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Robustness Metrics For Time-Dependent Quality Characteristics

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Outline

- Quality loss function
- Time-dependent quality loss function
- Robustness metrics
- Example

Quality Loss Function

- Nominal-the-best type performance *Y*

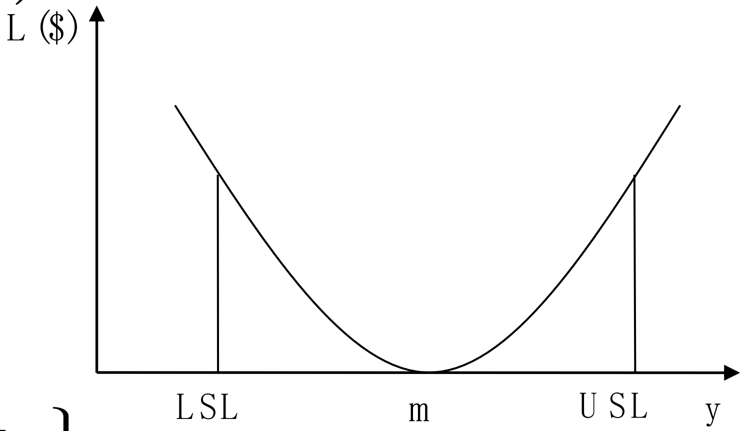
Constant

Target

- Quality loss $L = A(Y - m)^2$
- Robustness metric
  - Expected *L*

$$E_L = A[(\mu_Y - m)^2 + \sigma_Y^2]$$

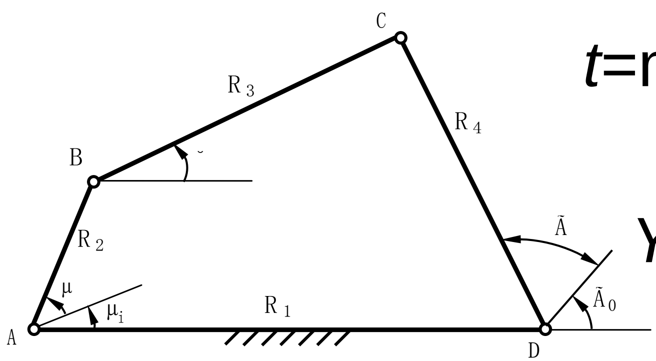
$$\min E_L \Rightarrow \mu_Y \to m \text{ and } \min\{\sigma_Y\}$$



Traditional robust design: *Y* is time invariant, and so is *L*.

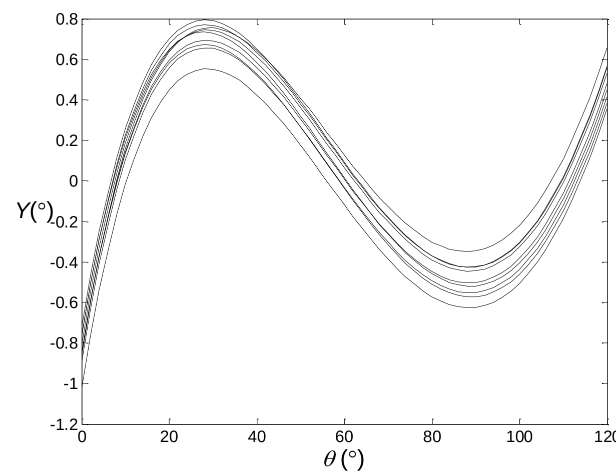
Reality: Time-Dependent Performances

Y=g(X(t),t) with input X(t)

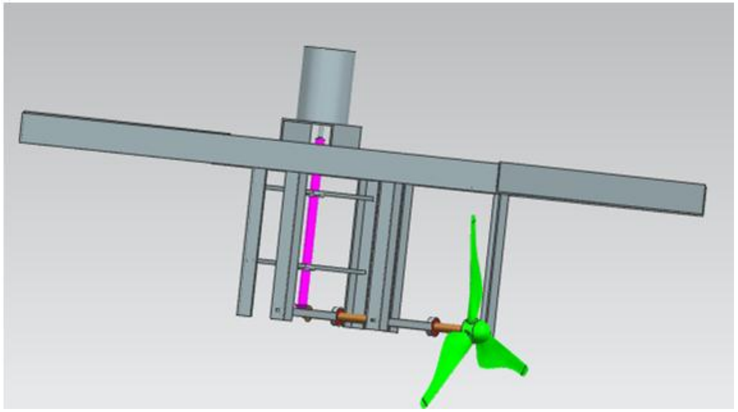


t=motion input θ

Y=motion error



Hydrokinetic turbine



Time-dependent random river speed

Challenge: We have stochastic processes now

- Input

  $\mathbf{X}(t)$

- Performance:

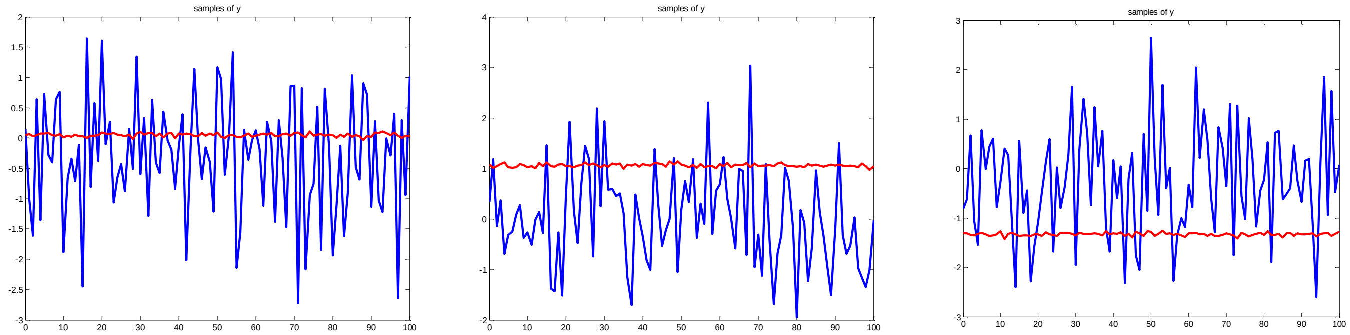
  $Y(t) = g[\mathbf{X}(t), t]$

- Quality loss

$$L(t) = A(t)[Y(t) - m(t)]^2$$

Why Is It a Challenge?

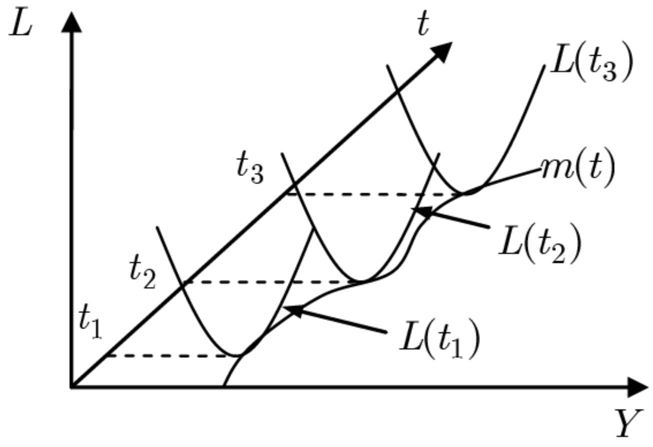
- Over $[t_0, t_f]$, for a stochastic process, we need to know
  - its instantaneous distributions at any t
  - its auto-dependence of any pair t_1 and t_2
- Example: two Gaussian processes
  - same instantaneous distributions (standard normal)
  - different auto correlation coefficients **ρ=0.999** (red) and **ρ=0.01** (weak).
- Both processes behave totally differently.



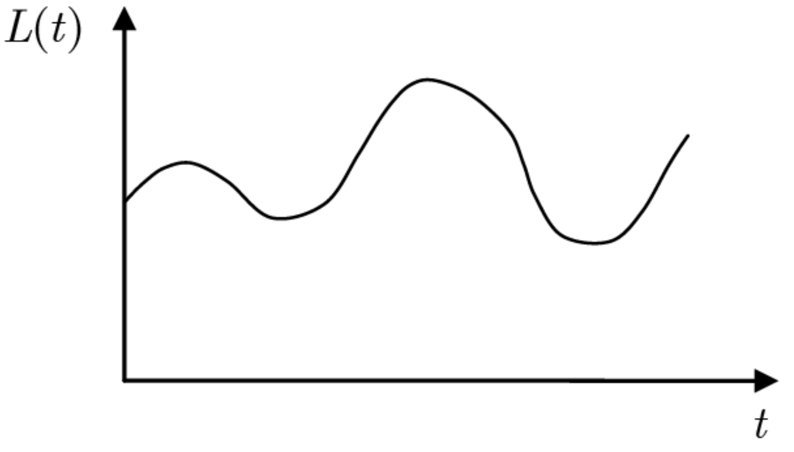
- Point expected QLF is not a good metric.

$$E_L(t) = A(t)[(\mu_Y(t) - m(t))^2 + \sigma_Y^2(t)]$$

Quality Loss Process

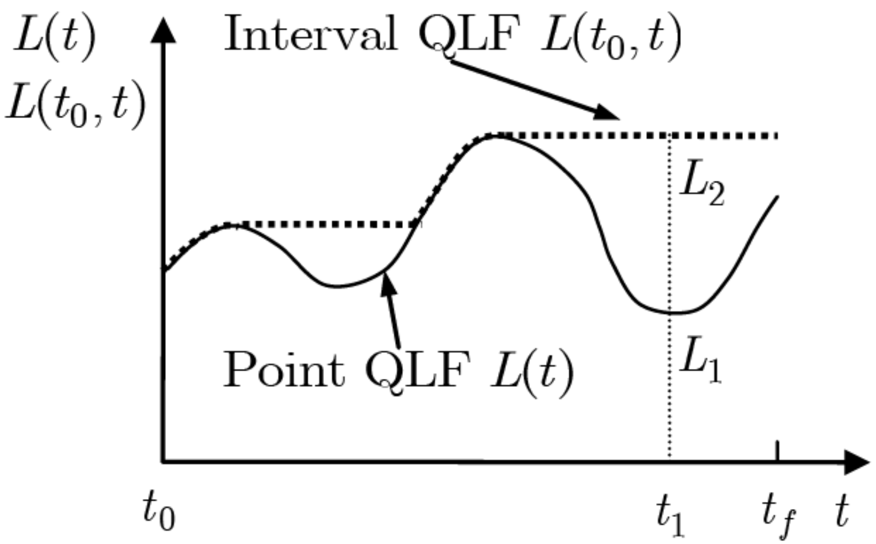


A sample curve



Interval Quality Loss Function

- QL is irreversible – once it occurred; there is no way to go back.
- Over [t_0,t_f], QL is the maximal instantaneous QL.



$$L(t_0, t_f) = \max_{\tau} \; A(\tau)[Y(\tau) - m(\tau)]^2 \quad , t_0 \le \tau \le t_f$$

New Metrics $E_L(t_0, t_f)$

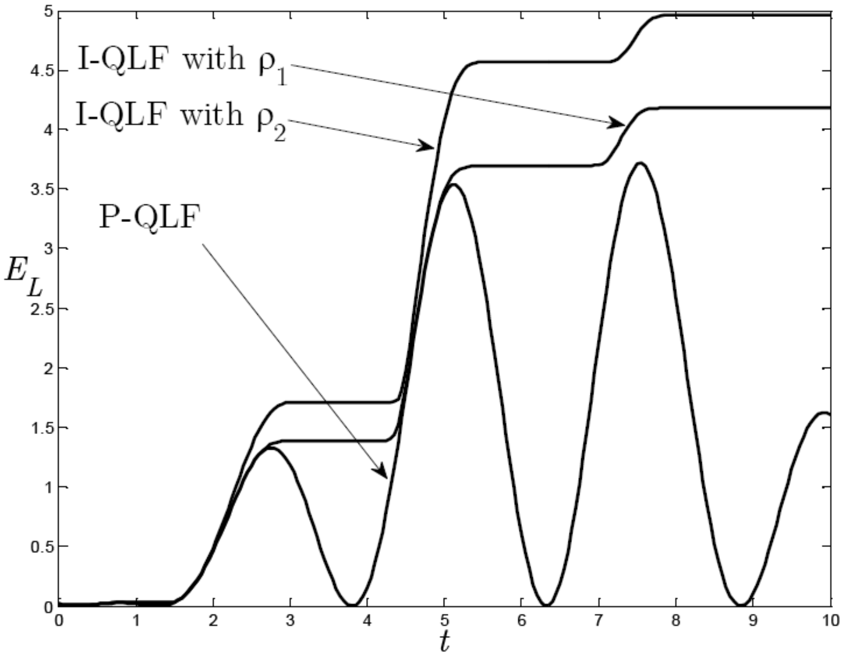
- True quality loss
- Can account for auto-dependence of *L*(*t*)

Two L(t) processes

- Same distributions at t
- Different autocorrelation coefficients

Result in

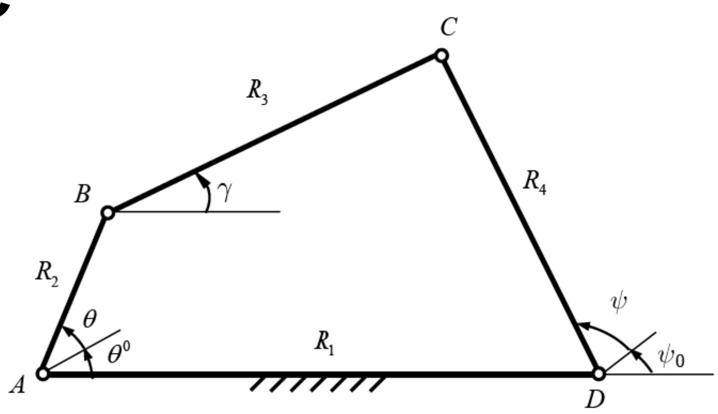
- Same P-QLF at t
- Different I-QLF over [t_0,t_f].



Example

Required motion (time $t = \theta$ $[0^{\circ}, 120^{\circ}]$)

$$\psi_d(\theta) = \psi_0 + 50^{\circ} \sin\left[\frac{3}{4}(\theta + \theta_0)\right]$$



QC – motion error

$$Y(\mathbf{X}, \theta) = \psi(\mathbf{X}, \theta) + \psi_0 - \psi_d(\theta)$$

$$\mathbf{X} = (R_1, R_2, R_3, R_4)^T$$

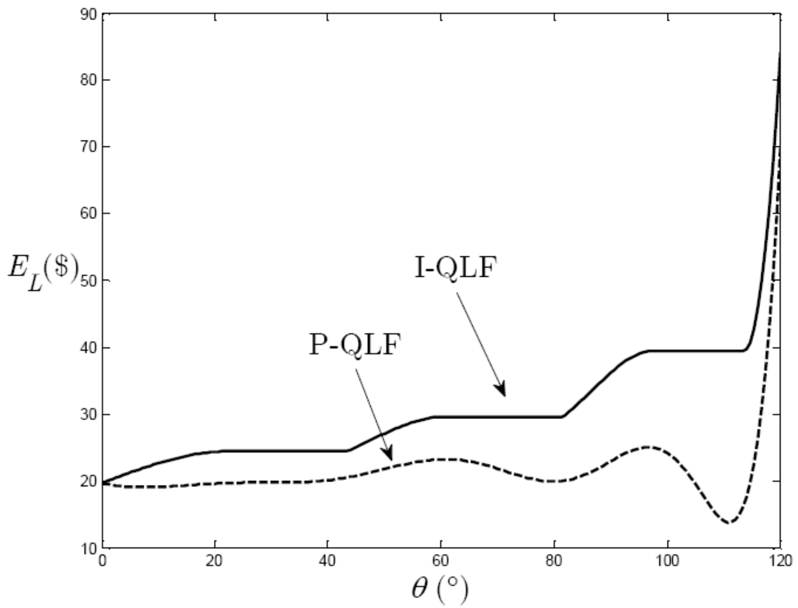
Traditional P-QLF	New I-QLD
$L(\theta)=A[Y(\theta)]^2$ $0° \le \theta \le 120°$	$L(0°,120°)=A \max\{L(\theta)\}$ $0° \le \theta \le 120°$
Min $\sum_{i=1 \text{ to } N} E_L(\theta_i)/N$ s.t. constraints	Min $E_L(0°,120°)$ s.t. constraints

Design variables $(\mu_2, \mu_3, \mu_4, \theta_0, \psi_0)$

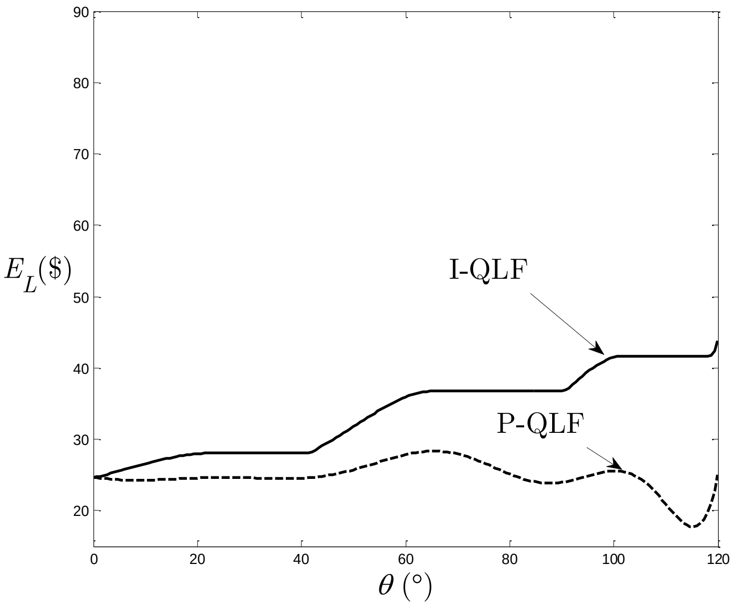
Results

Method	P-QLF	I-QLF
μ_{R_2} (mm)	48.14	43.08
μ_{R_3} (mm)	100.48	103.57
μ_{R_4} (mm)	74.13	66.51
θ_0 (deg)	100.46	97.15
ψ_0 (deg)	99.01	93.89
Average expected I-QLF $\bar{L}_E(\theta)$ ($)	21.60	24.76
Maximal expected I-QLF $\max L_E(\theta)$ ($)	70.0	28.33
Expected I-QLF $L_E(0^\circ, 120^\circ)$ ($)	84.21	43.87

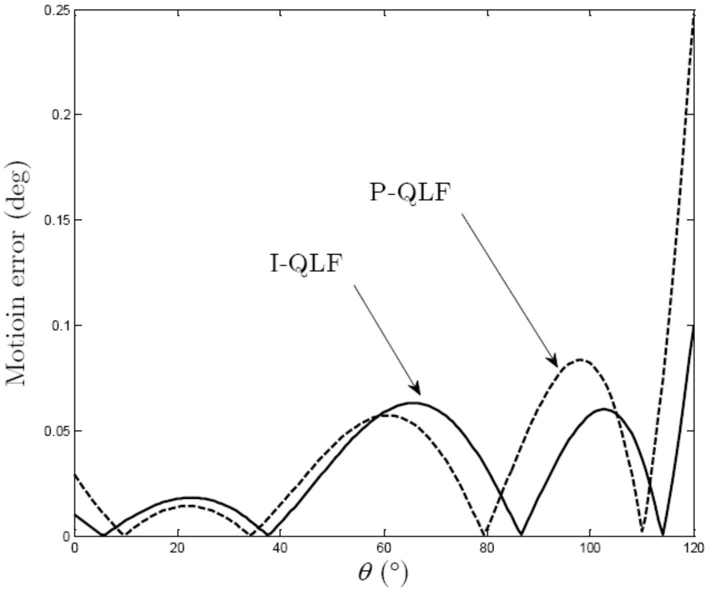
Traditional



New

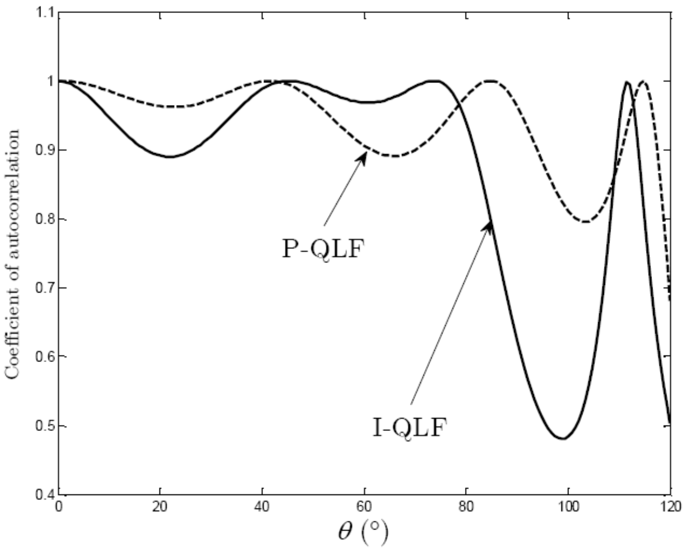


Ave. motion error: The new is better than the traditional.



How does auto correlation look like?

- Between 0° and θ (0°≤θ≤120°)



Conclusions

- Static robustness metrics are not good for time-dependent problems.
- New metrics should account for auto – dependence of time-dependent performances .
- The proposed metric is the only one of many possible metrics.