Interval Reliability Analysis

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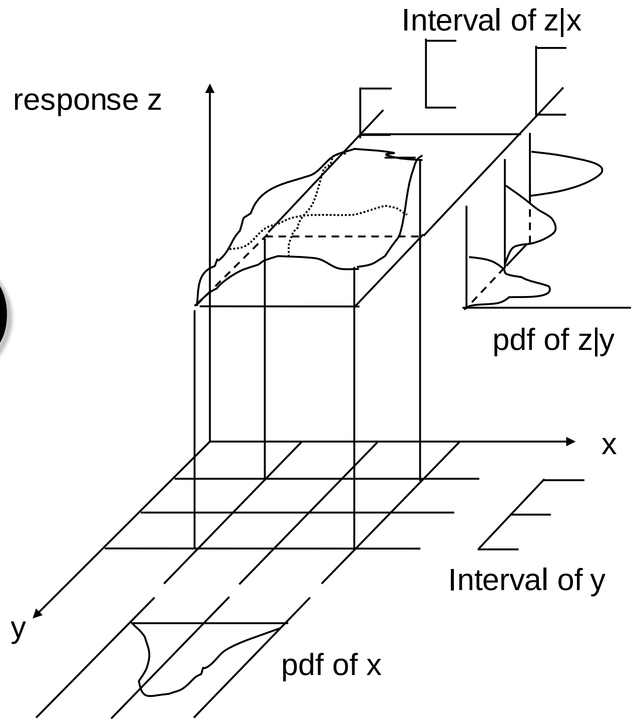
Outline

- Background
- Algorithm for reliability analysis
- Sensitivity analysis for intervals
- Examples
- Conclusions

Response (Performance)

- Let random variables be **x** and interval variables be **y.**
- Then response z=g(**x**, **y**) is also in mixture of randomness and intervals.



Examples of Existing Research

- Design optimization with only interval variables
  - Lombardi and Haftka, 1998
  - Rao and Cao, 2002
- Reliability analysis with random and interval variables
  - Penmetsa and Grandhi, 2002
- Reliability-based design with random and interval variables
  - Du and Sudjianto, 2003
- Fuzzy and epistemic uncertainty
  - Choi
  - Mourelatos

Our New Method

- Reliability analysis for reliability bounds
  - The extension of our reliability analysis algorithms (2003)
  - to the mixture of random and interval variables
- Sensitivity analysis in terms of intervals
  - The most important interval variables to reliability

Direct Reliability Analysis

- Given: $g = g(\mathbf{X}, \mathbf{Y})$
- Find: the probability of failure

$$p_f = \Pr\left\{ g(\mathbf{X}, \mathbf{Y}) < 0 \right\}$$

  - Minimum $p_f^{\max} = \Pr\left\{ \min_{\mathbf{Y}} g(\mathbf{X}, \mathbf{Y}) < 0 \right\}$
  - Maximum $p_f^{\min} = \Pr\left\{ \max_{\mathbf{Y}} g(\mathbf{X}, \mathbf{Y}) < 0 \right\}$

Maximum p_f

$$p_f^{\max} = \Pr\left\{\min_{\mathbf{Y}} g(\mathbf{X}, \mathbf{Y}) < 0\right\}$$

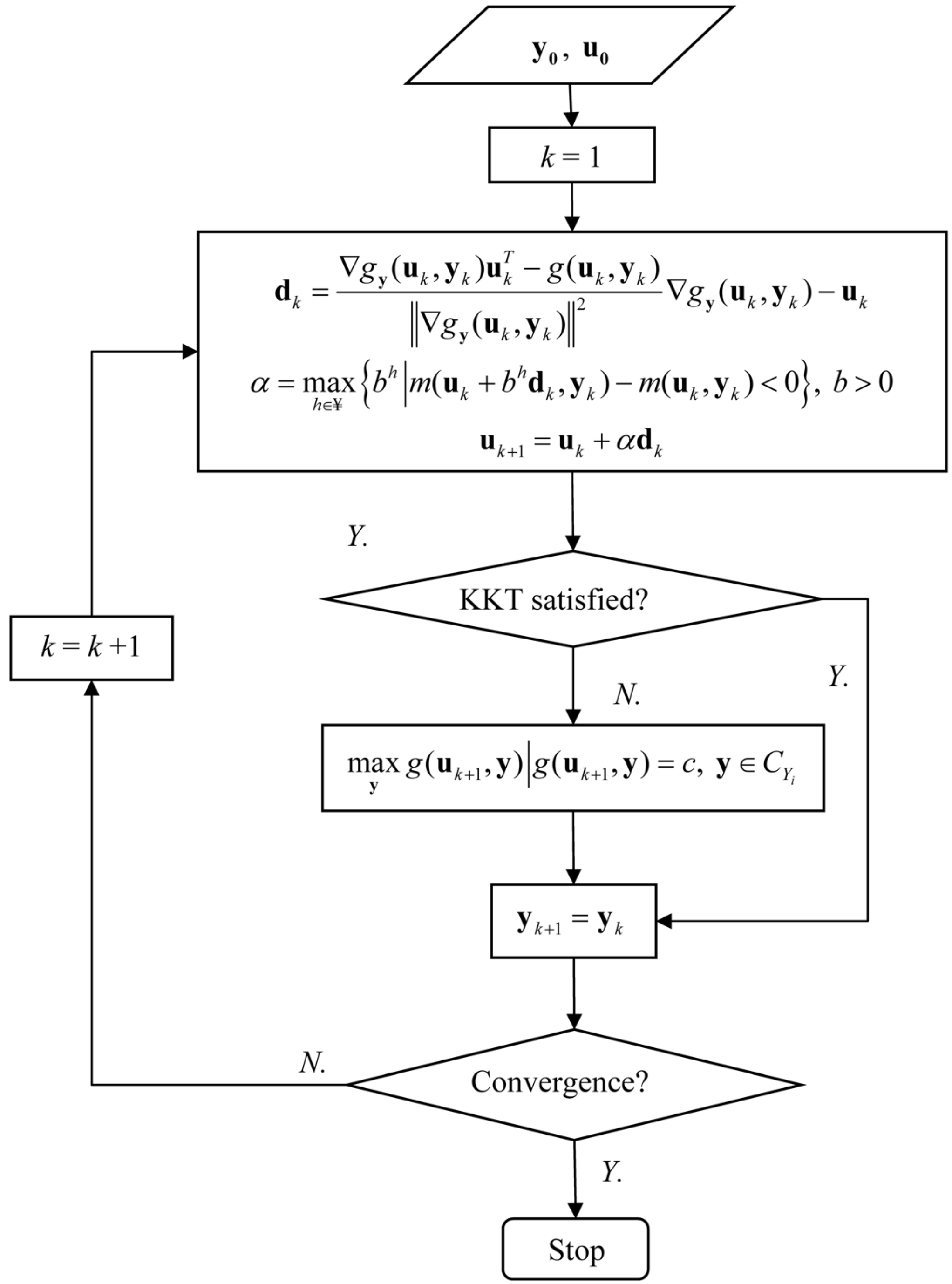
- FORM – First Order Reliability Method

$$\min_{\mathbf{u}} \|\mathbf{u}\| \Big| \min_{\mathbf{Y}} g(\mathbf{U}, \mathbf{Y}) = 0$$

- U – std. normal var. from X.

Flowchart

- Improved HL-RF algorithm
  - Line search
- KKT conditions
- Optimization for intervals



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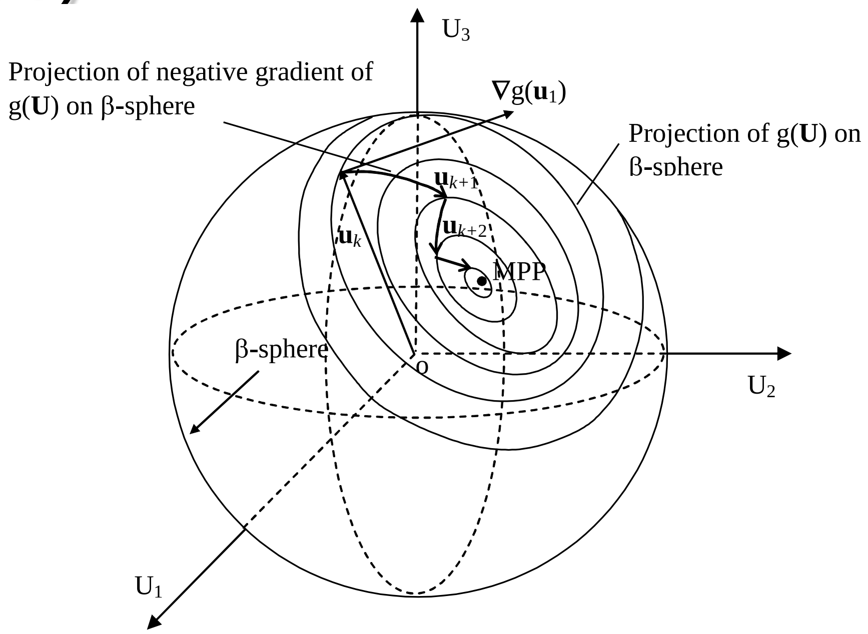
Inverse Reliability Analysis

- Given: the probability of failure p_f
- Find: c such that $p_f = \Pr\{g(\mathbf{X},\mathbf{Y}) < c\}$

- Modified our algorithm (2003)
  - Arc search
  - Optimization for intervals
  - KKT conditions



Sensitivity Analysis

- 6 types of sensitivity are proposed to identify the relationship between the p_f bounds and interval variables
- 1st type of sensitivity – derivative of the width of p_f with respect to the width of an interval variable

$$\frac{\partial \delta_p}{\partial \delta_i}$$

$$\delta_p = p_f^U - p_f^L$$

$$\delta_i = y_i^U - y_i^L$$

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1st type of sensitivity

- We consider only one way in our definition

- Derivation of equation:

Different cases give different equations, for example,

- Case 1 $y_i = y_i^U, reach\ p_f^U;$ $y_i = y_i^L, reach\ p_f^L$ ⇒

$$\frac{1}{2}\left(\frac{-\phi(-\beta^{\max})}{\|\nabla\|\mathbf{u}^U}\frac{\partial g}{\partial y}\Big|_{y=y_i^U} + \frac{-\phi(-\beta^{\min})}{\|\nabla\|\mathbf{u}^L}\frac{\partial g}{\partial y}\Big|_{y=y_i^L}\right)$$

- Case 2 $y_i = y_i^L, reach\ p_f^U;$ $y_i = y_i^U, reach\ p_f^L$ ⇒

$$-\frac{1}{2}\left(\frac{-\phi(-\beta^{\max})}{\|\nabla\|\mathbf{u}^U}\frac{\partial g}{\partial y}\Big|_{y=y_i^L} + \frac{-\phi(-\beta^{\min})}{\|\nabla\|\mathbf{u}^L}\frac{\partial g}{\partial y}\Big|_{y=y_i^U}\right)$$

1^{st} type of sensitivity

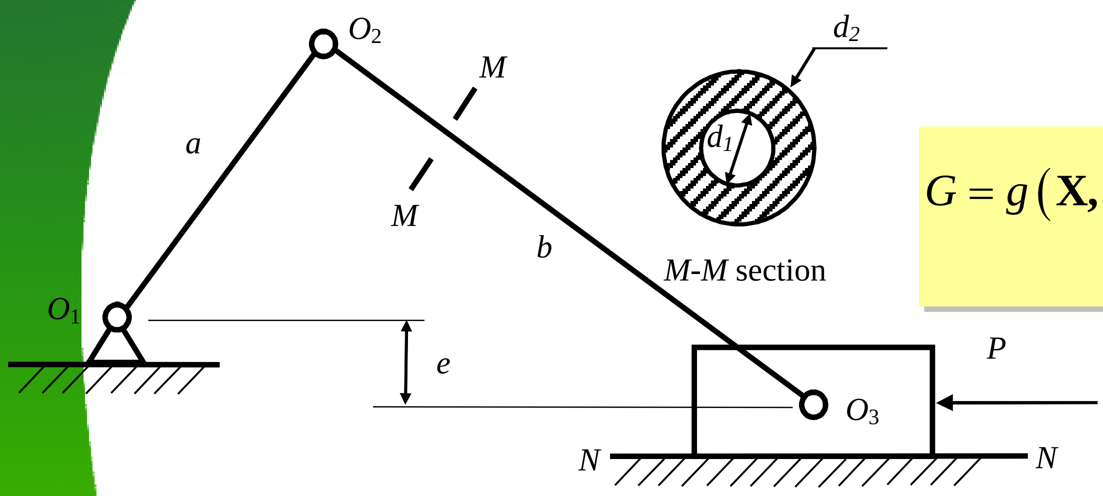
- Other possible cases are
  - Case 3 $y_i = y_i^U, reach\ p_f^U; interior, reach\ p_f^L$
  - Case 4 $y_i = y_i^L, reach\ p_f^U; interior, reach\ p_f^L$
  - Case 5 $y_i = y_i^U, reach\ p_f^L; interior, reach\ p_f^U$
  - Case 6 $y_i = y_i^L, reach\ p_f^L; interior, reach\ p_f^U$
  - Case 7 $interior, reach\ p_f^U\ and\ p_f^L$

Other types of sensitivities

- 2nd type of sensitivity $\frac{\partial \overline{p}_f}{\partial \delta_i}$
- 3rd type of sensitivity $\frac{\partial \delta_p}{\partial \overline{y}_i}$
- 4th type of sensitivity $\frac{\partial \overline{p}_f}{\partial \overline{y}_i}$
- 5th type of sensitivity $\frac{\partial \delta_p}{\partial q_i}$
- 6th type of sensitivity $\frac{\partial \overline{p}_f}{\partial q_i}$

Note: **<u>q denotes a distribution parameter of a random variable</u>**

Example



$$G = g(\mathbf{X}, \mathbf{Y}) = S - \frac{4P(a+b)}{\pi\left(\sqrt{(a+b)^2 - e^2} - \mu e\right)\left(d_2^2 - d_1^2\right)}$$

Interval variables

	Variable	Lower bound	Upper bound
Y_1	e (mm)	100	150
Y_2	μ	0.15	0.25

Random variables

Variable	Symbols in Figure	Mean	Standard deviation	Distribution
X_1	a	100 mm	0.01 mm	Normal
X_2	b	300 mm	0.01 mm	Normal
X_3	P	250 kN	25 kN	Normal
X_4	E	200 GPa	30 GPa	Normal
X_5	S	390 MPa	29 MPa	Normal

Reliability Analysis Result

Sensitivity Analysis Result

- Sensitivity of interval variables

	Y_1	Y_2
Sensitivity $\partial\delta_p / \partial\delta_i$	0.0000350	0.00453
Sensitivity $\partial\overline{p}_f / \partial\delta_i$	1.634e-005	0.00211
Sensitivity $\partial\delta_p / \partial\overline{y}_i$	6.537e-005	0.00842
Sensitivity $\partial\overline{p}_f / \partial\overline{y}_i$	3.450e-005	0.00453

Sensitivity Analysis Result

- Sensitivity of random variables

		Sensitivity $\partial \delta_p / \partial p$	Sensitivity $\partial \bar{p}_f / \partial p$
X_1	$p = \mu_1$	3.365e-005	1.780e-005
	$p = \sigma_1$	2.370e-008	1.228e-008
X_2	$p = \mu_2$	-3.365e-005	1.780e-005
	$p = \sigma_2$	2.370e-008	1.228e-008
X_3	$p = \mu_3$	2.333e-008	1.287e-008
	$p = \sigma_3$	4.009e-008	2.237e-008
X_4	$p = \mu_4$	-3.399e-011	-1.901e-011
	$p = \sigma_4$	9.449e-011	5.454e-011
X_5	$p = \mu_5$	0	0
	$p = \sigma_5$	0	0

Conclusions

- The methods can give us more accurate results about reliability.
- The methods can allow us to make more reliability decisions.
- The methods can give us directions of collecting more information to reduce the effects of uncertainty.

- Acknowledgements: