A General Approach to Robust Design Optimization with Multilevel Uncertainty

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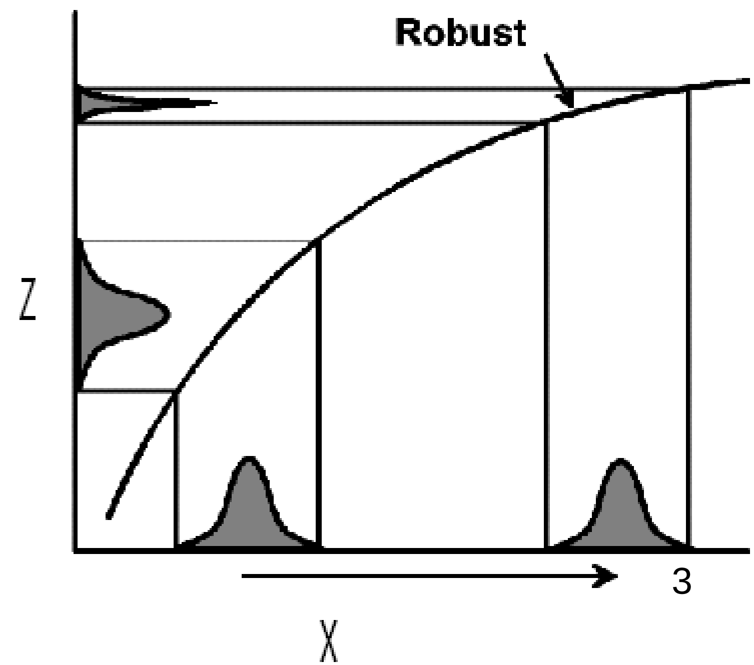
Outline

- Background
  - Robustness
  - Random and interval variables
- Robustness assessment
- Robust design
- Examples
- Conclusion

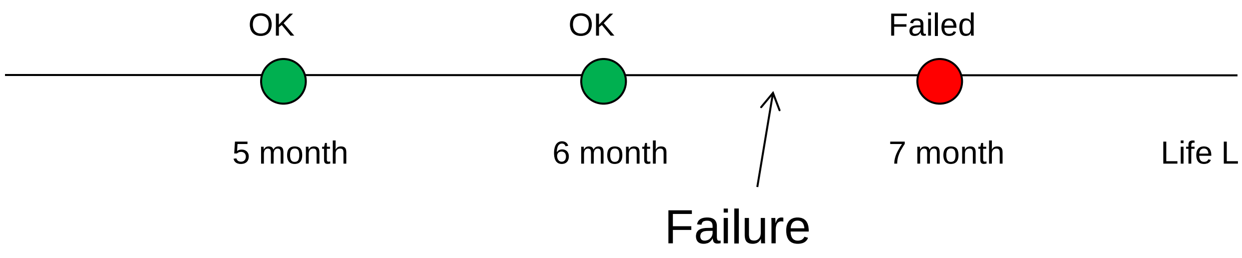
Robustness

- Products function regardless of variations in environments.
- Degree of insensitivity to uncertainties.
- Performance $Z = f(\mathbf{X})$



Uncertainties in Input **X**

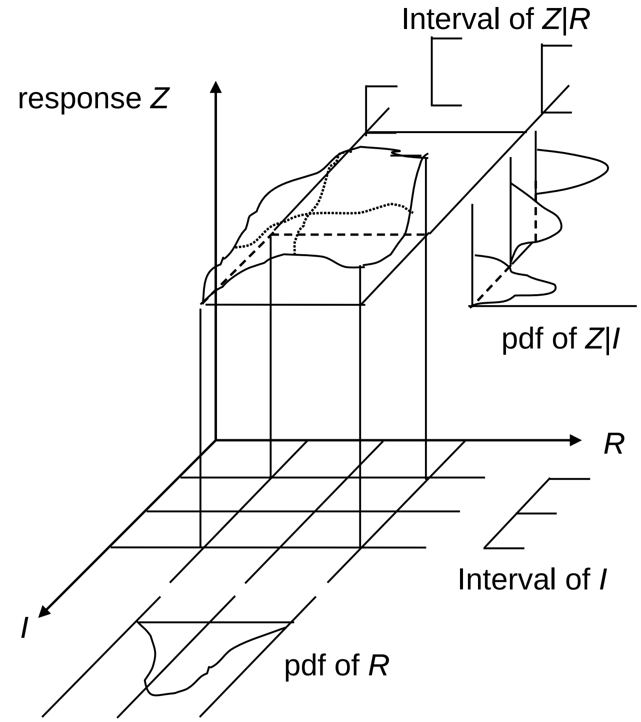
- **X**=(**R**,**I**)
- Random variables **R**
- Interval variables **I**
  - Tolerance, d = 10±0.01 mm
  - Periodic condition monitoring, 6 mo <L<7 mo



Robustness with **R** and **I**

- Robustness measured by standard deviation (std) of *Z*, σ_Z
- Because of **I**,

$$\sigma_Z^{\min} \le \sigma_Z \le \sigma_Z^{\max}$$



Robustness Assessment

- Given
  - Distributions of **R**
  - Width of **I**
- Find
  - Effect of **R**: average std

$$\overline{\sigma}_Z = \frac{1}{2}\left(\sigma_Z^{\max} + \sigma_Z^{\min}\right)$$

  - Effect of **I**, std difference

$$\delta\sigma_Z = \sigma_Z^{\max} - \sigma_Z^{\min}$$

Semi-2nd-Order Approximation

$$\Delta f(\mathbf{R}, \mathbf{I}) = \nabla_{\mathbf{R}}^{T} \Delta \mathbf{R} + \nabla_{\mathbf{I}}^{T} \Delta \mathbf{I} + \frac{1}{2} \Delta \mathbf{X}^{T} \nabla^{2} \Delta \mathbf{X}$$

Interaction terms of **R** and **I** are kept.

$$\nabla^2 = \begin{pmatrix} & & & \frac{\partial^2 g}{\partial R_1 \partial I_1} & \cdots & \frac{\partial^2 g}{\partial R_1 \partial I_{nI}} \\ & 0 & & \cdots & \cdots & \cdots \\ & & & \frac{\partial^2 g}{\partial R_{nR} \partial I_1} & \cdots & \frac{\partial^2 g}{\partial R_{nR} \partial I_{nI}} \\ \frac{\partial^2 g}{\partial R_1 \partial I_1} & \cdots & \frac{\partial^2 g}{\partial R_1 \partial I_{nI}} & & & \\ \cdots & \cdots & \cdots & & 0 & \\ \frac{\partial^2 g}{\partial R_{nR} \partial I_1} & \cdots & \frac{\partial^2 g}{\partial R_{nR} \partial I_{nI}} & & & \end{pmatrix}$$

σ_z Bounds

$$\sigma_z = \sqrt{\sum_{i=1}^{nR}\left(\left.\frac{\partial f}{\partial R_i}\right|_{\bar{\mathbf{R}},\bar{\mathbf{I}}} + \sum_{j=1}^{nI}\left.\frac{\partial^2 f}{\partial R_i \partial I_j}\right|_{\bar{\mathbf{R}},\bar{\mathbf{I}}}\Delta I_j\right)^2 \sigma_{Ri}^2}$$

Calls f function $(nR + 3\times nR \times nI + 1)$ times

For $\sigma_Z^{\min}$

$$\min_{\mathbf{I}} \sum_{i=1}^{nR}\left(\left.\frac{\partial f}{\partial R_i}\right|_{\bar{\mathbf{R}},\bar{\mathbf{I}}} + \sum_{m=1}^{nI}\left.\frac{\partial^2 f}{\partial R_i \partial I_j}\right|_{\bar{\mathbf{R}},\bar{\mathbf{I}}}\Delta I_j\right)^2 \sigma_i^2$$

$$s.t.\ \ \mathbf{I}^l \le \mathbf{I} \le \mathbf{I}^u$$

No f function calls

Robust Design

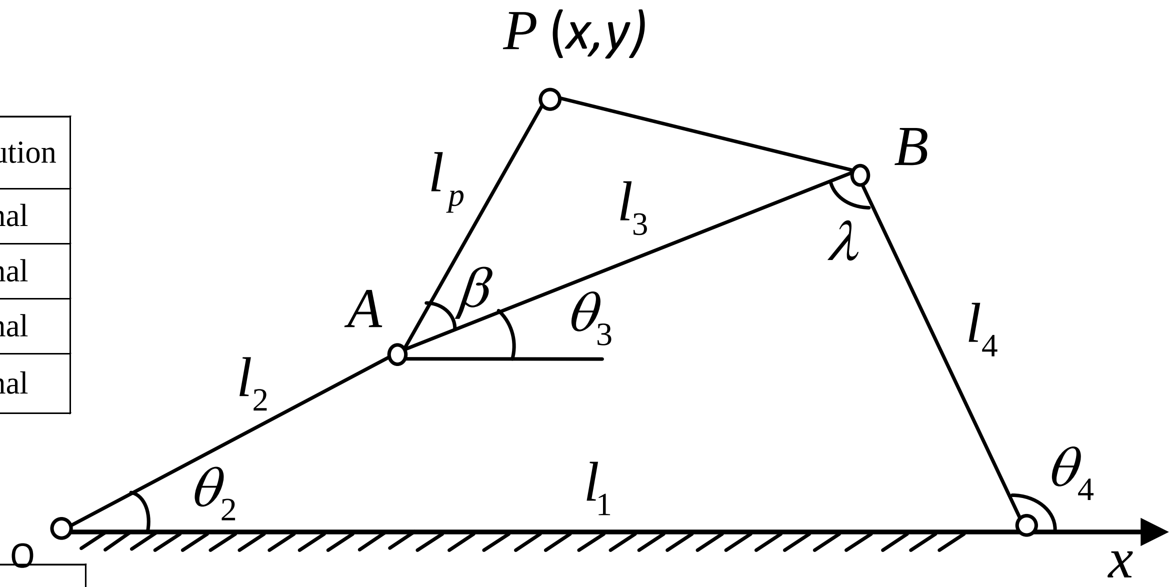
Example 1: 4-Bar Linkage Single Discipline

Random variables

Variable	Mean	Standard Deviation	Distribution
$R_1\left(l_2\right)$	51.68mm	0.20 mm	Normal
$R_2\left(l_3\right)$	297.06mm	0.20 mm	Normal
$R_3\left(l_4\right)$	197.08mm	0.20 mm	Normal
$R_4\left(l_p\right)$	99.11mm	0.20 mm	Normal

Interval variables

Variable	I^L	I^U
$I_1\left(l_1\right)$	$\bar{l}_1-0.5\text{mm}$	$\bar{l}_1+0.5\text{ mm}$
$I_2\left(\beta\right)$	$\bar{\beta}-1^\circ$	$\bar{\beta}+1^\circ$



Output: P(x,y)

4-Bar Linkage - Results

	GRA	MCS
at 10°	(0.178,0.203) mm	(0.172,0.207) mm
at 10°	(0.177 ,0.203) mm	(0.171,0.207) mm
at 60°	(0.149,0.260) mm	(0.148,0.261) mm
at 60°	(0.148 ,0.259) mm	(0.146 ,0.259) mm

GRA: General robustness assessment

Multidisciplinary Cylinder Problem

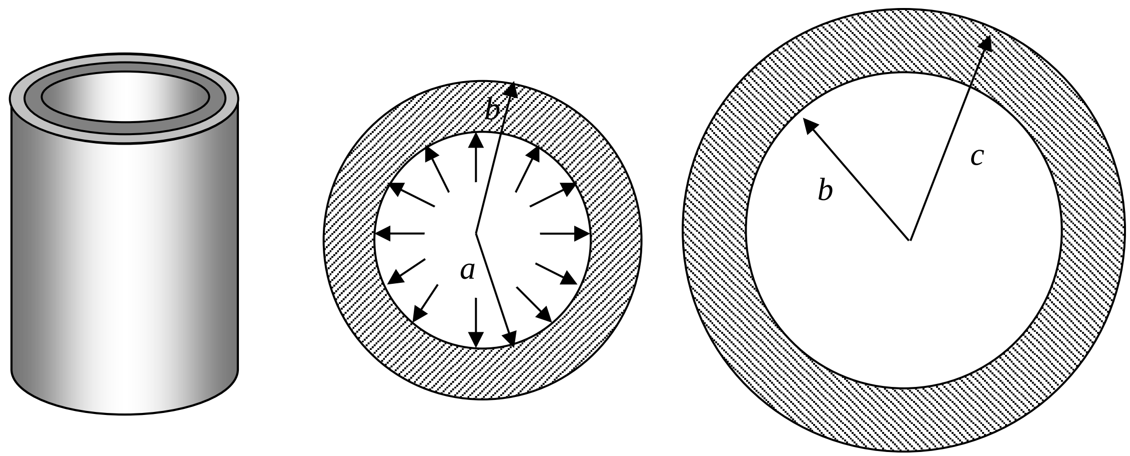
Output: P(x,y)

$$Z_1^{(1)} = \sigma_a - S$$

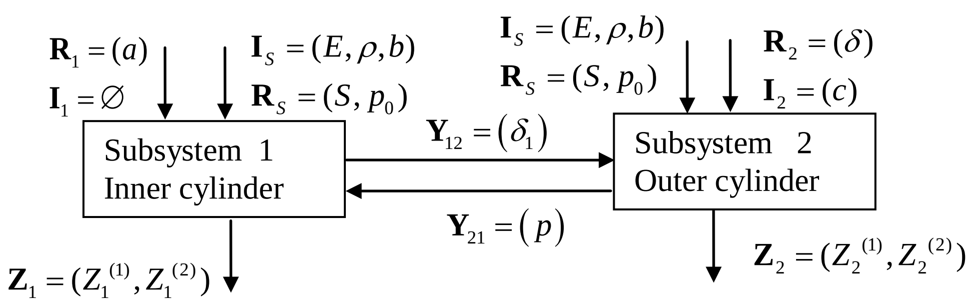
$$Z_1^{(2)} = \sigma_b - S$$

σ_a and σ_b: stresses at a and b

S: strength



Compound cylinder system   Subsystem 1: Inner cylinder   Subsystem 2: Outer cylinder



Cylinder Problem

Random variables

Variables	Mean	Standard Deviation	Distribution
S	10.0×10^3 psi	1.0×10^3 psi	Normal
p_0	20.0×10^3 psi	2.0×10^3 psi	Normal
a	75 in	0.1 in	Normal
δ	0.004 in	0.0004 in	Normal

Interval variables

Variables	I^L	I^U
E	$30\times10^6\times(1-2\%)$ psi	$30\times10^6\times(1+2\%)$ psi
ρ	$0.3\times(1-2\%)$	$0.3\times(1+2\%)$
b	9.95 in	10.05 in
c	14.95 in	15.05 in

Result

	GRA	MCS
$\sigma_{\mathbf{z}_1}^{\max}(psi)$	$[3.485\times10^3\ 2.444\times10^3]$	$[3.597\times10^3\ 2.570\times10^3]$
$\sigma_{\mathbf{z}_1}^{\min}(psi)$	$[3.484\times10^3\ 2.419\times10^3]$	$[3.562\times10^3\ 2.515\times10^3]$
$\sigma_{\mathbf{z}_2}^{\max}(psi)$	$[2.40\times10^3\ 1.667\times10^3]$	$[2.544\times10^3\ 1.750\times10^3]$
$\sigma_{\mathbf{z}_2}^{\min}(psi)$	$[2.373\times10^3\ 1.667\times10^3]$	$[2.50\times10^3\ 1.724\times10^3]$

Conclusions

- The robustness assessment is efficient.
- The accuracy is good when standard deviations and widths are small.
- The method is not applicable when the performance function is expanded at a saddlepoint.
- Future work will be the use of the method for robust design.

Acknowledgement

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