by the structural displacement computed from subsystem 2 – the structural model.

The objective of the aerodynamic-structural integrated wing design is to maximize the cruise range at the given values of total weight of the aircraft and wing area. The cruise range is given by the Breguet range equation (Raymer 1992) as

$$v = 603K \left(\frac{\eta_p}{c_p} \right) \ln \left(\frac{W_0}{W_a + W_w} \right) \quad (11)$$

where K is the ratio of lift to drag, $\eta_p = 0.75$ is the propeller efficiency in cruise, $c_p = 0.6$ is the specific fuel consumption of the engine, W_0 is the total weight (takeoff weight) of the aircraft, W_a is the airframe weight excluding the weight of the wing, and W_w is the weight of the wing.

At the system level, the sharing design variable is $\mathbf{d}_s = (AR)$, where AR is the aspect ratio of the wing, and the sharing random variables are $\mathbf{x}_s = (V, H)$, in which V is the flight speed, and H is the flight altitude.

The distributions of all the random variables are given in table 2. All the design variables and their lower and upper bounds are given in table 3.

Subsystem 1: aerodynamics

Subsystem 1 involves aerodynamics. Since the light aircraft flies at a subsonic speed and its wing span ratio is relatively large, the lifting line method is used to predict the aerodynamic characteristics, including lift distributions, lift coefficients, and induced drag. The total drag is the sum of induced drag and parasite drag. In this problem the parasite drag coefficient is assumed to be 0.015.

The disciplinary design variables are $\mathbf{d}_1 = (\theta, \alpha)$, where θ is a wing twist (washout), and α is the angle of attack. The random disciplinary variable is $\mathbf{x}_1 = (W_0)$, where W_0 is the total weight of the light aircraft.

The input linking variable to aerodynamics subsystem is $\mathbf{y}_{21} = (\delta)$, where δ is the twist deformation of the wing under the aerodynamic loads. δ is computed from subsystem 2 (structural subsystem). The output linking variables are aerodynamic lift distributions that are represented by a cubic polynomial function with the coefficients defined by $\mathbf{y}_{12} = (a_0, a_1, a_2, a_3)$, which are part of input to subsystem 2. The output of aerodynamics subsystem is $\mathbf{z}_1 = (K)$, where K is the ratio of lift to drag.

There is one constraint in subsystem 1, representing that the lift generated by the wing must be equal to the total weight of the aircraft. This constraint is treated as a deterministic constraint and is evaluated by

$$g_1(1) = \overline{W}_0 - \frac{1}{2} \overline{\rho} \overline{V}^2 \overline{S} \overline{C}_L = 0 \quad (12)$$

where \overline{W}_0 is the mean value of the takeoff weight, $\overline{\rho}$ is the mean value of air density at the given flight altitude, \overline{V} is the mean value of flight speed, \overline{S} is the wing area, and \overline{C}_L is the nominal lift coefficient of the aircraft computed by the aerodynamic model. The subscript of $g_1(1)$ indicates discipline 1, and the index in the brackets indicates the first constraint in discipline 1.

Subsystem 2: structure

Subsystem 2 involves structural analysis. The wing is structurally divided into seven sections along the wingspan (see figure 1). Each section is a single box beam that consists of several components including the spar caps, the front shear web, the rear shear webs, and the skin. For each section, the bending stress in the spars, the shear stress in the skin, the shear stress in the front and rear webs, and the wing twist deformation are calculated using the beam theory (Allen and Haisler 1984).

The disciplinary design variables are $\mathbf{d}_2 = (w_1, w_2, \dots, w_{10})$, where $w_i (i=1, \dots, 7)$ is the area of spar cap in section i , w_8, w_9 and w_{10} are the thickness of the skin, the front web, and the rear web, respectively. The random disciplinary variables are $\mathbf{x}_2 = (G, F, S_1, S_2, S_3)$, where G is shear modulus of the web and skin material, F is a gust load factor, and S_1, S_2, S_3 are the bending strength of the material of the spar caps, the shear strength of the skin, and the shear strength of the spar web, respectively.

The input linking variables are aerodynamic lift distributions defined by the cubic polynomial function $\mathbf{y}_{12} = (a_0, a_1, a_2, a_3)$, where a_0, a_1, a_2, a_3 are the coefficients of the cubic polynomial function. $\mathbf{y}_{12} = (a_0, a_1, a_2, a_3)$ is part of the input to subsystem 1. The output linking variable is $\mathbf{y}_{21} = (\delta)$, where δ is the twist deformation of the wing under the aerodynamic loads and is part of input to subsystem 1.

All the constraints for subsystem 2 are considered as reliability constraints. These constraints are given by

$$\Pr\{G_2(i) \leq 0\} = \Pr\{\sigma_i - S_1 \leq 0\} \geq R_2(i) \quad (i=1, 2, \dots, 7) \quad (13)$$

$$\Pr\{G_2(8) \leq 0\} = \Pr\{\tau_{skin} - S_2 \leq 0\} \geq R_2(8) \quad (14)$$

$$\Pr\{G_2(9) \leq 0\} = \Pr\{\tau_{wf} - S_3 \leq 0\} \geq R_2(9) \quad (15)$$

$$\Pr\{G_2(10) \leq 0\} = \Pr\{\tau_{wr} - S_3 \leq 0\} \geq R_2(10) \quad (16)$$

$$\Pr\{G_2(11) \leq 0\} = \Pr\{\delta - \delta_a \leq 0\} \geq R_2(11) \quad (17)$$

where δ_a is the allowable twist deformation of the wing and is set as 2° , σ_i ($i=1,2, \dots, 7$) are the bending stresses in the spar cap for each section, τ_{skin} is the maximum shear stress in the skin, τ_{wf} is the shear stress in the web, τ_{wr} is the shear stress in the rear web, δ is the twist deformation of the wing, and $R_2(i)$, $i = 1,2,\dots,11$, are required reliabilities, which are set as $R_2(1) = R_2(2) = \dots = R_2(11) = 0.9987$. The required reliability of 0.9987 is correspondent to a reliability index $\beta = 3$. For the constraint $\Pr\{G_2(i) \leq 0\} \geq R_2(i)$, the subscript 2 stands for discipline 2, the index i stands for the i -th constraint.

The overall RBMDO wing design problem is modeled as

$$\left\{ \begin{array}{l}
 \text{Given : Distributions of } H, V, W_0, G, F, S_1, S_2, S_3 \\
 \text{Find : } AR, \theta, \alpha, w_1, w_2, \dots, w_{10} \\
 \text{Maximize } \bar{v} = 603\bar{K} \left(\frac{\eta_p}{c_p} \right) \ln \left(\frac{\bar{W}_0}{\bar{W}_a + \bar{W}_w} \right) \\
 \text{s.t.} \\
 g_1(1) = - \left(\bar{W}_0 - \frac{1}{2} \bar{\rho} \bar{V}^2 \bar{S} \bar{C}_L \right) \leq 0 \\
 g_1(2) = \bar{W}_0 - \frac{1}{2} \bar{\rho} \bar{V}^2 \bar{S} \bar{C}_L \leq 0 \\
 R_2(i) - \Pr\{G_2(i) \leq 0\} = R_2(i) - \Pr\{\sigma_i - S_1 \leq 0\} \leq 0 \quad (i = 1, 2, \dots, 7) \\
 R_2(8) - \Pr\{G_2(8) \leq 0\} = R_2(8) - \Pr\{\tau_{skin} - S_2 \leq 0\} \leq 0 \\
 R_2(9) - \Pr\{G_2(9) \leq 0\} = R_2(9) - \Pr\{\tau_{wf} - S_3 \leq 0\} \leq 0 \\
 R_2(10) - \Pr\{G_2(10) \leq 0\} = R_2(10) - \Pr\{\tau_{wr} - S_3 \leq 0\} \leq 0 \\
 R_2(11) - \Pr\{G_2(11) \leq 0\} = R_2(11) - \Pr\{\delta - \delta_a \leq 0\} \leq 0
 \end{array} \right. \quad (18)$$

The above design model involves two subsystems, 13 deterministic design variables, 9 random variables, 5 linking variables, 2 deterministic constraint functions, and 9 reliability constraint functions. Next, we will discuss how to solve a general RBMDO problem by SORA and then discuss how to solve the model specified in equation (18).

3 Solve RBMDO problems

As demonstrated in figure 4, solving a RBMDO problem such as the one given in equation (18) involves a double-loop procedure, where the outer loop is the overall optimization, and the inner loop is the reliability analysis. If the multidisciplinary analysis (MDA) requires an iterative process, an extra loop will be added under the reliability analysis loop; and then the overall RBMDO will involve a triple-loop procedure as shown in figure 4. To alleviate the computational burden, we propose to use the Sequential Optimization and Reliability Assessment (SORA) (Wu et al. 2001, Du and Chen, 2004) to solve the wing design problem efficiently.

3.1 Proposed RBMDO procedure

The proposed RBMDO procedure that implements SORA under the MDO environment includes the following two aspects.

1) Model a RBMDO problem in such a formulation that the problem can be solved efficiently. SORA has been successfully applied to single-disciplinary RBD (Du and Chen 2004, Du et al. 2005). The same strategy will be used under the RBMDO environment. The central idea is to employ sequential cycles of deterministic MDO and reliability analysis. In each cycle MDO and reliability analysis are decoupled from each other; the reliability analysis is only conducted after the MDO. The idea is outlined in figure 5. It is seen that through this procedure, the number of reliability analyses is equal to the number of cycles. The design is expected to converge in a few cycles, and therefore the computational efficiency is much higher than a procedure where reliability analysis is applied directly in MDO.

(2) Perform reliability analysis only up to the necessary level by using the inverse reliability strategy (percentile performance) (Der Kiureghian et al. 1994, Li and Foschi 1998, Chen and Hasselman 1997, Wu and Wang 1998, Tu et al. 1999, Wu et al. 2001, Du et al., 2004). This strategy is also called Performance Measure approach (Tu et al. 1999). In general, evaluating the percentile performance is more efficient than evaluating an actual reliability (Tu et al. 1999, Du et al. 2004). On the other hand, since the system failure modes are correlated due to the common variables (materials, dimensions, and loads) of the limit-state functions (constraint functions), if reliability constraints of some failure modes are satisfied, other reliability constraints may be satisfied automatically and may never be active. However, those never-active reliability constraints may unfortunately dominate the computational effort in the RBD process (Murotsu et al. 1994). To this end, we seek a procedure that evaluates a percentile performance only up to a necessary level, namely, the required reliability level. In this paper, the percentile performance is computed by the First Order Reliability Method (FORM) (Hasofer and Lind, 1974).

For simplicity, let a constraint function G in one discipline be $G = G(\mathbf{Y})$, where \mathbf{Y} is the vector of random variables consisted of the random sharing variables, random disciplinary variables and linking variables outputted from other disciplines. At first \mathbf{Y} is transformed into a random vector \mathbf{U} whose elements follow a standard normal distribution (Hasofer and Lind, 1974). Next, the Most Probable Point (MPP) \mathbf{u}^* is identified by solving the following optimization problem.

$$\begin{cases} \max_{\mathbf{u}} G(\mathbf{u}) \\ s.t. \|\mathbf{u}\| = \Phi^{-1}(R) \end{cases} \quad (19)$$

where Φ^{-1} is the inverse distribution function of a standard normal variable, and R is a required reliability. The percentile performance corresponding to R is then calculated at the MPP \mathbf{u}^* as

$$G^R = G(\mathbf{u}^*) \quad (20)$$

The reliability requirement $\Pr\{G(\mathbf{Y}) \leq 0\} \geq R$ is equivalent to $G^R = G(\mathbf{u}^*) \leq 0$ (Tu et al. 1999). Therefore, a reliability constraint function in the RBMDO model in equation (8) can be replaced by a constraint function $G^R = G(\mathbf{u}^*) \leq 0$.

The overall optimization is conducted with sequential cycles of optimization and reliability analysis where the deterministic MDO is followed by the reliability analysis. In each cycle, four steps are involved and are shown as follows:

Step 1: Solve the deterministic MDO. In the first cycle, the optimization is a conventional MDO without any consideration of uncertainty. From the second cycle, based on the information obtained in the previous cycle, the MDO is formulated in such a way that the optimality and reliability requirement will be gradually achieved.

Step 2: Perform reliability analysis. At the optimal point obtained in Step 1, the Most Probable Points (MPPs) of all the reliability constraints are identified, and the percentile performance values corresponding to the required reliability are also calculated.

Step 3: Check the convergence. If the reliability requirement is satisfied and the system objective function becomes stable, the whole optimization process stops; otherwise, continue to Step 4.

Step 4: Formulate a new deterministic MDO model for the next cycle based on the MPP information from Step 3.

The percentile formulation of the MDO problem in cycle k is given by

$$\begin{cases} \min_{(\mathbf{d}_s, \mathbf{d})} v(\bar{\mathbf{v}}_1, \bar{\mathbf{v}}_2, \bar{\mathbf{v}}_3) \\ s.t. \\ \mathbf{G}_i(\mathbf{d}_s, \mathbf{d}_i, \mathbf{u}_s^{*,k-1}, \mathbf{u}_i^{*,k-1}, \mathbf{y}_i^{*,k-1}) \leq 0, \quad i = 1, 2, 3 \\ \mathbf{g}_i(\mathbf{d}_s, \mathbf{d}_i, \bar{\mathbf{x}}_s, \bar{\mathbf{x}}_i, \bar{\mathbf{y}}_i) \leq 0, \quad i = 1, 2, 3 \end{cases} \quad (21)$$

where $\mathbf{u}_s^{*,k-1}$ and $\mathbf{u}_i^{*,k-1}$ are components of the MPP obtained in the reliability analysis in cycle $k-1$, and $\mathbf{y}_i^{*,k-1}$ is the vector of linking variables at the MPP.

The above strategy is applicable for any MDO formulations. Next we will limit our discussions on the application of the SORA method into two MDO formulations – single-loop method and double-loop method. The application of the SORA method to other MDO formulations will be our future work.

3.2 Deterministic MDO

The task of the deterministic MDO is to solve the optimization model specified in equation (21). We use two methods to perform the deterministic MDO. Depending on whether the linking variables are treated as part of the design variables, the methods are classified as the single loop-method and double-loop method. The single loop-method can be considered as the Simultaneous Analysis and Design (SAND) method (Alexandrov and Lewis 2000). In addition to the original design variables, it also accommodates the linking variables as design variables. The system consistency is also formulated as part of system constraints. The double-loop method is also termed the Multidisciplinary Feasible method (MDF) (Alexandrov and Kodiyalam 1998). Its formulation is a conventional method for solving MDO problems, where only the original design variables are used at the system level (outer loop). The linking variables are solved by MDA, which is the inner loop embedded under the overall optimization outer loop.

Single-loop method

In the single-loop method, the deterministic MDO model in the k th cycle is given by

$$\left\{ \begin{array}{l} \min_{(\mathbf{d}_s, \mathbf{d}_i, \bar{\mathbf{y}}^*)} v(\bar{\mathbf{v}}_1, \bar{\mathbf{v}}_2, \bar{\mathbf{v}}_3) \\ s.t. \\ \mathbf{G}_i(\mathbf{d}_s, \mathbf{d}_i, \mathbf{u}_s^{*(i),k-1}, \mathbf{u}_i^{*(i),k-1}, \mathbf{y}_i^{*(i)}) \leq 0, \quad i = 1, 2, 3 \\ \mathbf{g}_i(\mathbf{d}_s, \mathbf{d}_i, \bar{\mathbf{x}}_s, \bar{\mathbf{x}}_i, \bar{\mathbf{y}}_i) \leq 0, \quad i = 1, 2, 3 \\ \bar{\mathbf{y}}_{ij} = \mathbf{y}_{ij}(\mathbf{d}_s, \mathbf{d}_i, \bar{\mathbf{x}}_s, \bar{\mathbf{x}}_i, \bar{\mathbf{y}}_i), \quad i, j = 1, 2, 3, i \neq j \\ \mathbf{y}_{jm}^{*(i)} = \mathbf{y}_{jm}^{*(i)}(\mathbf{d}_s, \mathbf{d}_j, \mathbf{u}_s^{*(i),k-1}, \mathbf{u}_j^{*(i),k-1}, \mathbf{y}_j^{*(i)}), \quad i = 1, 2, 3, j = 1, 2, 3, m = 1, 2, 3, j \neq m \end{array} \right. \quad (22)$$

In the above model, the design variables also include the nominal values of all the linking variables, which are given by

$$\bar{\mathbf{y}} = (\bar{\mathbf{y}}_{ij}), \quad i = 1, 2, 3, j = 1, 2, 3, i \neq j \quad (23)$$

The linking variables corresponding to each of the reliability constraints, \mathbf{y}_{ij}^* , are also part of design variables and are given by

$$\mathbf{y}_{jm}^* = (\mathbf{y}_{jm}^{*(1)}, \mathbf{y}_{jm}^{*(2)}, \mathbf{y}_{jm}^{*(3)}), \quad j = 1, 2, 3, m = 1, 2, 3, j \neq m \quad (24)$$

where $\mathbf{y}_{jm}^{*(i)}$ is the vector of the linking variables for each of the reliability constraints in discipline i ($i=1, 2, 3$), and it is the output of discipline j and the input to discipline m .

The system consistency is also considered at the means of all the random variables and the MPPs of all the reliability constraints as additional constraints as shown in the last two lines in equation (22). Therefore, the optimal solution can be identified within a single-loop procedure since everything is taken care of in the above optimization model.

Double-loop method

The double-loop method only takes the original design variables without any additional design variables in its optimization outer loop. The linking variables are identified in the inner loop where the system consistency equations are solved. The outer loop deterministic MDO model in the k th cycle is given by

$$\left\{ \begin{array}{l} \min_{(\mathbf{d}_s, \mathbf{d})} v(\bar{\mathbf{v}}_1, \bar{\mathbf{v}}_2, \bar{\mathbf{v}}_3) \\ s.t. \\ \mathbf{G}_i(\mathbf{d}_s, \mathbf{d}_i, \mathbf{u}_s^{*(i),k-1}, \mathbf{u}_i^{*(i),k-1}, \mathbf{y}_i^{*(i)}) \leq 0, \quad i=1, 2, 3 \\ \mathbf{g}_i(\mathbf{d}_s, \mathbf{d}_i, \bar{\mathbf{x}}_s, \bar{\mathbf{x}}_i, \bar{\mathbf{y}}_i) \leq 0, \quad i=1, 2, 3 \end{array} \right. \quad (25)$$

Since the linking variables are not included in the outer loop optimization model, they have to be obtained from the inner loop system consistency equations. The inner loop MDA for solving the linking variables is given by the simultaneous equations

$$\left\{ \begin{array}{l} \bar{\mathbf{y}}_{jm} = \mathbf{y}_{jm}(\mathbf{d}_s, \mathbf{d}_j, \bar{\mathbf{x}}_s, \bar{\mathbf{x}}_j, \bar{\mathbf{y}}_j) \\ \mathbf{y}_{jm}^{*(i)} = \mathbf{y}_{jm}^{*(i)}(\mathbf{d}_s, \mathbf{d}_j, \mathbf{u}_s^{*(i),k-1}, \mathbf{u}_j^{*(i),k-1}, \mathbf{y}_j^{*(i)}), \quad i=1, 2, 3, j=1, 2, 3, m=1, 2, 3, j \neq m \end{array} \right. \quad (26)$$

Depending on the problems, different numerical methods, including optimization methods, can be used to solve the above simultaneous equations. In this paper, we use Sequential Quadratic Programming (SQP), to solve the simultaneous equations. The model of the MDA is given by

$$\left\{ \begin{array}{l} \min_{\mathbf{y}_{ij}^*} \sum_{\substack{j,m=1,2,3 \\ j \neq m}} \left[\bar{\mathbf{y}}_{jm} - \mathbf{y}_{jm}(\mathbf{d}_s, \mathbf{d}_j, \bar{\mathbf{x}}_s, \bar{\mathbf{x}}_j, \bar{\mathbf{y}}_j) \right]^2 + \sum_{\substack{j,m=1,2,3 \\ j \neq m}} \left[\mathbf{y}_{jm}^{*(i)} - \mathbf{y}_{jm}^{*(i)}(\mathbf{d}_s, \mathbf{d}_j, \mathbf{u}_s^{*(i),k-1}, \mathbf{u}_j^{*(i),k-1}, \mathbf{y}_j^{*(i)}) \right]^2 \\ s.t. \\ \bar{\mathbf{y}}_{jm} = \mathbf{y}_{jm}(\mathbf{d}_s, \mathbf{d}_j, \bar{\mathbf{x}}_s, \bar{\mathbf{x}}_j, \bar{\mathbf{y}}_j) \\ \mathbf{y}_{jm}^{*(i)} = \mathbf{y}_{jm}^{*(i)}(\mathbf{d}_s, \mathbf{d}_j, \mathbf{u}_s^{*(i),k-1}, \mathbf{u}_j^{*(i),k-1}, \mathbf{y}_j^{*(i)}), \quad i=1, 2, 3, j=1, 2, 3, m=1, 2, 3, j \neq m \end{array} \right. \quad (27)$$

It is noted the above use of SQP is just for analysis purpose, namely, MDA. It may not be necessary to model the MDA as an optimization problem, and other numerical methods can be used for MDA.

3.3 Reliability analysis under MDO environment

Once the optimal solution is obtained from the deterministic MDO, reliability analysis is conducted. There are two purposes of reliability analysis. The first purpose is to evaluate the percentile values of the reliability constraint functions and check whether the reliability requirement is met; the second purpose is to use the reliability analysis result to build the deterministic MDO model for the next cycle if the reliability requirement in current cycle is not satisfied.

Since FORM is used, the reliability analysis is essentially an MDO problem for the MPP search. Therefore, similar to the deterministic MDO, there are two ways to conduct reliability analysis, namely, the single-loop method and the double-loop method.

Single-loop method

In the single-loop method, the system consistency is also included in the optimization model for the MPP search, and the design variables in the MPP search include the sharing random variables and disciplinary random variables, and all the linking variables as well. Since the system consistency is treated as constraints, there is no need to perform MDA as an inner loop. The optimization model for a reliability constraint G_i in discipline i is given by

$$\left\{ \begin{array}{l} \max_{(\mathbf{u}_s, \mathbf{u}, \mathbf{y})} G_i(\mathbf{d}, \mathbf{u}_s, \mathbf{u}_i, \mathbf{y}_i) \\ s.t. \\ \left\| (\mathbf{u}_s, \mathbf{u}) \right\| = \Phi^{-1}(R) \\ \mathbf{y}_{jm}^* = \mathbf{y}_{jm}^*(\mathbf{d}_s, \mathbf{d}_j, \mathbf{u}_s, \mathbf{u}_j, \mathbf{y}_j), \quad i = 1, 2, 3, j = 1, 2, 3, m = 1, 2, 3, j \neq m \end{array} \right. \quad (28)$$

where $\mathbf{u} = (\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3)$, and R is the required reliability.

The solutions are the MPP in u-space $(\mathbf{u}_s^*, \mathbf{u}^*)$ and the linking variables $\mathbf{y}^* = (\mathbf{y}_{jm}^*)$ at the MPP. Percentile value of the constraint function is calculated at the MPP $(\mathbf{u}_s^*, \mathbf{u}^*)$. The MPP will also be used for formulating the deterministic MDO model for the next cycle, as shown in equation (21).

Double-loop method

The design variables in the double-loop method are only the sharing random variables and the disciplinary random variables in transformed normal space. The linking variables are not included as design variables and will be obtained from the inner loop MDA. The optimization model of the outer loop is given by

$$\begin{cases} \max_{(\mathbf{u}_s, \mathbf{u})} G_i(\mathbf{d}, \mathbf{u}_s, \mathbf{u}_i, \mathbf{y}_i) \\ s.t. \\ \left\| (\mathbf{u}_s, \mathbf{u}) \right\| = \Phi^{-1}(R) \end{cases} \quad (29)$$

The inner loop is to solve the linking variables. Similarly to the double-loop MDO, for the purpose of demonstration, the inner loop problem is solved by an optimization model, which is formulated as

$$\begin{cases} \min_{\mathbf{y}_{jm}} \sum_{\substack{j,m=1,2,3 \\ j \neq m}} \left[\mathbf{y}_{jm} - \mathbf{y}_{jm}(\mathbf{d}_s, \mathbf{d}_j, \mathbf{u}_s, \mathbf{u}_j, \mathbf{y}_j) \right]^2 \\ s.t. \\ \mathbf{y}_{jm} = \mathbf{y}_{jm}(\mathbf{d}_s, \mathbf{d}_j, \mathbf{u}_s, \mathbf{u}_j, \mathbf{y}_j), \quad i = 1, 2, 3, j = 1, 2, 3, m = 1, 2, 3, j \neq m \end{cases} \quad (30)$$

3 Solve RBMDO Problems

The above RBMDO methods, the single-loop method (single-loop MDO + single-loop reliability analysis) and the double-loop method (double-loop MDO + double-loop reliability analysis), are used to solve the wing design problem described in section 2. The Sequential Quadratic Programming (SQP) method provided by MATLAB optimization toolbox was used for the deterministic MDO, MPP search, and MDA. The stopping criteria such as the differences of function values, design variable at two consecutive design points and the constraint violations are set to the default values of the MATLAB optimization toolbox. The optimal solutions and convergence history from both the single-loop and the double-loop methods are given in tables 4 and 5, respectively. The optimal points are given in table 6. In tables 4 and 5, five representative reliability constraints $G_2(3)$, $G_2(4)$, $G_2(5)$, $G_2(6)$, and $G_2(7)$ are also given, which are the most critical reliability constraints in this application, representing the constraints for the front beam spar cap at section 3, 4, 5, 6, and 7, respectively. At the optimal point, the percentile values of constraints $G_2(3)$, $G_2(4)$, $G_2(5)$, $G_2(6)$, and $G_2(7)$ are very close to zero. This indicates that the actual reliabilities at the optimal point are equal to the required reliabilities. The percentile values of the other constraints are negative, which indicates that the reliabilities of those constraint functions are greater than the required reliabilities.

In the first cycle, the deterministic MDO is performed, and the maximum objective (flight range) is found. Then the reliability analysis is conducted and shows that the optimal solution does not satisfy the reliability requirement since the percentile values of some constraint function are greater than zero (see cycle 1 in tables 4 and 5). In the second cycle, the MDO model is reconstructed with the information obtained from the reliability analysis in cycle 1. In cycle 2 after the MDO, the reliability analysis at the optimal point shows that

the design is improved significantly since the percentile values of the reliability constraints are reduced considerably (see cycle 2 in tables 4 and 5). The optimization solution is identified in cycle 4. Both the single-loop method and the double-loop method converge to the same solution after four cycles, but the numbers of subsystem analyses required for the convergence are substantially different. The single-loop method requires 900 analyses in subsystem 1 and 4301 analyses in subsystem 2, while the double-loop method requires 15021 analyses in both subsystems 1 and 2. For this application, the single-loop method is much more efficient than the double-loop method. As shown in figure 4, if the reliability-based design was directly applied to MDO, a triple-loop procedure would be involved, and therefore the number of subsystem analyses would be much larger than that of the proposed methods.

5. Conclusions

The purpose of this paper is to investigate the feasibility of using the SORA method for RBMDO. With the SORA method, the RBMDO is conducted through a series of cycle of deterministic MDO and reliability analysis. In each cycle, the reliability analysis is decoupled from the deterministic MDO, and the model of the deterministic MDO is formulated based on the reliability analysis results from the previous cycle. The updated optimization model ensures that the violated reliability constraints be improved automatically. Since the decoupling strategy is used for separating multidisciplinary reliability analysis from the MDO, it is applicable for any formulations of MDO problems. The strategy can be used in the framework of Collaborative Optimization (CO), Concurrent Subsystem Optimization (CSSO), IDF, MDF, etc.

A simple integrated aerodynamic/structural wing design for a light aircraft was used for the demonstration. In this preliminary study, only two MDO methods have been considered, namely, single-loop method and double-loop method. In the single loop method, in addition to the original design variables for deterministic MDO and MPP search for reliability analysis, the linking variables are also considered as design variables. The system consistency is considered as additional constraints in the deterministic MDO and reliability analysis. Because everything is taken care of in a single model for deterministic MDO and reliability analysis, the entire RBMDO is performed in a series of single loops. On the other hand, in double-loop method, no additional design variables are considered as design variables in the model of either deterministic MDO or reliability analysis (the outer loop); the system consistency for solving the linking variables has to be formulated separately as the inner loop and is nested with the outer loop.

Even though the wing design problem we used is quite simple and the numbers of design variables and random variables are relatively small, it still gives us useful observations. As indicated by results, both the single-loop method and double-loop methods were able to find the same optimal solution, but their efficiency differs dramatically. The efficiency depends on the number of linking variables and the number of reliability constraint functions. As a rule of thumb, if the numbers of linking variables, reliability constraint functions, and random variables are small, one may consider choosing the single-loop method; otherwise, one should consider using the double-loop method to avoid increasing the problem scale significantly. In the wing design problem, the single-loop method is more efficient than the double-loop method since the numbers of linking variables, reliability constraint functions, and random variables are not large.

It should be noted that in the above simple wing MDO problem, the lifting line theory was used for the aerodynamic model, and the beam theory was used for the structure model. For more complicated wing MDO problems (such as transport jet wing design), nonlinear flow theories (such as Euler equations) can be used to solve aerodynamic models, and Finite Element Method (FEM) can be used to solve structural models. In that

case, the coupling between aerodynamics and structure is much more complicated and lead to a large number of linking variables. The reduced basis modeling approaches (Manning 1999) could be applied to reduce the number of linking variables. Also, the aerodynamic computations based on nonlinear flow theories and the structure analyses using FEM would lead to the increase of computational burden.

It should also be noted that the proposed method uses the First Order Reliability Method (FORM). FORM depends on the existence of a unique MPP and the global solution of the MPP. If a limit-state (constraint) function is highly nonlinear in the transformed normal space, FORM may results in a larger error (Du and Sudjianto 2004). Therefore, the MPP based RBMDO may fail or may have a significant error in following cases: (1) There are multiple MPPs; (2) a global MPP is hard to find; and (3) there is a large error of reliability assessment. In these cases, other reliability analysis methods can be used such as Monte Carlo Simulation. The proposed the strategy of decoupling reliability analysis and MDO is still applicable if other reliability analysis methods are used.

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Table 1 Notations

Variables	Definition
\mathbf{d}_s	Sharing design variables, which are common design variables to all disciplines
\mathbf{d}_i	Local design variables in discipline i
\mathbf{x}_s	Sharing random input variables, which are common input variables of all disciplines
\mathbf{x}_i	Local random input variables to discipline i
\mathbf{y}_{ij}	Linking variables, which are the output of discipline i and the input of discipline j

Table 2 Distributions of random variables

Variables	Mean	Standard deviation	Distribution
Flight altitude H	3000 m	300 m	Normal
Flight speed V	200 km/h	20 km/h	Normal
Wing area S	10 m ²	0.5 m ²	Normal
Take-off weight W_0	700 kg	70 kg	Normal
Shear modulus G	2.7×10^{10} N/mm ²	2.7×10^9 N/mm ²	Normal
Gust load factor F	4.0	0.4	Normal
Bending strength S_1	450 N/mm ²	45 N/mm ²	Normal
Shear strength of the skin S_2	200 N/mm ²	20 N/mm ²	Normal
Shear strength of the web S_3	250 N/mm ²	25 N/mm ²	Normal

Table 3 Design variables

Design variables	Lower value	Upper value	Disciplines
Aspect ratio, AR	5.5	9.5	System
Twist angle, θ	0 (deg)	4 (deg)	Aerodynamics
Angle of attack, α	0 (deg)	6 (deg)	Aerodynamics
Area of spar cap in section i , $w_i (i = 1, \dots, 7)$	50 (mm ²)	700 (mm ²)	Structure
Thickness of the skin, w_8	1 (mm)	2 (mm)	Structure
Thickness of the front web, w_9	1 (mm)	2 (mm)	Structure
Thickness of the rear web, w_{10}	1 (mm)	2 (mm)	Structure

Table 4 Convergence history of the single-loop method

<i>Cycle</i>	<i>v (km)</i>	$G_2(3)$	$G_2(4)$	$G_2(5)$	$G_2(6)$	$G_2(7)$	n_1	n_2
1	1500.2	335.90	335.00	334.01	333.03	332.16		
2	1363.9	7.4423	7.4058	7.3659	7.3259	7.2887	900	4301
3	1359.2	0.01085	0.01073	0.01060	0.01040	0.01031		
4	1359.2	-1.074×10^{-8}	5.414×10^{-6}	2.203×10^{-7}	-2.508×10^{-7}	1.408×10^{-6}		

n_1 – number of function evaluations in subsystem 1

n_2 – number of function evaluations in subsystem 2

Table 5 Convergence history of the double-loop method

<i>Cycle</i>	<i>v (km)</i>	$G_2(3)$	$G_2(4)$	$G_2(5)$	$G_2(6)$	$G_2(7)$	n_1	n_2
1	1500.2	335.90	335.00	334.01	333.03	332.16		
2	1363.9	7.4418	7.4061	7.3657	7.3256	7.2890	15021	15021
3	1359.2	0.01086	0.01072	0.01061	0.01041	0.01030		
4	1359.2	8.93×10^{-8}	6.32×10^{-6}	7.06×10^{-6}	8.58×10^{-6}	-1.73×10^{-7}		

Table 6 The optimum points

Design variables	Double-loop method	Single-loop method
Aspect ratio, AR	5.7823	5.7824
Angle of attack, α	3.0877 (deg)	3.0877
Wing twist, θ	0.8040 (deg)	0.8041
Area of spar cap in section 1, w_1	50.0 (m ²)	50.0 (m ²)
Area of spar cap in section 2, w_2	54.81 (mm ²)	54.81 (mm ²)
Area of spar cap in section 3, w_3	122.07 (mm ²)	122.07 (mm ²)
Area of spar cap in section 4, w_4	215.22 (mm ²)	215.23 (mm ²)
Area of spar cap in section 5, w_5	333.13 (mm ²)	333.13 (mm ²)
Area of spar cap in section 6, w_6	472.82 (mm ²)	472.83 (mm ²)
Area of spar cap in section 7, w_7	628.41 (mm ²)	628.42 (mm ²)
Thickness of the skin, w_8	1.0 (mm)	1.0 (mm)
Thickness of the front web, w_9	1.0 (mm)	1.0 (mm)
Thickness of the rear web, w_{10}	1.0 (mm)	1.0 (mm)

Figure 1. The wing structure model

Figure 2. A multidisciplinary system

Figure 3. Wing design with the integration of aerodynamic and structural subsystems

Figure 4. Triple loop for reliability-based MDO

Figure 5. The procedure of SORA under RBMDO environment