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A Reliability Approach to Inverse Simulation Under Uncertainty

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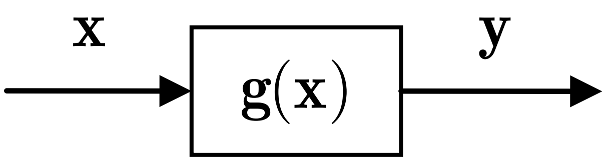
Outline

- Inverse simulation
- Inverse simulation under Uncertainty
  - A reliability and optimization approach
- Example
- Conclusions

Inverse Simulation

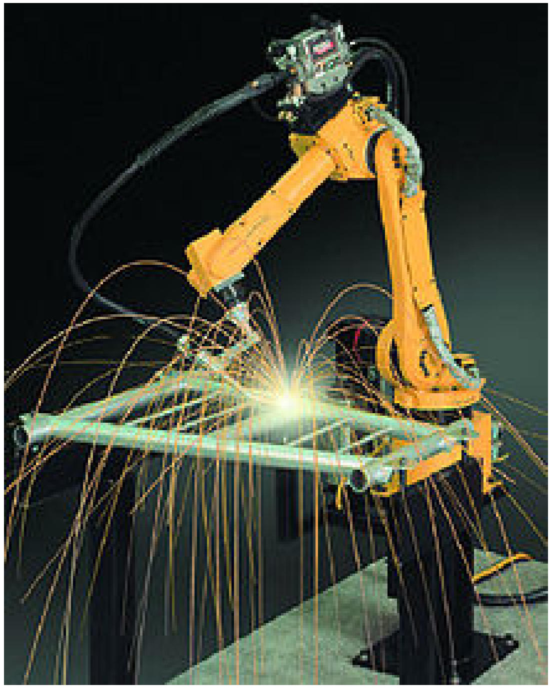
- Direct simulation: Given x find y



- Inverse simulation
  - An inverse process to the direct simulation
  - Given y find part of x

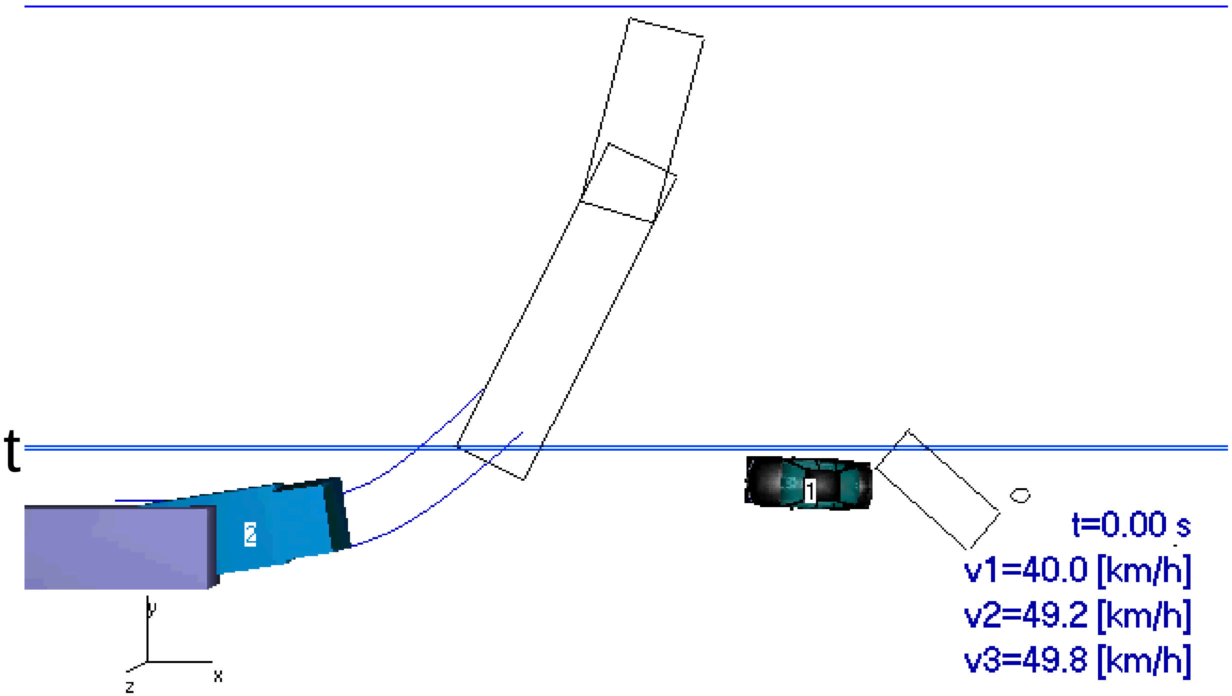
Example: Inverse Kinematics

- Direct modeling
  - Uses joint parameters to compute motion output
- Inverse modeling
  - Determines the joint parameters to achieve desired motion output



Source: http://en.wikipedia.org/wiki/Inverse_kinematics

Example: Accident Reconstruction



Direct simulation

- Input: vehicle speed, position, etc.
- Output: accident consequences

Inverse simulation: accident reconstruction

– Given measured accident consequences

– Find vehicle speed of collision

Source: From Dr. Xaioyun Zhang

Methodology: Model

Direct simulation equations

$$\mathbf{y} = \mathbf{g}(\mathbf{x})$$

$$\mathbf{x} = (\mathbf{x}_{\text{unkn}}, \mathbf{x}_{\text{kn}}, \mathbf{x}_{\text{unc}})$$

$$\begin{cases} y_1 = g_1(\mathbf{x}) \\ y_2 = g_2(\mathbf{x}) \\ \cdots \\ y_m = g_m(\mathbf{x}) \end{cases}$$

Unknown input $\mathbf{x}_{\text{unkn}} = (x_{\text{unkn},1}, \ldots, x_{\text{unkn},n_1})$

Precisely known input $\mathbf{x}_{\text{kn}} = (x_{\text{kn},1}, \ldots, x_{\text{kn},n_2})$

Random known input $\mathbf{x}_{\text{unc}} = (x_{\text{unc},1}, \ldots, x_{\text{unc},n_3})$

Methodology: Task and Approach

Given: CDF of $x_{\text{unc},i}$ ($i = 1, \ldots, n_3$) $F_{\text{unc},i}(x)$,

$\mathbf{x}_{\text{kn}} = (x_{\text{kn},1}, \ldots, x_{\text{kn},n_2})$,

$\mathbf{y} = (y_1, \ldots, y_m)$, and

$\mathbf{g}(\cdot) = (g_1(\cdot), \ldots g_m(\cdot))$

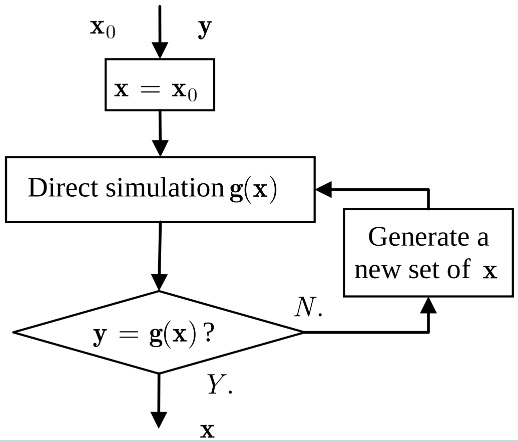
Find: CDF of $x_{\text{ukn},j}$ ($j = 1, \ldots, n_1$) $F_{\text{ukn},j}(x)$

- Assume a unique solution
- Use First Order Reliability Method (FORM) for the CDF
- Use optimization

Methodology: Challenge and Solution

- Challenge: double loop

$$\begin{cases} \min_{\mathbf{u}} \quad \beta = \|\mathbf{u}\| \\ \text{subject to} \\ x_{\text{unkn}} = g^{-1}(\mathbf{x}_{\text{unkn}}, \mathbf{x}_{\text{kn}}, \mathrm{T}(\mathbf{u})) > x \end{cases}$$



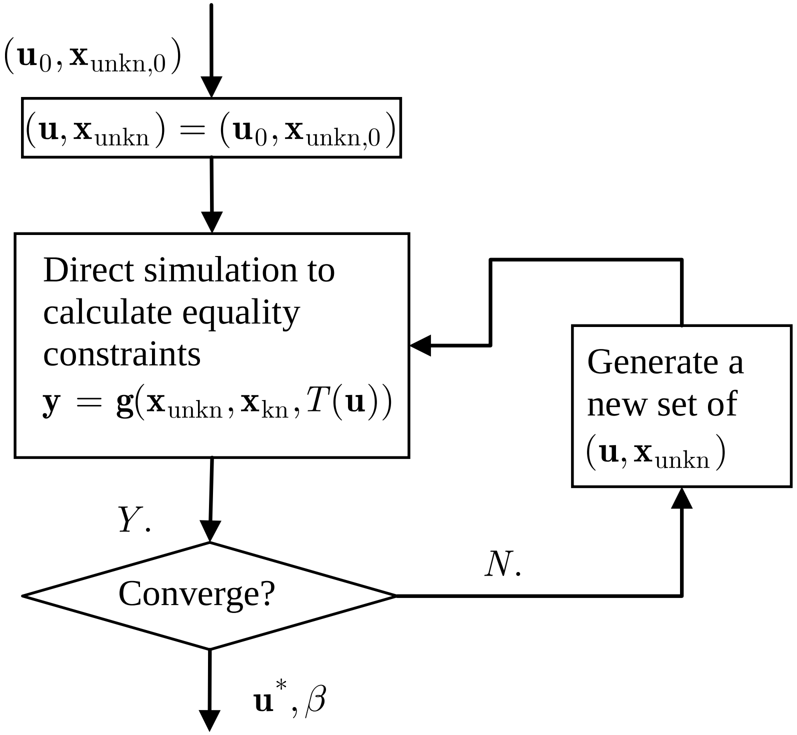
Outer loop: CDF evaluation (reliability analysis FORM)

Embedded inner loop: Inverse simulation

- Solution: combine the two loops → single loop

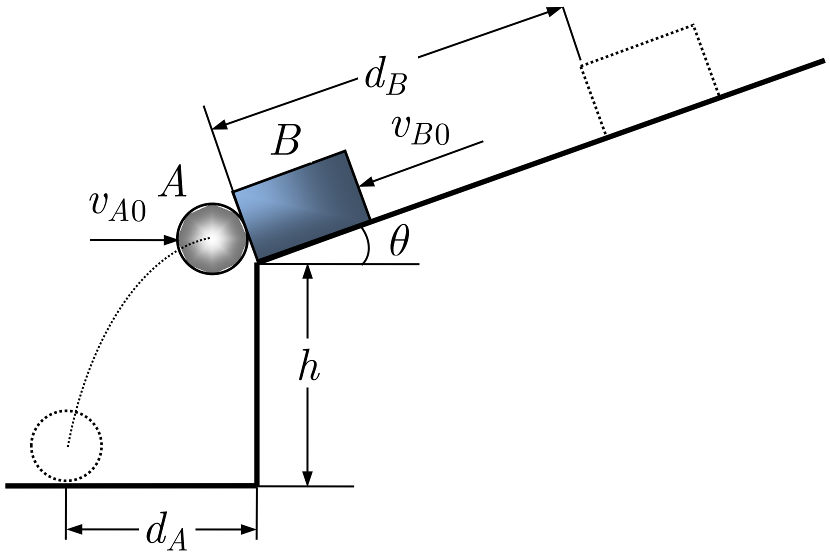
Methodology: Single Loop

- KKT conditions → Single loop solution = Double loop solution
- Much more efficient



Example: Particle Impact

- A hits B with v_{A0}
- After impact
  - A rebounds with d_A
  - B slides with d_B
- Task of inverse simulation
  - What are v_{A0} and v_{B0}?
  - d_A and d_B are measured (observed).



Problem Formulation

- Direct simulation

$$\mathbf{y} = \mathbf{g}(\mathbf{x}_{\text{unkn}}, \mathbf{x}_{\text{kn}}, \mathbf{x}_{\text{unc}})$$

- Unknown input

$$\mathbf{x}_{\text{unkn}} = (v_{A0}, v_{B0})$$

- Known input

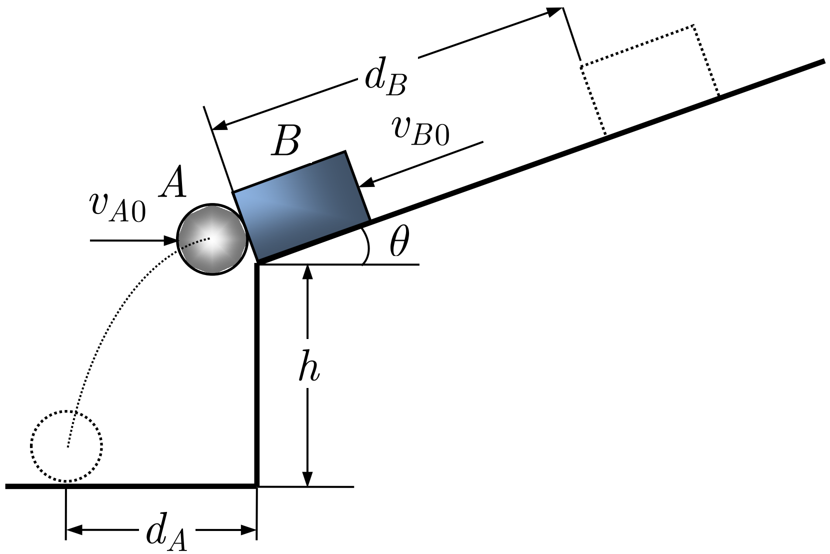
$$\mathbf{x}_{\text{kn}} = (m_A, m_B, h, \theta)$$

- Random input

$$\mathbf{x}_{\text{unc}} = (e, \mu_k) = (\text{coeff of restitution,coeff of friction})$$

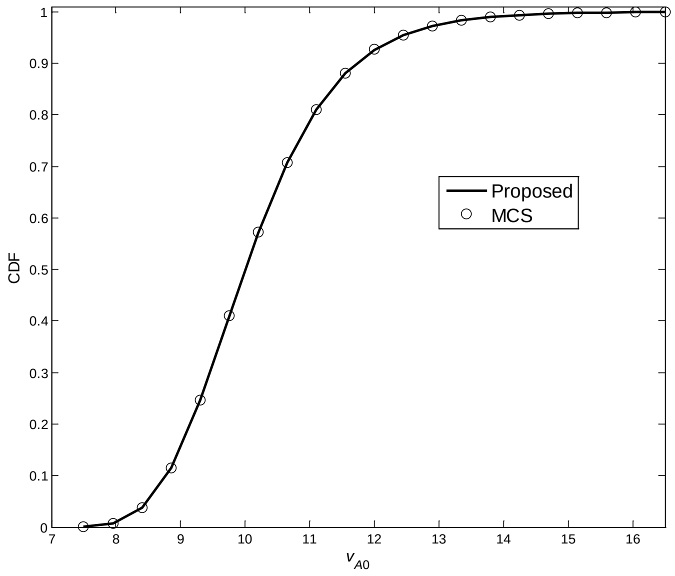
- Output

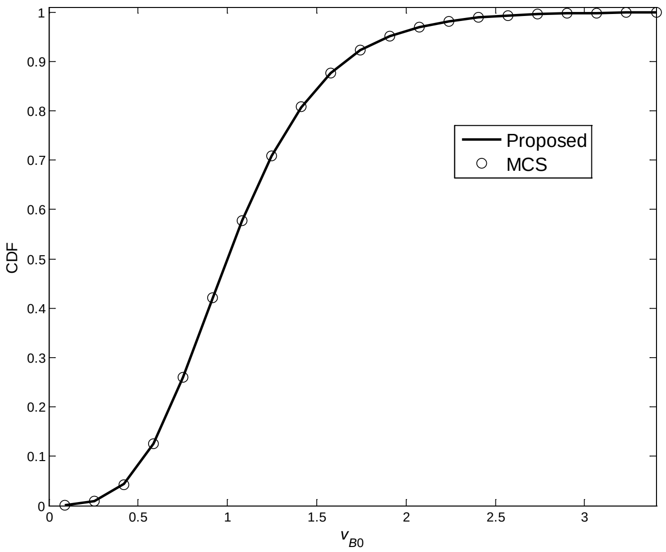
$$\mathbf{y} = (d_A, d_B) = (0.582, 0.708)\ \text{m}$$



Results

- v_{A0}: Mean = 10.16 m/s, std =1.22 m/s
- v_{B0}: Mean = 1,06 m/s, std =0.46 m/s





Conclusions

- As direct simulation, inverse simulation also has uncertainties.
- Considering uncertainty gives more information
  - Distribution
  - Mean and Std
- The proposed method is efficient.
- May not be accurate for highly nonlinear simulation models.

Future Work

- We are working on more advanced methodologies
  - More general problems
  - Maximum likelihood
- Vehicle accident reconstruction
  - Commercial crash simulations
  - Real accident cases