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Quantifying Model Uncertainty with ASME Measurement Uncertainty Standards

Xiaoping Du and Harsheel Shah

Missouri University of Science and Technology

Outline

- Background
- Basic concepts
- ASME Measurement Uncertainty Guidelines
- Model uncertainty qualification
- Example

Our Interest

- The model builder is not able to performe validation testing.
- Or doing so is expensive and time-consuming.
- The testing job is outsourced to an external lab.

Complexities

- What is the agreement between model builder and experimenter?
- Many communications between them.
- How do they communicate effectively?

Solution

- Find a common framework between them.
- The common framework could be certain experimental standards.
- We use the ASME standards
  – Guidelines for the evaluation of dimensional measurement uncertainty
  – B89.7.3.2 – 2007

A General Model

$$y^m = f(\mathbf{x}, \mathbf{w})$$

Model prediction

Model

Model input

Model parameter

Example

- *f* - a dynamics simulation model
- y^m – velocity, acceleration, etc.
- **x** – forces, dimensions, etc.
- **w** – gravitational acceleration

Model Error

$$y^m = f(\mathbf{x}, \mathbf{w})$$

$$y = y^m + \varepsilon^m$$

True value

Model error

Sources of model error if **x** is precise.
- *f*
- **w**

- We never know the model error, but we can estimate it through experiments.
- The estimation is the model uncertainty.

ASME Standards

- Uncertainty of measurement
  - associated with the result of a measurement
  - characterizes the dispersion of the values that are attributed to the measurand
- The true value *y* is being measured

$$\underbrace{\tilde{y}^e}_{\text{Measurement result}} \pm \underbrace{U_y}_{\text{Expanded uncertainty (95\% confidence)}}$$

Measurement Errors and Uncertainties

$$y = \tilde{y}^e + \varepsilon^e$$

True value

Experimental error

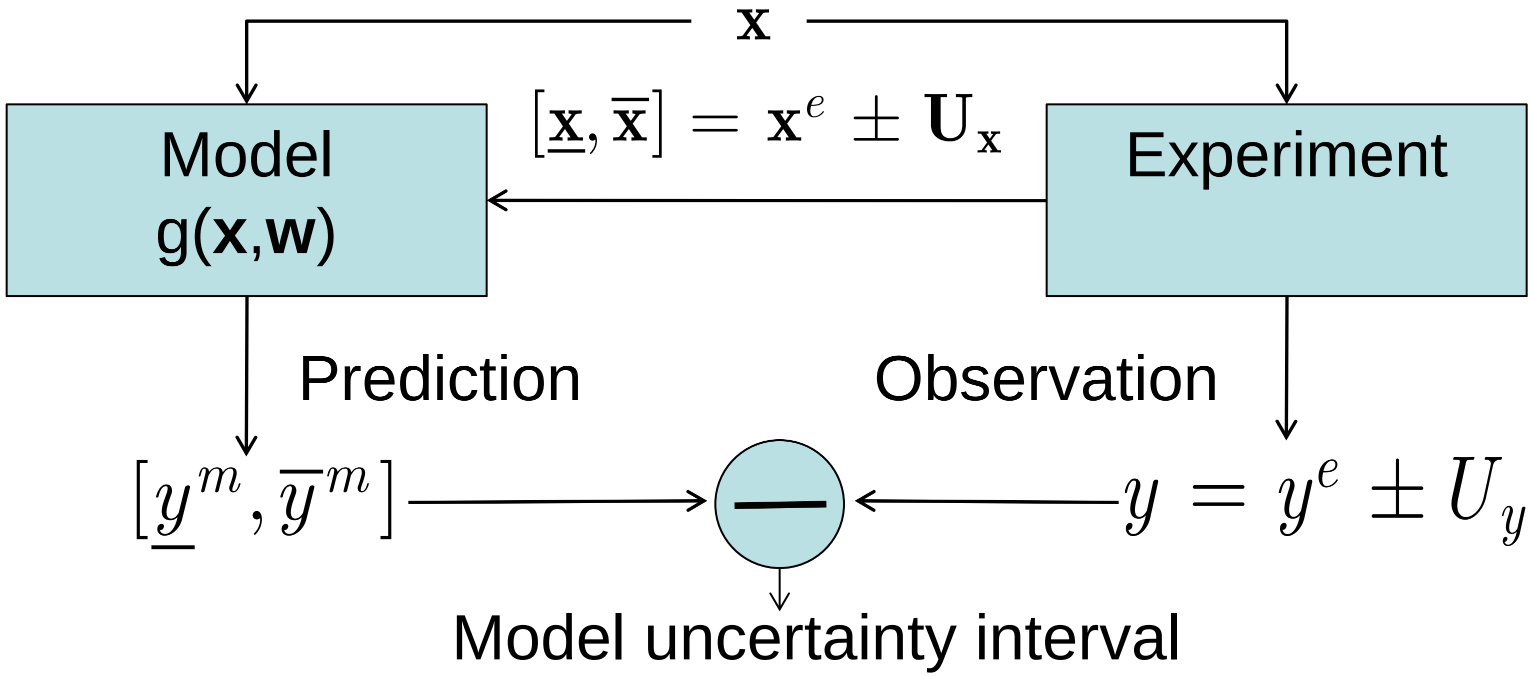
- We never know the experimental error, but we can estimate it by the expanded uncertainty.
- The estimation is the experimental uncertainty.

$$y = \tilde{y}^e \pm U_y$$ (95% confidence)

Experimental uncertainty

Proposed Methodology

- Model uncertainty = discrepancy between prediction and observation
  - Condition: Model simulation and experiment performed at the same input point x



Implementation

Experiment

$$\mathbf{x}^e = \tilde{\mathbf{x}}^e \pm \mathbf{U}_x = [\underline{\mathbf{x}}^e, \overline{\mathbf{x}}^e] \text{ (95\% conf.)}$$

$$y = y^e = \tilde{y}^e \pm U_y = [\underline{y}^e, \overline{y}^e] \text{ (95\% conf.)}$$

Simulation

$$y^m = g([\underline{\mathbf{x}}^e, \overline{\mathbf{x}}^e], \mathbf{w}) = [\underline{y}^m, \overline{y}^m]$$

Model uncertainty

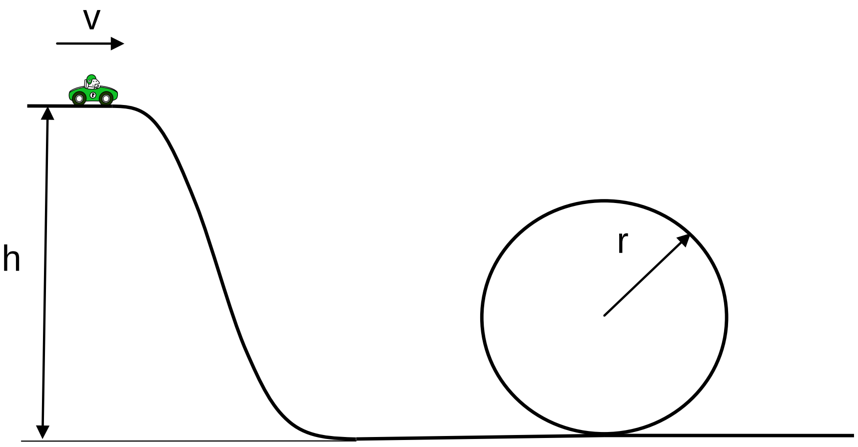
$$U^m = \tilde{y}^e \pm U_y - y^m = [\underline{U}^m, \overline{U}^m]$$

Confidence of Model Uncertainty Interval

- Assume 1-α (95%) confidence for
  - $\mathbf{x}^e = [\underline{\mathbf{x}}^e, \overline{\mathbf{x}}^e]$
  - $y^e = [\underline{y}^e, \overline{y}^e]$
- Confidence level on $[\underline{U}^m, \overline{U}^m]$?
  - At least $(1-\alpha)^n$ (n: size of **x**); too conservative
  - If bounds of $[\underline{\mathbf{x}}^e, \overline{\mathbf{x}}^e]$ are narrow, the confidence of $[\underline{U}^m, \overline{U}^m]$ will be $\geq 1-\alpha$.

A Simple Example: Roller-Coaster Car

- Assumptions of the model
  - No energy loss
  - No friction
  - Particle



$$y^m = h = f(\mathbf{x}, \mathbf{w}) = \frac{5}{2} r - \frac{1}{2} \frac{v^2}{g}$$

$$\mathbf{x} = (v, r), \mathbf{w} = (g)$$

Experiments

- Performed at $\mathbf{x} = (v, r) = (3\ \mathrm{m/s}, 10\ \mathrm{m})$
- To simulate the experiments, we assumed
  - Experimental uncertainties in measurements of *v* and *r*
  - 95% to 99% energy loss with a uniform distribution
  - 95% confidence for observations

$$[\underline{x}_1^e, \overline{x}_1^e] = [\underline{v}^e, \overline{v}^e] = [2.980, 3.020]\ \mathrm{m/s}$$

$$[\underline{x}_2^e, \overline{x}_2^e] = [\underline{r}^e, \overline{r}^e] = [9.9980, 10.0020]\ \mathrm{m}$$

$$[\underline{y}^e, \overline{y}^e] = [\underline{h}^e, \overline{h}^e] = [24.6930, 25.8658]\ \mathrm{m}$$

Model Uncertainty

Experiment

$$[\underline{x}_1^e, \overline{x}_1^e] = [\underline{v}^e, \overline{v}^e] = [2.980, 3.020]$$

$$[\underline{x}_2^e, \overline{x}_2^e] = [\underline{r}^e, \overline{r}^e] = [9.9980, 10.0020]$$

$$[\underline{y}^e, \overline{y}^e] = [\underline{h}^e, \overline{h}^e] = [24.6930, 25.8658]$$

Simulation

$$[\underline{y}^m, \overline{y}^m] = \frac{5}{2}[\underline{r}^e, \overline{r}^e] - \frac{1}{2}\frac{[\underline{v}^e, \overline{v}^e]^2}{g} = [24.5301, 24.5524]$$

Model uncertainty

$$U_m = y^e - y^m = [0.1406, 1.3357] \text{ m}$$

$$y^m = f(\mathbf{x}, \mathbf{w}) + \varepsilon^m$$ 95% confidence

Conclusions

- Advantages
  - Easy to use when experiments are performed externally
  - Easy to understand for model users and experimenters
- Future work
  - Account for multiple and dependent output variables
  - Better ways to calculate model uncertainty intervals