Journal of Mechanical Design | Volume 132 | Issue 1 | Research Paper

Optimal Design Accounting for Reliability, Maintenance, and Warranty

Zhonglai Wang
wangyang713@163.com
University of Electronic Science and Technology of China

Hong-Zhong Huang
hzhuang@uestc.edu.cn
University of Electronic Science and Technology of China

Xiaoping Du*
Missouri University of Science and Technology

*Corresponding Author
400 West 13th Street
Department of Mechanical and Aerospace Engineering
Rolla, Missouri 65409 – 4494
E-mail: dux@mst.edu

ABSTRACT

Reliability-based design (RBD) ensures high reliability with a reduced cost. Most of the RBD methodologies do not account for maintenance and warranty actions. As a result, the RBD result may not be truly optimal in terms of lifecycle reliability. This work attempts to integrate reliability, maintenance, and warranty during RBD. Three RBD models are built. The total cost of production, maintenance, and warranty is minimized. The computational procedures for solving the RBD models are developed. As demonstrated by two examples, the proposed RBD models meet not only the initial reliability requirement, but also the maintenance and warranty requirements with reduced costs.

1 INTRODUCTION

With the use of new technologies, engineering systems have become increasingly complex, and so has the risk of failure [1, 2]. To this end, reliability-based design (RBD) has become a major task of engineering design. In RBD, reliability is viewed as the probability of success, and the state of success is determined by computational models. Reliability can then be conveniently evaluated without directly using product life data [3].

RBD is computationally expensive. Many efforts have been devoted to efficient RBD methods, including the performance measure method [4, 5], single-loop method [6-9], safety-factor based method [10, 11], and sequential optimization and reliability assessment (SORA) method [12-14]. Some of the methods can also deal with system reliability [15, 16] when multiple failure modes exist. Both continuous and discrete design variables could be incorporated in RBD [17, 18]. While static reliability is considered in most of RBD methods, a few studies have taken time-variant reliability into consideration [19, 20].

In reliability engineering, maintenance and warranty actions are also implemented. Maintenance is an important measure to maintain and extend the product service life. It is categorized into corrective maintenance (CM) and preventive maintenance (PM) [21]. Corrective maintenance is used to maintain or restore product functions after a failure occurs. When products enter the predetermined unsafe domain, preventive maintenance takes place.

Warranty is also an important intervention in the product service life. It is a contractual agreement between consumers and producers [22]. From the consumer’s point of view, the main role of warranty is protectional - if the product fails to perform as intended, the producer will repair or replace the failed product for free or at reduced costs [23]. The other role of warranty is informational. A longer warranty period indicates higher quality. From a producer’s point of view, the role of warranty is also protectional and informational. The condition of use is specified in the warranty terms for which the product is intended, and limited coverage or no coverage is provided at all in the case of misuse of the product. Warranty has also been used as an advertising tool for producers [24]. Among many warranty policies are the two basic ones: free repair or replacement warranty (FRW) and pro-rata warranty (PRW) [23, 25]. With PRW, product maintenance is provided at a prorated cost.

RBD, maintenance, and warranty share a common purpose – maintaining the probability of success (reliability). But the latter two reliability actions have seldom been considered during RBD. Doing so will undoubtedly further the benefits of RBD and produce a true optimal design in terms of lifecycle reliability and cost. Exploratory work has been reported in [26-29], where the lifecycle cost and maintenance have been considered for structural systems. Another preliminary study was our previous work [30], where three RBD models have been proposed for three different depths of maintenance policies: nonrepairable products, perfect maintenance, and minimal maintenance. The present research attempts to further explore the feasibility of integrating RBD with maintenance and warranty actions.

In Section 2, the traditional RBD methods are briefly reviewed. In Section 3, three RBD models are proposed. The numerical procedure of solving the models is described in Section 4. Two examples are given in Section 5. Conclusions and future work are provided in Section 6.

2 RELIABILITY-BASED DESIGN

The typical RBD is modeled by [12]

$$\begin{aligned} &\min_{\mathbf{d},\boldsymbol{\mu}_X} \ \mathrm{Cost}(\mathbf{d},\mathbf{X},\mathbf{P}) \\ &\text{s.t.} \quad \Pr\left\{g_i(\mathbf{d},\mathbf{X},\mathbf{P}) \ge 0)\right\} \ge \left[R_i\right] \quad i = 1,2,\ldots,n_g \\ &\qquad \mathbf{d}^L \le \mathbf{d} \le \mathbf{d}^U, \boldsymbol{\mu}_X^L \le \boldsymbol{\mu}_X \le \boldsymbol{\mu}_X^U \end{aligned} \tag{1}$$

$\mathbf{d}$ is the vector of deterministic design variables. $\mathbf{X}$ is the vector of random design variables, whose mean values $\boldsymbol{\mu}_X$ are to be determined. $\mathbf{P}$ is the vector of random parameters. $g_i(\mathbf{d},\mathbf{X},\mathbf{P})$ is a constraint function, and $\Pr\left\{g_i(\mathbf{d},\mathbf{X},\mathbf{P}) \ge 0)\right\} \ge \left[R_i\right]$ means that the probability of constraint satisfaction $g_i(\mathbf{d},\mathbf{X},\mathbf{P}) \ge 0$ should be greater than or equal to the desired reliability $\left[R_i\right]$. $\mathbf{d}^L$ and $\mathbf{d}^U$ are lower and upper bounds of $\mathbf{d}$, respectively. Likewise, $\boldsymbol{\mu}_X^L$ and $\boldsymbol{\mu}_X^U$ are lower and upper bounds of $\boldsymbol{\mu}_X$, respectively. In this paper, we assume all the random variables in $(\mathbf{X},\mathbf{P})$ are independent.

Reliability $\Pr\left\{g_i(\mathbf{d},\mathbf{X},\mathbf{P}) \ge 0\right\}$ can be computed by

$$\Pr\{g_i(\mathbf{d},\mathbf{X},\mathbf{P}) \ge 0\} = \int_{g_i(\mathbf{d},\mathbf{X},\mathbf{P}) \ge 0} f_{\mathrm{X,P}}(\mathbf{x},\mathbf{p})\mathrm{d}\mathbf{x}\mathrm{d}\mathbf{p} \tag{2}$$

The First Order Reliability Method (FORM) is commonly used to evaluate the probability integral. The FORM at first transforms random variables $\mathbf{Z} = (\mathbf{X}, \mathbf{P})$ into standard normal random variables $\mathbf{U}_Z = (\mathbf{U}_X, \mathbf{U}_P)$ by

$$F_{Z_j}(z_j) = \Phi(u_j),\ j = 1, 2, \cdots, n_X + n_P \tag{3}$$

where $F_{Z_j}(z_j)$ is the cumulative distribution function (CDF) of Z_j, $\Phi(u_j)$ is the CDF of U_j, n_X is the length of $\mathbf{X}$, and n_P is the length of $\mathbf{P}$. Then the Most Probable Point (MPP) is obtained by solving the following optimization problem:

$$\mathbf{u}_Z^* = \left\{ \mathbf{U}_Z : \min \overline{\mathbf{U}}_Z \middle| g_i(\mathbf{d}, \mathbf{U}_Z) = 0 \right\} \tag{4}$$

where $\overline{\mathbf{U}}_Z$ stands for the magnitude of $\mathbf{U}_Z$, and $\mathbf{u}_Z^*$ is the MPP. The reliability index is given by $\beta_i = \overline{\mathbf{u}}_Z^*$. The reliability is then computed by

$$\Pr\{g_i(\mathbf{d}, \mathbf{X}, \mathbf{P}) \geq 0\} = \Phi(\beta_i) \tag{5}$$

3 RELIABILITY-BASED DESIGN WITH MAINTENANCE AND WARRANTY

In this section we develop three RDB models that include time-variant reliability, maintenance, and warranty.

3.1 Proposed RBD models

In Model I, failures must be controlled under an invariably low level. This model is for products whose failures may lead to catastrophic consequences. High inherent (initial) reliability must be designed into the product. Preventive maintenance actions should also be taken to prevent breakdowns and failures during operations. Typical preventive maintenance actions include systematic inspection, detection, and correction of incipient failures either before they occur or before they develop into major defects. After a failure, the product will be discarded. The model is given by

$$
\begin{aligned}
&\text{RBD model I} \\
&\min_{DV=(\mathbf{d},\boldsymbol{\mu}_{\mathbf{X}})} \; C = C_I\left[\mathbf{d},\mathbf{X}(t)\right]+[n]C_p \\
&\text{s.t. } \Pr\left\{g_i\left(\mathbf{d},\mathbf{X}(0),\mathbf{P}(0);0\right)\ge 0\right\}\ge [R_i] \qquad i=1,2,\dots,n_g \\
&\quad T_{[n]}=\alpha^{[n]}T_1\ge t_r \\
&\quad n=\ln\left[1-(1-\alpha)\frac{T}{T_1}\right]\Big/\ln\alpha \\
&\quad \mathbf{d}^L\le\mathbf{d}\le\mathbf{d}^U;\boldsymbol{\mu}_{\mathbf{X}}^L\le\boldsymbol{\mu}_{\mathbf{X}}\le\boldsymbol{\mu}_{\mathbf{X}}^U
\end{aligned}
\tag{6}
$$

T is the service life. n the number of preventive maintenance. During the optimization process, n is treated as a continuous value and is round to the nearest integer $[n]$ for the cost calculation. If n happens to be an integer, $[n]=n-1$ will be used. Therefore, $[n]$ is the actual number of preventive maintenance. In this paper, all the costs are average costs. The total cost C includes the initial cost C_I (design, development and production costs) and the preventive maintenance cost $[n]C_p$, where C_p is the cost per preventive maintenance.

The first constraint indicates that the initial reliability $\Pr\left\{g_i\left(\mathbf{d},\mathbf{X}(0),\mathbf{P}(0);0\right)\ge 0\right\}$ should be greater than or equal to the desired reliability $[R_i]$. The time-dependent reliability $\Pr\left\{g\left(\mathbf{d},\mathbf{X}(t),\mathbf{P}(t);t\right)\ge 0\right\}$ during operation does not explicitly appear in the RBD model; but it is used to predict the work time (uptime), T_j, between the (j-1)-th and

j-th preventive maintenance. The second constraint shows that the uptime $T_{[n]}$ should be greater than or equal to the desired uptime t_r.

The preventive maintenance takes place once the predicted reliability reaches a predetermined threshold [31]. In this work, we use the most common maintenance types: as-good-as-new (perfect maintenance), as-bad-as-old (minimal maintenance), and general (between old and new state). Then a coefficient $\alpha\left(0<\alpha<1\right)$ is assigned to describe the capacity of preventive maintenance [32] so that the two consecutive uptimes satisfy

$$T_j = \alpha T_{j-1} \tag{7}$$

And the total uptime is equal to

$$\sum_{j=1}^{n} T_j = \frac{1\text{-}\alpha^n}{1\text{-}\alpha} T_1 \tag{8}$$

For many commercial products, failures are unavoidable and are allowed if customer compensation, such as warranty, takes place in case of failures. Producers are interested in the extra revenue, which should exceed the warranty servicing cost. From the perspective of a producer, the second RBD model is proposed with the warranty consideration as follows:

$$\begin{aligned}
&\text{RBD model II} \\
&\min_{DV=\left(\mathbf{d},\boldsymbol{\mu}_{\mathbf{X}}\right)} \quad C = C_I\left[\mathbf{d},\mathbf{X}(t)\right] + C_W\left[\mathbf{d},\mathbf{X}(t)\right] \\
&s.t. \quad \Pr\left\{g_i\left(\mathbf{d},\mathbf{X}(0),\mathbf{P}(0);0\right)\geq 0\right\} \geq \left[R_i\right] \\
&\qquad\qquad \mathbf{d}^L \leq \mathbf{d} \leq \mathbf{d}^U ; \boldsymbol{\mu}_{\mathbf{X}}^L \leq \boldsymbol{\mu}_{\mathbf{X}} \leq \boldsymbol{\mu}_{\mathbf{X}}^U
\end{aligned} \tag{9}$$

In this model, the total cost, including the warranty servicing cost, is minimized. The inherent (initial) reliability is included as a constraint. Maintenance is also implicitly included in the warranty cost C_W . In addition to preventive maintenance, corrective maintenance is also involved. Corrective maintenance consists of the repair or replacement of the failed product. It is obvious that the higher is the reliability, the lower is the warranty servicing cost, and the higher is the initial cost. The calculation of warranty cost and corrective maintenance cost will be discussed in Sec. 3.4.

After warranty, customers have to repair or replace the failed product at their own expenses, where both preventive maintenance and corrective maintenance may be implemented. To maximize the lifecycle value of the product, manufacturers may also be interested in post-warranty maintenance. For this situation, from the perspectives of both manufacturer and customer, Monga and Zuo [33] have included the total cost incurred over the product service time in the objective function. For the same reason, we also propose RBD model III to incorporate the total cost during the product service time. The model is given by

$$\begin{aligned}
&\text{RBD model III}\\
&\min_{DV=(\mathbf{d},\boldsymbol{\mu}_{\mathbf{X}})} \quad C = C_I\left[\mathbf{d},\mathbf{X}(t)\right] + C_W\left[\mathbf{d},\mathbf{X}(t)\right] + C_{PW}\left[\mathbf{d},\mathbf{X}(t)\right]\\
&s.t. \quad \Pr\left\{g_i\left(\mathbf{d},\mathbf{X}(0),\mathbf{P}(0)\right)\geq 0\right\}\geq\left[R_i\right]\\
&\qquad T_1 \geq W\\
&\qquad \mathbf{d}^L \leq \mathbf{d} \leq \mathbf{d}^U ; \boldsymbol{\mu}_{\mathbf{X}}^L \leq \boldsymbol{\mu}_{\mathbf{X}} \leq \boldsymbol{\mu}_{\mathbf{X}}^U
\end{aligned} \tag{10}$$

The total cost includes the initial cost C_I , warranty cost C_W , and post-warranty maintenance cost C_{PW} . Since the first preventive maintenance occurs in the post-warranty period, the time to the first preventive maintenance, T_1, should be greater than or equal to the warranty period W .

Next we first present maintenance, warranty, and cost models that we borrow from reliability engineering, and then we develop numerical procedures for solving the three RBD models.

3.2 Maintenance model

Decisions on maintenance are generally based on reliability functions $R(t)$. When the product reliability reaches a predetermined critical threshold $[R]$, preventive maintenance takes place. The condition is given by

$$\Pr\left\{g\left(\mathbf{d}, \mathbf{X}(T_i), \mathbf{P}(T_i); T_i\right) \geq 0\right\} = [R] \tag{11}$$

where T_i is the time to the *i*-th preventive maintenance. This preventive maintenance is included in RBD Model I.

RBD Models II and III involve warranty. During the warranty period, corrective maintenance takes place in the event of failure. The time-dependent reliability is shown in Fig. 1, where T_1 is the time to the first preventive maintenance, and W is the warranty period. RBD Model II covers the warranty period while RBD Model III covers both the warranty and the post-warranty periods.

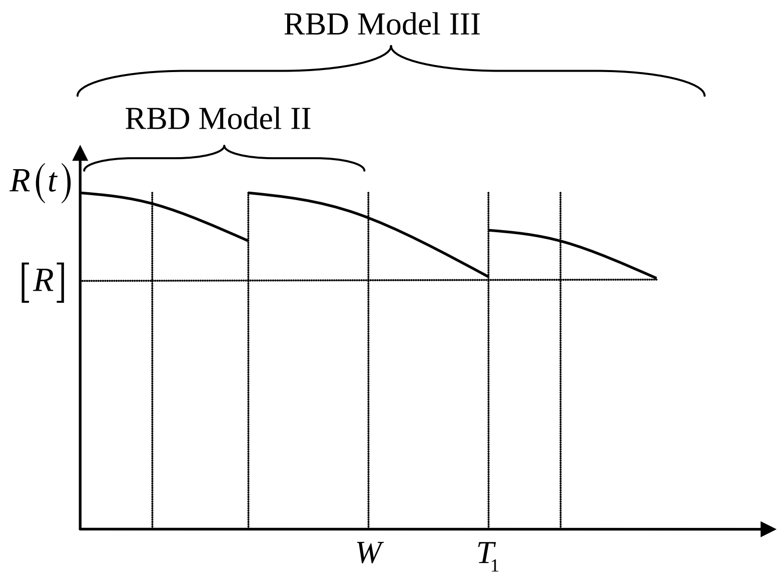

Fig. 1 The reliability curves for RBD models II and III

After warranty expires, products enter into post-warranty period, and preventive maintenance and the minimal maintenance may be performed. When the time to failure T_f is greater than the time to preventive maintenance T_p, the preventive maintenance takes place; otherwise, the minimal corrective maintenance is performed. This procedure is shown in Fig. 2.

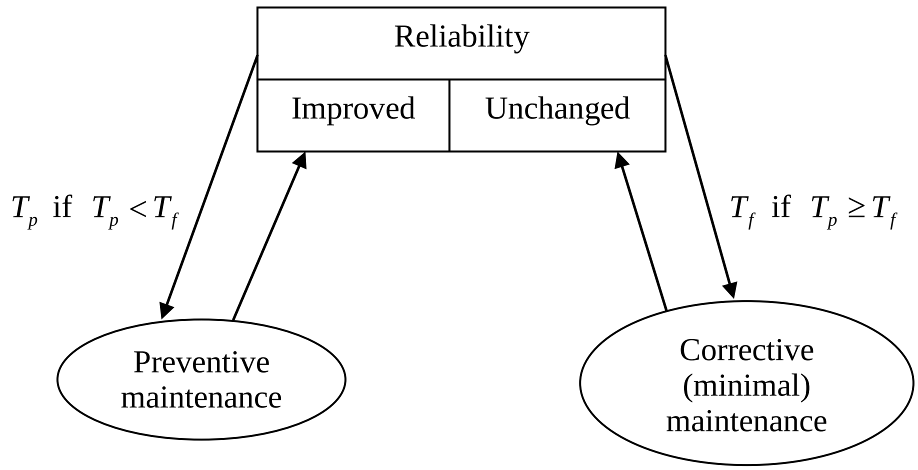

Fig. 2 Two types of maintenance during post-warranty period

The imperfect preventive maintenance restores the product uptime to $\alpha\left(0<\alpha<1\right)$ times that before the maintenance. The expected number of failures during $\left[W,T_1\right]$ is given by [33]

$$\begin{aligned} N_1 &= \int_0^{T_1-W} r\left(t\right)dt = -\int_0^{T_1-W} \frac{R'\left(W+t\right)}{R\left(W+t\right)}dt \\ &= -\ln\left[R\left(T_1\right)\right] + \ln\left[R\left(W\right)\right] \end{aligned} \tag{12}$$

where $r\left(t\right) = -R'\left(W+t\right)/R\left(W+t\right)$ is the failure rate and $R'\left(W+t\right) = dR\left(W+t\right)/dt$. The expected number of failures between the (*i*-1)-th and *i*-th preventive maintenance is given by

$$N_i = -\int_0^{T_i-T_{i-1}} \frac{R'\left(T_{i-1}+t\right)}{R\left(T_{i-1}+t\right)}dt = -\ln\left[R\left(T_{i-1}\right)\right] + \ln\left[R\left(\alpha T_{i-1}\right)\right] \tag{13}$$

where i is an integer and $i>1$. Then the total expected number of failures during the post-warranty period is

$$N(PW) = \ln\left[\frac{R(W)}{R(\alpha^{n-1}T_1)}\right] \tag{14}$$

where n is the number of preventive maintenance during the post-warranty period.

3.3 Warranty model.

In this work, we consider the following two types of failure, which are introduced in [34].

(1) type I: Failures is removed by the perfect maintenance or replacement

(2) type II: Failures is removed by the minimal maintenance

The perfect maintenance restores a product to an as-good-as-new state. The minimal maintenance restores a product to an as-bad-as-old state; in other words, the failure rate of a product is not disturbed after the minimal maintenance.

The two key variables in the warranty model are the cost per failure and the expected number of failures. The cost per failure includes maintenance cost, transportation cost, and service cost. This unit cost is not related to the product quality. The expected number of failures, however, is mainly determined by reliability. For the warranty model, the expected number of type I failures, $m_1(W)$, and the expected number of type II failures, $m_2(W)$, during the warranty period, should be provided.

In the traditional warranty model, $m_1(W)$ and $m_2(W)$ are usually obtained from field data. In this work, we use the time-dependent reliability obtained from computational models. $m_1(W)$ is given by the following renewal equation [35]:

$$m_1(W)=G(W)+\int_0^W m_1(W-T)dG(T) \tag{15}$$

where $G(T)$ is the CDF of the product lifetime and is given by the following equation:

$$G(T)=1-\exp\left\{-\int_0^T p_{\mathrm{I}}(t)r(t)\,dt\right\} \tag{16}$$

where $p_{\mathrm{I}}(t)$ is the probability of type I failure at time t.

$m_2(W)$ is given by [36]

$$m_2(W)=\int_0^W p_{\mathrm{II}}(t)r(t)\,dt \tag{17}$$

where $p_{\mathrm{II}}(t)=1-p_{\mathrm{I}}(t)$ is the probability of type II failure at time t. With the relationship between the reliability and failure rate, Eqs. (16) and (17) can be rewritten as

$$G(T)=1-\exp\left\{\int_0^T p_{\mathrm{I}}(t)\frac{dR(t)}{R(t)}\right\} \tag{18}$$

and

$$m_2(W)=-\int_0^W p_{\mathrm{II}}(t)\frac{dR(t)}{R(t)} \tag{19}$$

respectively. The reliability $R(t)$ in the equations can be computationally evaluated by the FORM in this work.

For the special case when $p_{\mathrm{I}}(t)=p_{\mathrm{I}}=\text{constant}$, Eqs. (18) and (19) become

$$G(T)=1-\left[R(t)\right]^{p_{\mathrm{I}}} \tag{20}$$

and

$$m_2(W)=-p_{\mathrm{II}}\ln R(t) \tag{21}$$

3.4 Cost model

The total cost is minimized. In RBD Model I, the total cost during the service life is the sum of the initial cost and preventive maintenance cost and is given by

$$C = C_I + [n] C_p \tag{22}$$

The initial cost C_I includes the design, development, and production costs, which are a function of design variables. The preventive maintenance cost is the product of the actual number of preventive maintenance, $[n]$, and the cost per maintenance, C_p. In RBD Model II, the total cost during the warranty period is the sum of the initial cost and warranty cost; namely

$$C = C_I + c_1 m_1(W) + c_2 m_2(W) \tag{23}$$

where $c_1 m_1(W) + c_2 m_2(W)$ is the warranty cost. It includes the repair or replacement cost, transportation cost, and service cost.

In RBD Model III, the total cost during the service life includes the initial cost, warranty cost, and post-warranty maintenance cost. The total cost is given by

$$C = C_I + c_1 m_1(W) + c_2 m_2(W) + c_p n(PW) + c_2 N(PW) \tag{24}$$

where $n(PW)$ and $N(PW)$ are the numbers of preventive maintenance and failures during the post-warranty period, respectively; c_p and c_2 are the cost per preventive maintenance and the cost per corrective maintenance during the post-warranty period, respectively. $c_p n(PW) + c_2 N(PW)$ is therefore the total cost incurred by the post-warranty maintenance. Details about c_1, c_2 and c_p are given in [37].

4. NUMERICAL PROCEDURE

We now develop numerical procedures for solving the three RBD models.

4.1 RBD Model I

In RBD Model I, when the product reliability reaches a threshold $[R_i]$, preventive maintenance takes place. The condition is given by

$$\Pr\left\{g_i\left(\mathbf{d}, \mathbf{X}(t), \mathbf{P}(t); t\right) \geq 0\right\} = \Pr\left\{g_i\left(\mathbf{d}, \mathbf{U}(t); t\right) \geq 0\right\} = [R_i] \tag{25}$$

where $\mathbf{U}(t)$ is the vector of standard normal variables transformed from $\mathbf{X}(t)$ and $\mathbf{P}(t)$.

The solution is the time to the *j*-th preventive maintenance; namely

$$T_j = \left\{t : \Pr\left\{g_i\left(\mathbf{d}, \mathbf{U}(t); t\right) \geq 0\right\} = [R_i]\right\} \tag{26}$$

If the FORM is used, Eqs. (25) and (26) are equivalent to

$$\beta_i(T_j) = \Phi^{-1}\left([R_i]\right) \tag{27}$$

where $\beta_i\left(T_j\right) = \Phi^{-1}\left(\Pr\left\{g_i\left(\mathbf{d}, \mathbf{U}(t); t\right) \geq 0\right\}\right)$, which is the reliability index at T_j. The maintenance analysis can then be formulated as

$$\begin{cases} \beta_i(t) = \Phi^{-1}\left(\Pr\left\{g_i\left(\mathbf{d}, \mathbf{u}^*(t); t\right) \ge 0\right\}\right) = \Phi^{-1}\left(\left[R_i\right]\right) \\ \beta_i(t) = \overline{\mathbf{u}}^*(t) \\ \mathbf{u}^*(t) = \left\{\mathbf{u} : \min \overline{\mathbf{u}} \middle| g_i\left(\mathbf{u}(t); t\right) = 0\right\} \end{cases} \tag{28}$$

To solve the first equation in Eq. (28), an iterative process is required. At each intermediate point t during the process, the MPP $\mathbf{u}^*(t)$ must be identified. We then propose the following procedure for the maintenance analysis:

$$\begin{cases} \text{Outer loop: } t = \left\{t : \Pr\left\{g_i\left(\mathbf{d}, \mathbf{u}^*(t); t\right) \ge 0\right\} = \left[R_i\right]\right\} \\ \text{Inner loop: } \mathbf{u}^*(t) = \left\{\mathbf{u} : \min \overline{\mathbf{u}} \middle| g_i\left(\mathbf{d}, \mathbf{u}(t); t\right) = 0\right\} \end{cases} \tag{29}$$

Solving RBD Model I involves a triple-loop procedure as shown in Fig. 3. The outer loop is the overall RBD where the double loop procedure for Eq. (29) is embedded.

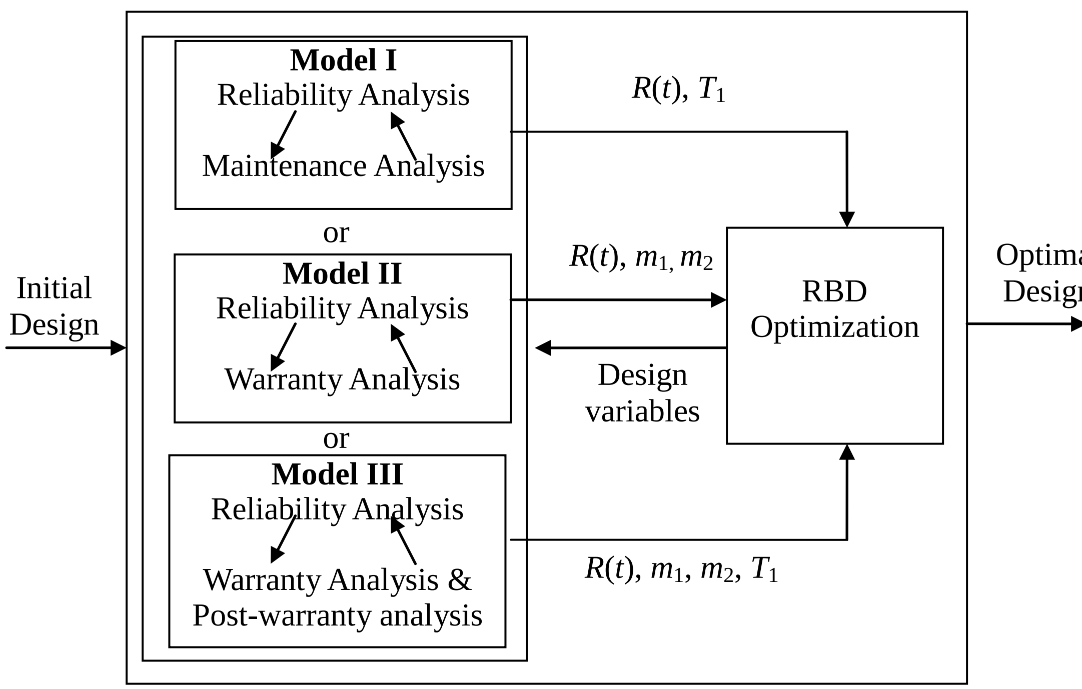

Fig. 3 Triple-loop procedure for RBD Models I, II, and III

4.2 RBD Model II

The initial reliability requirement is treated as a constraint. The reliability can be calculated by the FORM when $t=0$. As shown in Eq. (23), the expected number of perfect maintenance $m_1(W)$ in Eq. (15) must be computed for the warranty cost in the objective function. Since $m_1(W)$ is an implicit function of the reliability, it is difficult, or even impossible, to obtain a closed-form solution to $m_1(W)$. A numerical method is therefore used, and $m_1(W)$ can be computed iteratively by [38]

$$m_1(t_i)=\frac{\left[G(t_i)+S_i-G\left(t_i-\frac{t_i-t_{i-1}}{2}\right)m(t_{i-1})\right]}{1-G\left(t_i-\frac{t_i-t_{i-1}}{2}\right)} \tag{30}$$

where

$$S_i=\sum_{j=1}^{i-1}G\left(t_i-\frac{t_i-t_{i-1}}{2}\right)\left[m(t_j)-m(t_{j-1})\right] \tag{31}$$

As shown in Eq. (18), $G(t_i)=1-\sum_{j=1}^{i}\exp\left\{\int_{t_{j-1}}^{t_j}p_{\mathrm{I}}(t)\frac{dR(t)}{R(t)}\right\}\quad(j=1,2,\ldots,i)$. The FORM is called repeatedly to evaluate the integration in Eq. (30). The warranty analysis therefore involves a double-loop procedure, where warranty analysis and reliability analysis are nested.

Similarly to RBD Model I, a triple-loop procedure is required to solve RBD Model II (Fig. 3). The time step size t_i-t_{i-1} in Eq. (31) affects both of accuracy and efficiency.

One could determine the step size by the following strategy: At first, find out how many reliability analyses one can afford, and then set the step size $t_i - t_{i-1}$ equal to the product lifetime divided by the number of reliability analyses.

4.3 RBD Model III

RBD Model III includes both of warranty and post-warranty periods. During warranty period, corrective maintenance is performed when failures occur. Then we use Eqs. (17) and (30) to obtain the expected number of minimal maintenance and perfect maintenance. During post-warranty period, the minimal maintenance is performed when failures occur, and the imperfect preventive maintenance takes place when reliability reaches a required level. The expected number of minimal maintenance, $N\left(PW\right)$, is given in Eq. (14). The expected number of preventive maintenance, $n\left(PW\right)$, is obtained from RBD model I. The numerical procedure is given in the above subsection, and the triple-loop procedure is provided in Fig. 3. Since preventive maintenance takes place in the post-warranty period, $T_1 \geq W$ should be satisfied.

5. EXAMPLES

RBD Model I is applied to Example One while RBD Models II and III are used for Example Two.

5.1 Pressure tank design

Fig. 4 shows a pressure tank, whose leakage may lead to catastrophic consequences. High initial reliability and reliability-centered preventive maintenance are therefore required. We hence use RBD Model I. In Fig. 4, h is the thickness, H is the radius, L is the height, and P_b is the bursting pressure of the tank. If the hoop stress exceeds the ultimate strength, the tank is considered not functioning; therefore

$$g(t) = S_U - \frac{P_b H}{rh(t)}\left(1 - \frac{H^2}{2L^2}\right)$$

where S_U is the material ultimate strength, and r is the ratio of bursting pressure to the internal pressure. The thickness decreases with time in a stochastic manner due to corrosion. It is given by $h(t) = h_0 - 3.4\times10^{-2} t^{0.65}$, where h_0 is the random initial thickness [39]. The distributions of random variables are given in Table 1, where COV is the coefficient of variation.

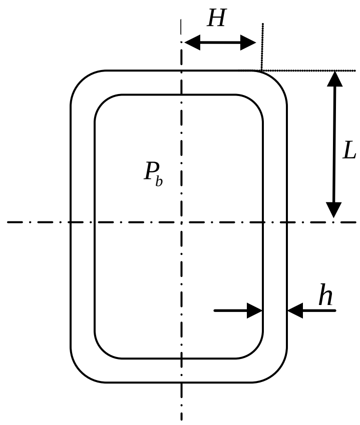

Fig. 4 The pressure tank

Table 1 Distributions of stochastic variables

Variables		Mean	COV	Distribution
X	H (cm)	μ_H	0.01	Normal
	L (cm)	μ_L	0.01	Normal
	h_0 (cm)	μ_{h_0}	0.0377	Normal
P	S_U (MPa)	387.0	0.05	Normal
	P_b (MPa)	14.495	0.1	Normal

The initial cost is assumed to be directly proportional to the volume of the pressure tank and is given by $C_I = 2\pi\rho(2\mu_{h_0}\mu_H\mu_L - 2\mu_{h_0}^2\mu_H - \mu_{h0}^2\mu_L + \mu_{h_0}^3 + \mu_{h_0}\mu_H^2)$, where $\rho = \$5\times10^{-2}/\text{cm}^3$. The maintenance cost C_{PM} is the product of the actual number of preventive maintenance, $\left[n\right]$, and the cost per preventive maintenance $C_P = \$1000$; namely, $C_{PM} = \left[n\right]C_P$.

The design model is then given by

$$\begin{aligned}
\min_{\mu_{h_0},\mu_H,\mu_L} \quad & C = C_I + \left[n\right]C_P \\
s.t. \quad & \Pr\left\{g\left(\mathbf{d},\mathbf{X}(0),\mathbf{P}(0);0\right) \geq 0\right\} \geq 0.99997 \\
& T_n = \alpha^n T_1 \geq t_r \\
& n = \ln\left[1-(1-\alpha)T/T\right] \big/ \ln\alpha \\
& 1.5 \leq \mu_{h_0} \leq 3.8,\ 142.65 \leq \mu_H \leq 174.35, \\
& 213.975 \leq \mu_L \leq 261.524
\end{aligned}$$

where α is the indicator of the capacity of the preventive maintenance. Preventive maintenance for this problem may include electroplating, painting, and so on. T_1 is the time to the first preventive maintenance. T is the desired service life of the product and $T = 60$ months. t_r is the allowable minimal uptime after the preventive maintenance and

$t_r = 3$ months. The first constraint indicates that the initial reliability should not be less than 0.99997; and the second constraint indicates that the minimum uptime after maintenance should not be less than t_r. When reliability decreases to the critical threshold $[R] = 0.999$, preventive maintenance is performed. The optimal designs with different α values are provided in Table 2.

Table 2 Design results for pressure tank

α	μ_{h_0} (mm)	μ_L (mm)	μ_H (mm)	n	T_1 (month)	C ($)	C_I/C_P	Function Calls
0.7	3.21	142.65	213.98	3.39	25.65	83,452	26.95	104,141
0.8	3.20	142.65	213.98	3.03	24.41	83,236	26.76	116,920
0.9	3.17	142.65	213.98	3.26	20.65	82,547	26.6	116,399

When the capacity of the preventive maintenance α increases, the initial thickness of the tank decreases, the preventive maintenance period becomes shorter, the total cost becomes slightly smaller, and the ratio of the initial cost over the maintenance cost C_I/C_P decreases. The number of preventive maintenance n is a real number and its rounded value $[n]$ is used to calculate the costs C and C_I/C_P. For $\alpha = 0.7$, $n = 3.39$, and $[n] = 3$. Therefore preventive maintenance needs to be performed three times. The first time is $T_1 = 25.65$ months; the second time is $T_2 = \alpha T_1 = 0.7(25.65) = 17.96$ months after the first maintenance. The third time is $T_3 = \alpha T_2 = 0.7(17.96) = 12.57$ months after the second maintenance. After the third maintenance, the tank could continue to work for $60 - (T_1 + T_2 + T_3) = 60 - (25.65 + 17.96 + 12.57) = 3.83$ months. Solving the problem is inefficient given the high numbers of function calls shown in Table 2.

The reliability function with $\alpha = 0.7$ is plotted in Fig. 5, which shows that reliability during the lifetime is always greater than or equal to 0.999. The occurrence time of preventive maintenance is also shown in the figure.

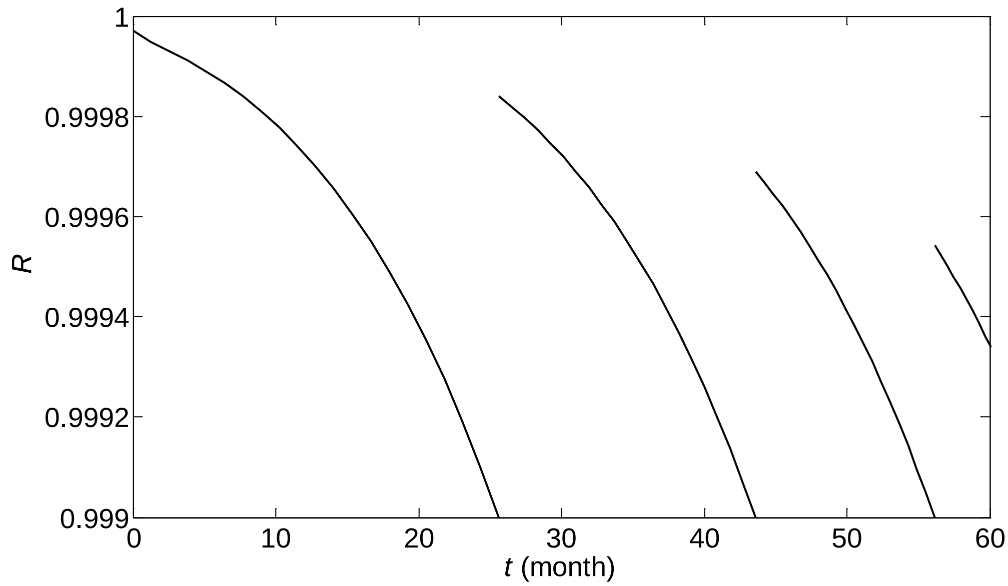

Fig. 5 Reliability function $R(t)$

5.2 Exposed single helical gear reducer design

In this example (Fig. 6), RBD Models II and III are employed because warranty is considered.

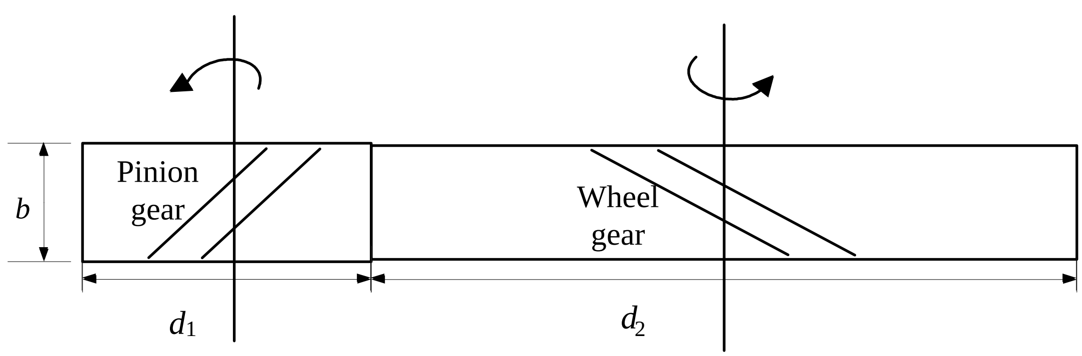

Fig. 6 A single helical gear reducer

One failure mode is wear, for which maintenance actions include adjust conversion, pile welding, inlays tooth, replacement, and so on. When the maximum amount of wear exceeds a threshold W_m, a failure might occur [40]; therefore,

$$g_1(\mathbf{d},\mathbf{X},\mathbf{P};t) = W_m - 4I_n r_1 st\sqrt{\frac{2\times 9.55\times 10^6 P}{\pi b\Psi m_n^2 z_1 \cos\alpha\cos\beta s}} \times\sqrt{\left(\frac{1-v^2}{E_1}+\frac{1-v^2}{E_2}\right)\frac{\rho_1\rho_2}{\rho_1+\rho_2}}$$

The second limit-state function indicates the difference between the allowable fatigue stress and the gear contact stress:

$$g_2(\mathbf{d},\mathbf{X},\mathbf{P}) = \sigma_{H\min} Z_N - Z_E Z_H\sqrt{\frac{2000\times 9.55P}{d_1{}^2 bs\cos\beta}\frac{u+1}{u}K_A}$$

where $Z_H = 2.25$, $K_A = 1.45$, $Z_N = 0.87$, and $u = 4$.

The third limit-state function is defined by the difference between the allowable bending stress and the bending stress:

$$g_3(\mathbf{d},\mathbf{X},\mathbf{P}) = \sigma_{F\min} Y_{ST} - \frac{2000\times 9.55P}{d_1 b m_n s\cos\beta}Y_F$$

where $Y_F = 1.98$ and $Y_{ST} = 2.32$.

The symbols in the three limit state functions are given below. I_n: wear rate; r_1: sliding coefficient of the gear and $r_1 = 1-\frac{\rho_1 z_2}{\rho_2 z_1}$; s: speed of pinion gear; t: working time; b: face width; Ψ: face width coefficient; m_n: normal module; z_1: number of pinion teeth; z_2: number of wheel teeth; d_1: reference diameter; P: input power; α: pressure angle; β: helix angle; E_1: elastic modules of the pinion gear; E_2: elastic modules of the wheel gear; v: Possion ratio; ρ_1: radius of curvature of the pinion gear; ρ_2: radius of curvature of the wheel gear; Z_E: coefficient of elasticity ; Z_H: geometry factor; K_A: load factor; u: transmission ratio; Z_N: life factor; Y_F: form factor; Y_{ST}: stress correction index; $\sigma_{H\min}$: allowable fatigue stress; $\sigma_{F\min}$: allowable bending stress.

The first limit state function is time dependent and is used to calculate time-dependent reliability. The second and third limit-state functions are time independent and are used to calculate the initial reliability. The input information is given in Table 3.

Table 3 Design variables and parameters

Variables	Variables	Mean	Std	Distribution
d	z_1	---	---	---
	m_n (mm)	---	---	---
X	b (mm)	μ_b	0.05	Normal
	β (degree)	μ_β	0.05	Normal
P	P (kw)	2000	200	Normal
	s (rpm)	1000	100	Normal
	Z_E ($\sqrt{\text{MPa}}$)	189.8	18.98	Normal
	$\sigma_{H\min}$ (MPa)	1400	140	Normal
	$\sigma_{F\min}$ (MPa)	480	48	Normal
	E_1 (MPa)	193.9	19.39	Normal
	E_2 (MPa)	159.8	15.98	Normal

Two cases are studied. The first case involves no post-warranty maintenance while the second case does. Therefore, RBD Model II is used for Case 1, and RBD Model III is used for Case 2. The initial cost C_I is directly proportional to the volume of the two gears in both cases; and the proportionality coefficient is $c = \$1\times10^{-4}/\text{mm}^3$. The warranty costs corresponding to type I and type II failures are $c_1 m_1(W)$ and $c_2 m_2(W)$, respectively, where $c_1 = \$6000$ and $c_2 = \$500$. The warranty period W is predetermined, and $W = 60$ months. RBD Model II for Case 1 is given by

$$\begin{aligned}
&\min\ f = c\frac{\pi m_n^2}{4}\left[z_1^2 + \left(uz_1\right)^2\right]\mu_b + c_1 m_1\left(W\right) + c_2 m_2\left(W\right) \\
&s.t.\ \ \Pr\left\{g_i\left(\mathbf{d}, \mathbf{X}, \mathbf{P}; 0\right) \ge 0\right\} \ge 0.999,\ i = 1, 2, 3 \\
&\qquad 0.3 \le \mu_b / d_1 \le 0.7 \\
&\qquad \mu_b \sin\left(\mu_\beta / \pi m_n\right) \ge 1 \\
&\qquad 17 \le z_1 \le 40; 2 \le m_n \le 10;\ 8^\circ \le \mu_\beta \le 16^\circ;\ 100 \le \mu_b \le 240
\end{aligned}$$

We assume that p and α are time-independent. In order to demonstrate the impact of p and α on the final design, the optimal design results for different scenarios of types I and II failures are given in Table 4.

Table 4 Design results for the exposed single helical gear reducer

p	z_1	m_n (mm)	μ_b (mm)	μ_β (degree)	$m_1(W)$	$m_2(W)$	C (\$)	$R(0)$	Function Calls
1	36	5.95	150.65	8.00	0.540	0	5,978.2	0.9998	195,752
0.9	36	5.94	150.03	8.00	0.508	0.079	5,793.9	0.9997	191,550
0.8	35	5.98	148.90	8.00	0.477	0.162	5,590.5	0.9997	170,540
0	32	5.82	131.30	8.00	0	1.235	2,431.5	0.9990	146,490

$R(0)$ is the initial system reliability. When $p_I = 1$, only type I failures occur, and then replacement (perfect maintenance) is applied. The expected number of perfect maintenance $m_1(W)$ is therefore the largest. When $p_I = 0$, only type II failures take place, and then the minimal maintenance is applied. The expected number of minimal maintenance $m_2(W)$ is therefore the largest. When $0 < p_I < 1$, both perfect maintenance and minimal maintenance are implemented. The minimal maintenance tasks for this problem include conversion adjustment, pile welding, and tooth inlay. The result also indicates that the total cost and required initial reliability decrease as the probability of type I failures decreases.

In Case 2, the post-warranty maintenance is considered. RBD Model III is therefore used, and the total cost is the total cost in Case 1 plus the post-warranty maintenance cost. The RBD model is given by

$$
\begin{aligned}
\min_{m_n, z_1, \mu_b, \mu_\beta} \; & f = \frac{1}{4}\pi c m_n^2 \left[z_1^2 + \left(u z_1 \right)^2 \right] \mu_b \\
& \quad + c_1 m_1 \left(W \right) + c_2 m_2 \left(W \right) + c_2 N \left(PW \right) + n c_3 \\
s.t. \; & \Pr\left\{ g_{1,2} \left(\mathbf{d}, \mathbf{X}, \mathbf{P}; 0 \right) \geq 0 \right\} \geq 0.999 \\
& \Pr\left\{ g_3 \left(0 \right) \left(\mathbf{d}, \mathbf{X}, \mathbf{P} \right) \geq 0 \right\} \geq 0.999 \\
& 0.3 \leq \mu_b / d_1 \leq 0.7 \\
& \mu_b \sin \mu_\beta / \pi m_n \geq 1 \\
& T_1 \geq W \\
& n = \ln\left[1 - \left(1 - \alpha \right) \left(P + PW \right) / T_1 \right] / \ln \alpha \\
& T_n = \alpha^n T_1 \geq t_r \\
& 17 \leq z_1 \leq 40, 2 \leq m_n \leq 10, 8^\circ \leq \mu_\beta \leq 16^\circ, 100 \leq \mu_b \leq 240
\end{aligned}
$$

where $c_3 = \$600$ and $PW = 60$. The preventive maintenance is performed when

$$
\Pr\left\{ g_1 \left(\mathbf{d}, \mathbf{X}\left(T_i \right), \mathbf{P}\left(T_i \right); T_i \right) \geq 0 \right\} = 0.975
$$

where T_i is the time to the *i*-th preventive maintenance. The optimal design for $p_I = 1$ is given in Table 5. The reliability function for $p_I = 1$, $\alpha = 0.7$ is plotted in Fig. 7.

Table 5 Results with $p_I = 1$

	z_1	m_n (mm)	μ_b (mm)	μ_β (degree)	T_1 (month)	n	C ($)	Function Calls
$\alpha = 0.9$	38	5.41	145.73	8.00	60.00	2.12	6,022.5	1,711,906
$\alpha = 0.8$	36	5.82	145.99	8.00	60.35	2.27	6,176.5	1,043,380
$\alpha = 0.7$	40	5.34	149.61	8.00	65.31	2.25	6,380.8	1,338,531

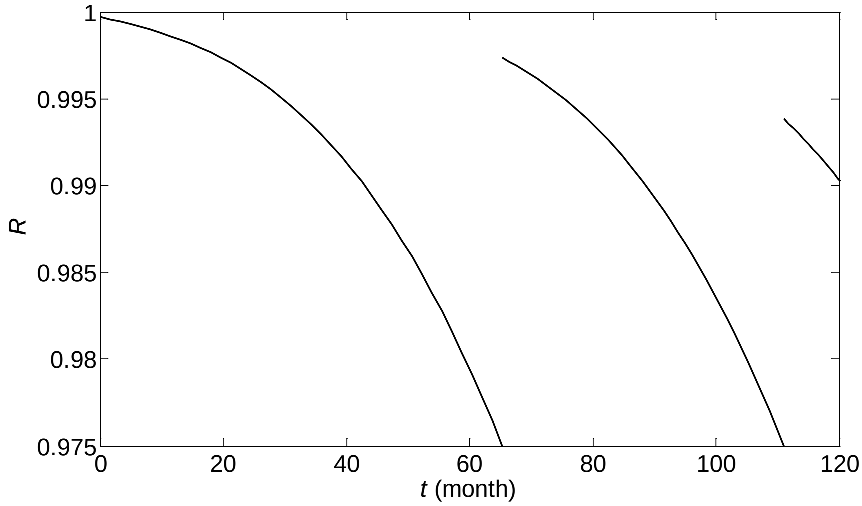

Fig. 7 $R(t)$ when $p_I = 1$ and $\alpha = 0.7$

As shown, the number of pinion teeth, normal module, face width, time to the first preventive maintenance, and number of preventive maintenance are not monotonic with respect to α. However, the total cost increases when α decreases. The total cost is also determined by the probability of the type I failure p_I. The total cost increases with the increase of p_I.

6. CONCLUDING REMARKS

This work is a preliminary study to show the feasibility of accounting for maintenance and warranty in reliability-based design (RBD). Three RBD models are proposed for the following situations: (1) highly reliable products with preventive maintenance, (2) warranty with corrective maintenance, and (3) warranty with post-warranty maintenance. The key to the new RBD models is the direct connection of design variables with reliability, warranty, and maintenance. The base of such connection is the time-dependent reliability function that is evaluated through computational models. The First Order Reliability Method (FORM) is used for the reliability analysis in this work.

This work is a starting point of extending RBD to design for lifecycle reliability. The following challenges should be addressed before the new RBD methodology can be confidently used. (1) Efficient algorithms are desired. As shown in Section 4, an expensive triple-loop procedure is required to solve the RBD models. (2) System reliability should be considered with multiple failure modes and multiple limit-state functions. (3) The physics-based reliability methods should be integrated with the empirical reliability methods when it is impossible to estimate the reliability function based on only computational models. (4) More advanced time-dependent reliability analysis methods are needed.

ACKNOWLEDGEMENT

This work was performed when the first author stayed at the Missouri University of Science and Technology (MST) as a visiting scholar, partially sponsored by the China Scholarship Council. This work is also partially supported by the University of Missouri Research Board grant 7116, the Intelligent Systems Center at MST, the National Natural Science Foundation of China under contract number 50775026, and the 111 project of China under grant number B08043.

References

[1] Blischke, W., and Murthy, D. N. P., 2000, Reliability Modeling, Prediction, and Optimization, pp. 3-6, John Wiley and Sons Inc., New York.

[2] Liu, Y., Huang H. Z., 2009, "Comment on ‘‘A Framework to Practical Predictive Maintenance Modeling for Multi-state Systems’’ by Tan C.M. and Raghavan N. [Reliab Eng Syst Saf 2008; 93(8): 1138–50]," Reliability Engineering and System Safety, 2009, 95(3), pp.776-780.

[3] Mahadevan, S., 1997, "Physics-Based Reliability Models," in Reliability-Based Mechanical Design, Cruse, T. A., ed., pp. 197-232, Dekker, New York.

[4] Tu, J., Choi, K. K., and Park, Y. H., 1999, "A New Study on Reliability-Based Design Optimization," Journal of Mechanical Design, 121(4), pp. 557-564.

[5] Youn, B. D., Choi, K. K., and Du, L., 2005, "Enriched Performance Measure Approach for Reliability-Based Design Optimization," AIAA Journal, 43(4), pp. 874-884.

[6] Yang, R. J., and Gu, L., 2004, "Experience with Approximate Reliability-Based Optimization Methods," Structural and Multidisciplinary Optimization, 26(1), pp. 152-159.

[7] Wang, L. P., and Kodiyalam, S., 2002, "An Efficient Method for Probabilistic and Robust Design with Non-Normal Distributions," 43rd AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference, Denver, Colorado, Apr. 22-25.

[8] Liang, J. H., Mourelatos, Z. P., and Nikolaidis, E., 2007, "A Single-Loop Approach for System Reliability-Based Design Optimization," Journal of Mechanical Design, 129(12), pp. 1215-1224

[9] Du, X., Sudjianto, A., and Huang, B. Q., 2005, "Reliability-Based Design with the Mixture of Random and Interval Variables," Journal of Mechanical Design, 127(6), pp.1068-1076.

[10] Wu, Y. T., Shin, Y., Sues, R., and Cesare, M., 2001, "Safety-Factor Based Approach for Probabilistic-Based Design Optimization," 42nd AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics and Materials Conference and Exhibit, Seattle, Washington, Apr. 16-19.

[11] Wu, Y. T., and Wang, W., 1998, "Efficient Probabilistic Design by Converting Reliability Constraints to Approximately Equivalent Deterministic Constraints," Journal of Integrated Design and Process Sciences, 2(4), pp. 13-21.

[12] Du, X., and Chen, W., 2004, "Sequential Optimization and Reliability Assessment for Probabilistic Design," Journal of Mechanical Design, 126(2), pp. 99-116.

[13] Du, X., 2008, "Saddlepoint Approximation for Sequential Optimization and Reliability Analysis," Journal of Mechanical Design, 130(1), pp. 011011-011022.

[14] Du, X., Guo, J., and Beeram, H., 2007, "Sequential Optimization and Reliability Assessment for Multidisciplinary Systems Design," Structural and Multidisciplinary Optimization, 35(2), pp. 117-130.

[15] McDonald, M., and Mahadevan, S., 2008, "Design Optimization with System-Level Reliability Constraints," Journal of Mechanical Design, 130(2), doi: 10.1115/1.2813782.

[16] Kokkolaras, M., Mourelatos, Z. P., and Papalambros, P. Y., 2006, "Design Optimization of Hierarchically Decomposed Multilevel Systems under Uncertainty," Journal of Mechanical Design, 128(2), pp. 503-508.

[17] Gunawan, S., and Papalambros, P. Y., 2007, "Reliability Optimization with Mixed Continuous-Discrete Random Variables and Parameters," Journal of Mechanical Design, 129(2), pp. 158-165.

[18] Gunawan, S., and Papalambros, P. Y., 2005, “Reliability Optimization Involving Mixed Continuous-Discrete Uncertainties,” International Design Engineering Technical Conferences and Computers and Information in Engineering Conference (IDETC/CIE), Long Beach, California, Sept. 24-28.

[19] Kuschel, N., and Rackwitz, R., 2000, “Optimal Design under Time-Variant Reliability Constraints,” Structural Safety, 22(2), pp. 113-127.

[20] Streicher, H., and Rackwitz, R., 2004, “Time-Variant Reliability-Oriented Structural Optimization and a Renewal Model for Life-Cycle Costing,” Probabilistic Engineering Mechanics, 19(1), pp. 171-183.

[21] Pascual, R., Meruane, V., and Rey, P. A., 2008, “On the Effect of Downtime and Budget Constraint on Preventive and Replacement Policies,” Reliability Engineering and System Safety, 93(1), pp. 144-151.

[22] Chukova, S., Arnold, R., and Wang, D. Q., 2004, “Warranty Analysis: An Approach to Modeling Imperfect Repairs,” International Journal of Production Economics, 89(1), pp. 57-68.

[23] Wu, C.-C., Chou, C.-Y., and Huang, C., 2007, “Optimal Burn-In Time and Warranty Length under Fully Renewing Combination Free Replacement and Pro-Rata Warranty,” Reliability Engineering and System Safety, 92(7), pp. 914-920.

[24] Murthy, D. N. P., and Djamaludin, I., 2002, “New Product Warranty: A Literature Review,” International Journal of Production Economics, 79(3), pp. 231-260.

[25] Murthy, D. N. P., 2006, “Product Warranty and Reliability,” Annals of Operations Research, 143, pp. 133-146.

[26] Vittal, S., and Hajela, P., 2004, “Probabilistic Design and Optimization of Aging Structures,” 10th AIAA/ISSMO Multidisciplinary Analysis and Optimization, New York, Aug. 30 - Sept. 1.

[27] Frangopol, D. M., and Maute, K., 2003, “Life-Cycle Reliability-Based Optimization of Civil and Aerospace Structures,” Computers & Structures, 81(7), pp. 397-410.

[28] Biondini, F., and Frangopol, D. M., 2008, “Probabilistic Limit Analysis and Lifetime Prediction of Concrete Structures,” Structure and Infrastructure Engineering, 4(5), pp. 399-412.

[29] Kong, J. S., and Frangogol, D. M., 2005, “Probabilistic Optimization of Aging Structures Considering Maintenance and Failure Costs,” Journal of Structural Engineering, 131(4), pp. 600-616.

[30] Wang, Z. L., Du, X., and Huang, H.-Z., 2008, “Reliability-Based Lifecycle Optimization with Maintenance Consideration,” *14th ISSAT Conference on Reliability and Quality in Design*, Orlando, Aug. 7-9.

[31] Abdul-Nour, G., Beaudoin, H., Ouellet, P., Rochette, R., and Lambert, S., 1998, “A Reliability-Based Maintenance Policy: A Case Study,” Computers & Industrial Engineering, 35(3), pp. 591-594.

[32] Wang, H. Z., and Pham, H., 1999, “Some Maintenance Models and Availability with Imperfect Maintenance in Production Systems,” Annals of Operations Research, 91(0), pp. 305-318.

[33] Monga, A., and Zuo, M. J., 1998, “Optimal System Design Considering Maintenance and Warranty,” Computers and Operations Research, 25(9), pp. 691-705.

[34] Nguyen, D. G., and Murthy, D. N. P., 1984, “A General Model for Estimating Warranty Costs for Repairable Products,” IIE Transactions, 16 (4), pp. 379-386.

[35] Ross, S. M., 1992, Applied Probability Models with Optimization Application, Courier Dover Publications, New York, pp. 34-36.

[36] Huang, H.- Z., Liu, Z. J., and Murthy, D. N. P., 2007, “Optimal Reliability, Warranty and Price for New Products,” IIE Transactions, 39 (8), pp. 819-827.

[37] Asiedu, Y., Gu, P., 1998, “Product Life Cycle Cost Analysis: State of the Art Review,” International Journal of Production Research, 36(4), pp. 883-908.

[38] Xie, M., 1989, “On the Solution of Renewal-Type Integral Equations,” Communications in Statistics Simulation and Computation, 18 (1), pp. 281-293.

[39] Sommer, A. M., Nowak A. S., and Thoft-Christensen P., 1993, “Probability-Based Bridge Inspection Strategy”, Journal of Structural Engineering, 119(12), pp. 3520-3536.

[40] Zhang, B., and Xie, Y. B., 1989, “Two Body Micro-Cutting Wear Model, Part III: Stable Profile Height Distribution of Worn Surface,” Wear, 129, pp. 59-66.