Reliability-Based Design with Mixture of Random and Interval Variables

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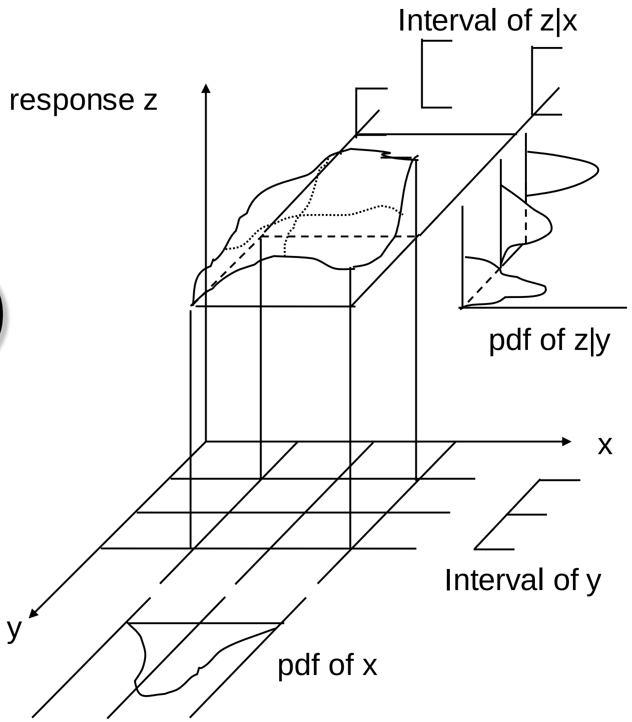
Motivations

- Distributions of some random variables are not precisely known.
  - Only intervals are known.
- Some uncertain variables are not from randomness
  - They are expressed in intervals.
- Therefore, we have mixture of random and interval variables.

Response (Performance)

- Let random variables be **x** and interval variables be **y.**
- Then response z=g(**x**, **y**) is also in mixture of randomness and intervals.



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Existing Research

- Reliability analysis with mixture of random and interval variables
  - Penmetsa and Grandhi, 2002
- Design optimization with only interval variables
  - Lombardi and Haftka, 1998
  - Rao and Cao, 2002

Issues?

1. How should we fully use the information available (distributions and intervals)?
2. In what sense should we make use of the reliability?
3. How can we solve RBD efficiently under such situation?

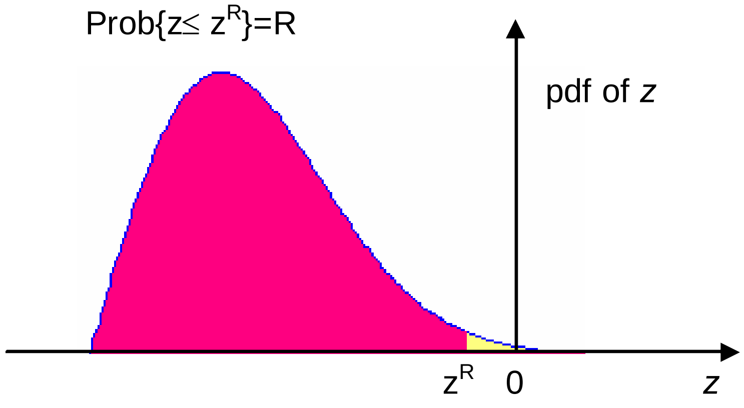
- This research tries to answer these three questions.

Answers

1. Use no assumptions.
2. Use reliability in the worst combinations of interval variables.
3. Use single-loop method to solve reliability-based design (RBD) problems?

Worst Case Reliability

- Inverse reliability (Der Kiureghian, et al, 1994; Li and Foschi, 1998; Wu, 1998; Tu and Choi, 1999; 2001; Wu, 2001; Du and Chen, 2001)
- Given reliability R, find corresponding response z: R-percentile performance z^R



Worst Case R-Percentile Performance by FORM

$$\begin{cases} \underset{\mathbf{u},\,\mathbf{y}}{\text{minimize}} & g(\mathbf{u}, \mathbf{y}) \\ \text{subject to} & \|\mathbf{u}\| = \Phi^{-1}(R) \end{cases}$$

- **u** – random variables transformed from x space to standard normal space
- Solution u^{MPP} – worst case MPP (Most Probable Point)
- y^{worst} – worst case combination of **y**
- Worst case R-percentile performance
  - $z^R = g(u^{MPP}, y^{worst})$

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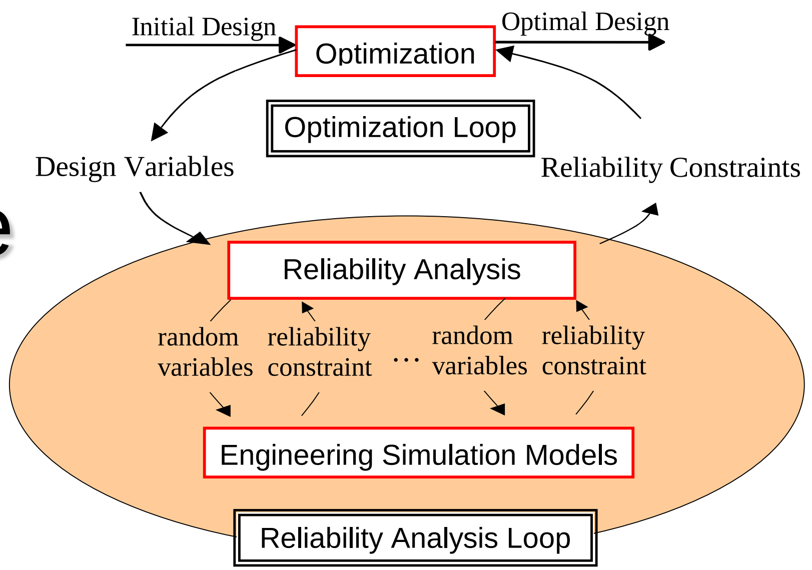
RBD Formulation

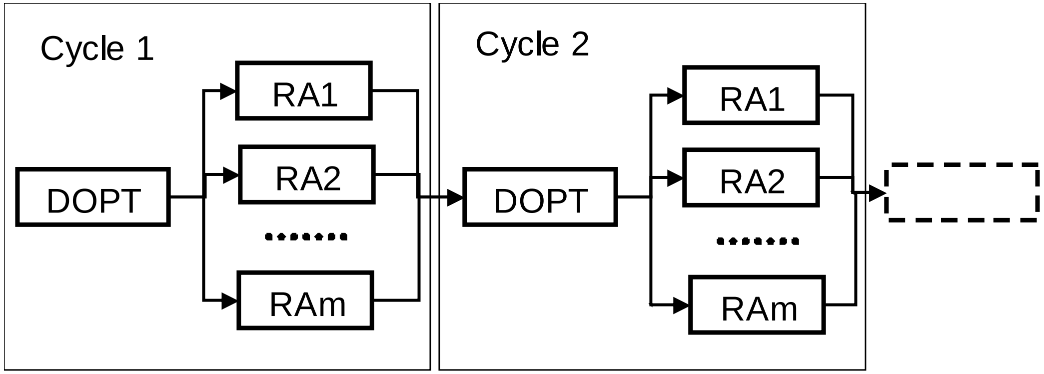
$$\begin{cases} \min\limits_{\mathbf{d}} & h(\mathbf{d},\ \boldsymbol{\mu}_x,\ \bar{\mathbf{y}}) \\ \text{subject to} & z_{worst,\,i}^{R_i} = g_i(\mathbf{d},\ \mathbf{u}_{worst,\,i}^{MPP,\,R_i},\ \mathbf{y}_{worst,\,i}) \geq 0,\ \text{i=1, 2, } \cdots, \text{ m} \end{cases}$$

- *h* – objective function
- **d** – design variables
- Double-loop procedure



Sequential Optimization and Reliability Assessment (SORA)

- Single loop strategy
- Decouple optimization from reliability analysis
- High efficiency



DOPT: deterministic optimization RA: reliability analysis

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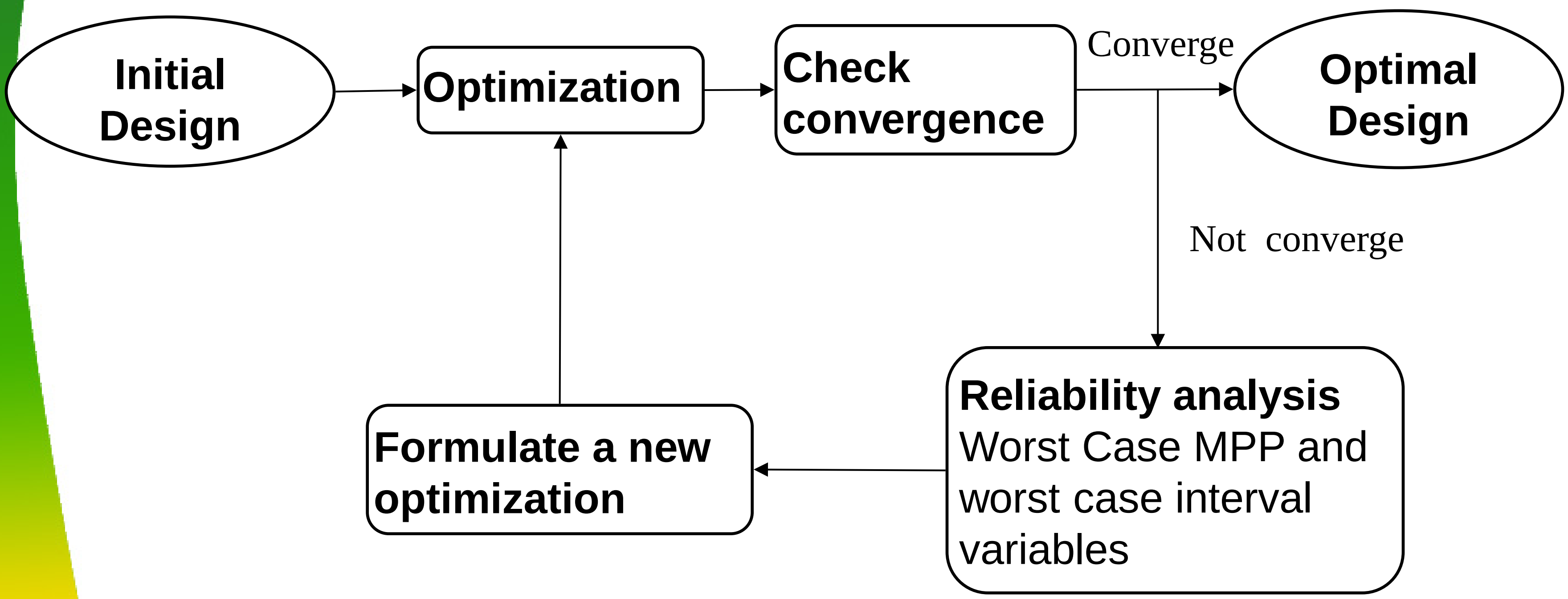
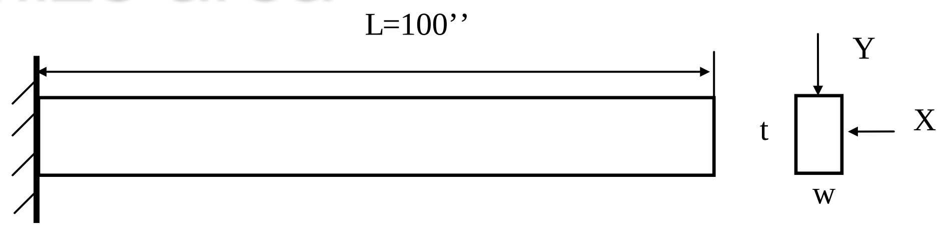
Procedure


Initial Design
Optimization
Check convergence
Converge
Optimal Design
Not converge
Reliability analysis
Worst Case MPP and worst case interval variables
Formulate a new optimization

Numerical Example

- Objective: Minimize area

$$h = wt$$



- Constraints

$$g_1(S, X, Y, w, t) = S - \left(\frac{600}{wt^2}Y + \frac{600}{w^2 t}X\right)$$

$$g_2(E, X, Y, w, t) = D_0 - \frac{4L^3}{Ewt}\sqrt{\left(\frac{Y}{t^2}\right)^2 + \left(\frac{X}{w^2}\right)^2}$$

- Random: X and Y; Interval: S and E

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Results

- Required reliability = 0.9978 (β=3)

Cycle	Design variables (w, t)	Objective	g_1^R	g_2^R
1	(3.1458, 2.2244)	6.9976	-0.3497	-0.3103
2	(4.0724, 2.0698)	8.4290	-0.0027	-0.0364
3	(3.9852, 2.1196)	8.4471	0.0541×10^{-3}	-0.2101×10^{-3}
4	(3.9848, 2.1199)	8.4474	0.0158×10^{-7}	0.4506×10^{-7}

- Total function calls=358 (double loop needs 4604)

Results (cont.)

- Let's compare the efficiency with traditional RBD (all variables are random)

	S and E: are Interval variables	S and E are uniformly distributed with same intervals
(t, w)	(3.9848, 2.119)	(3.8064, 2.1528)
Area $t \times w$	8.4438	8.1945

Starting point	mixture	Random variables
(d_1, d_2)	Number of function calls	Number of function calls
(4, 2)	318	409
(8, 3)	358	449
(2, 1)	339	430
(1.5, 0.5)	319	410

Same order of magnitude

Conclusions

- Worst case reliability
- Single-loop strategy
- Inverse reliability strategy
- Solution from worst case RBD is more conservative than traditional RBD
- The efficiency of proposed method is same as traditional RBD