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Time-Dependent Reliability Analysis by a Sampling Approach to Extreme Values of Stochastic Processes

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Outline

- Time-dependent reliability
- A sampling approach to extreme values of stochastic processes
- Reliability analysis
- Examples
- Conclusions
- Future work

Time-Dependent Reliability

- Limit-state functions change with time

$$G = g(\mathbf{X}, Y(t), t)$$

X: Random variables
Y: A stochastic process

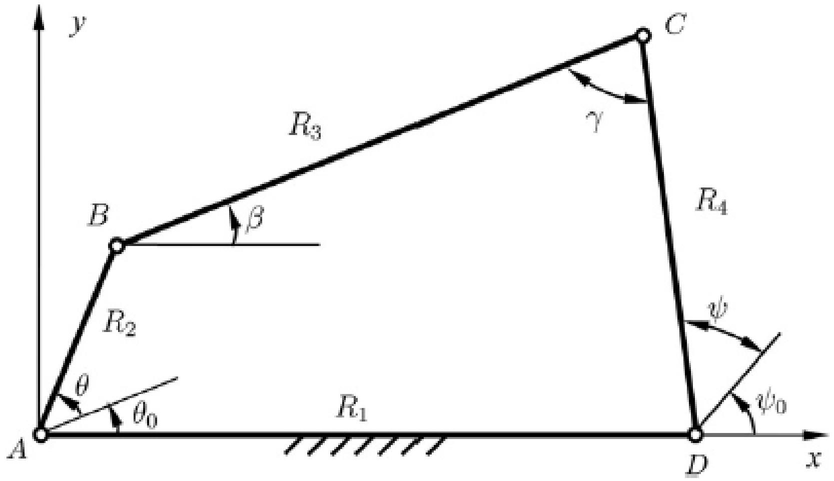
- Reliability

$$R(0, t_s) = \Pr\left\{ g(\mathbf{X}, Y(t)) < 0, \text{ for any } t \in [0, t_s] \right\}$$

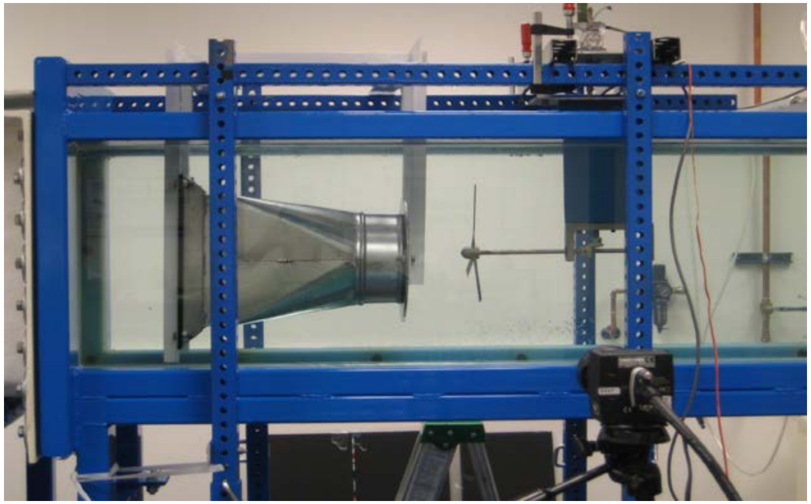
- Reliability is therefore time dependent
- Consistent with the reliability definition
  - Probability of success over a period of time

Examples

- Mechanisms
  - G-function: $G = g(\mathbf{X}, t)$
  - Defined over $[0, t_s]$ *or* $[\theta_0, \theta_s]$



- Hydrokinetic turbine
  - G-function: $G = g(\mathbf{X}, Y(t), t)$
  - Stochastic process
    - Water flow velocity



Challenges

- We need the distribution of extreme values of g(·) over $[0,t_s]$.

$$R(0,t_s)=\Pr\left\{g(\mathbf{X},Y(t))<0,\ \text{for any } t\in[0,t_s]\right\}$$

- Monte Carlo simulation is too expensive.
- The most commonly used method is inaccurate
  - Upcrossing rate method

New Methodology

- Limit-state function $G = g(\mathbf{X}, Y(t))$
- Decompose *Y* into Y_R and Y_S
  - Y_R: Generalized strength variables
  - Y_S: Generalized stress variables
- Worst case over $[0, t_s]$ with
  - Minimum Y_R and maximum Y_S
- Time-dependent $\rightarrow$ Time-independent

$$\begin{aligned} p_f(0, t_s) &= \Pr\left\{ g(\mathbf{X}, Y(t) > 0, \exists t \in [0, t_s] \right\} = \Pr\left\{ \max g(\mathbf{X}, Y_R(t), Y_S(t)) > 0, t \in [0, t_s] \right\} \\ &= \Pr\left\{ g(\mathbf{X}, Y_R^{\min}, Y_S^{\max}) > 0 \right\} \end{aligned}$$

Task and Approach

$$p_f(0, t_s) = \Pr\left\{g(\mathbf{X}, Y_R^{\min}, Y_S^{\max}) > 0\right\}$$

- Task: find distributions of $Y_R^{\min}$ and $Y_S^{\max}$.
- Approach
  - Use Monte Carlo simulation (MCS)
  - Sample on Y_R and Y_S over $[0,t_s]$
  - Obtain samples of $Y_R^{\min}$ and $Y_S^{\max}$.
  - It will not call the g-function
- Then time-independent analysis
  - FORM, SORM, etc.

Sampling Approach

- Expansion Optimal Linear Estimation method (EOLE) is used to generate samples for *Y*
- Saddlepoint Approximation (SPA) is employed to approximate the CDF of $Y_R^{\min}$ and $Y_S^{\max}$
- SPA maintains the robustness of reliability analysis

Procedure

- Identify Y_R and Y_S
- Find distributions of the extreme values of Y_R and Y_S by sampling
- Perform time-invariant reliability analysis

Example: Hydrokinetic Turbine Blades

- Random variables

$$\mathbf{X} = [l_1,\, t_1,\, t_2,\, \varepsilon_{allow}]$$

- Generalized stress variables

$$Y_s = v(t)$$

Limit-State Function

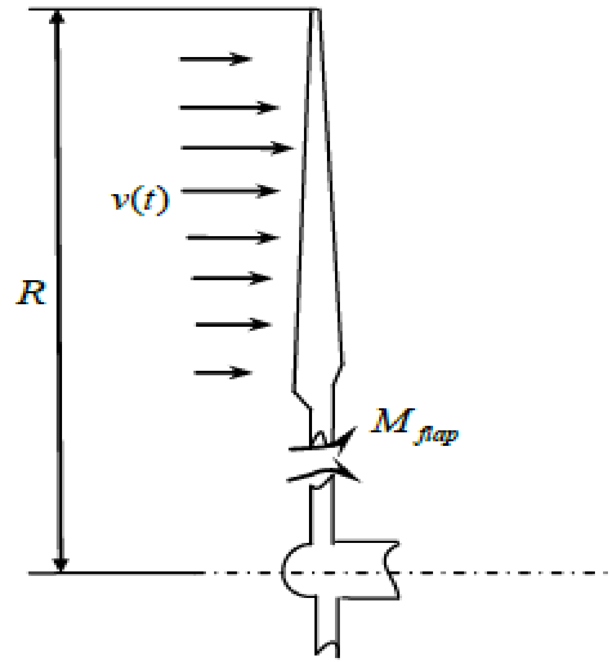
Bending moment

$$M_{flap} = \frac{1}{2}\rho v(t)^2 C_m$$

Limit-state function

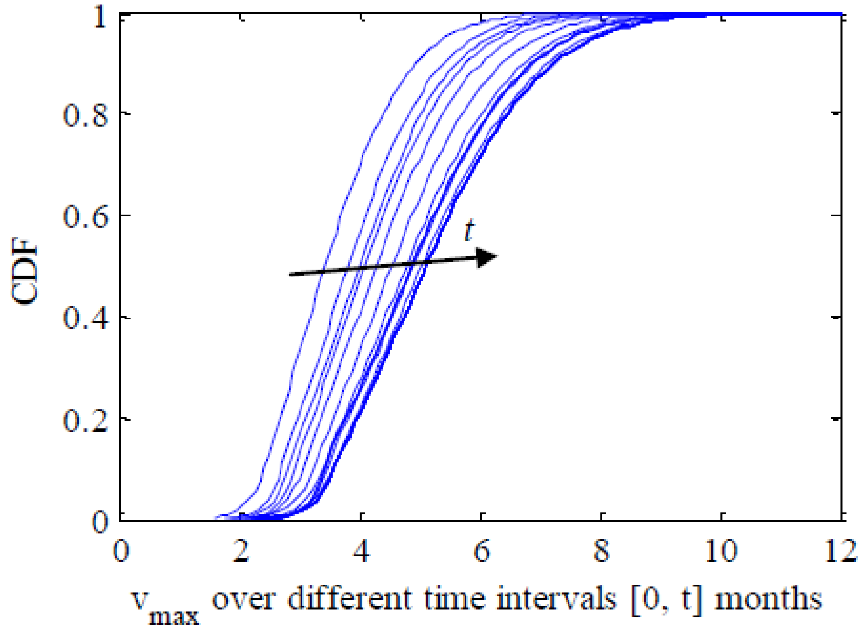
$$g = \varepsilon_{allow} - \frac{M_{flap} t_1}{EI} = \varepsilon_{allow} - \frac{\rho v(t)^2 C_m t_1}{2EI}$$

$$I = \frac{1}{12} l_1 ((2t_1)^3 - (2t_2)^3) = \frac{2}{3} l_1 (t_1{}^3 - t_2{}^3)$$



$v(t)$ is a non-stationary stochastic process ⇒ Mean and standard deviation are functions of time

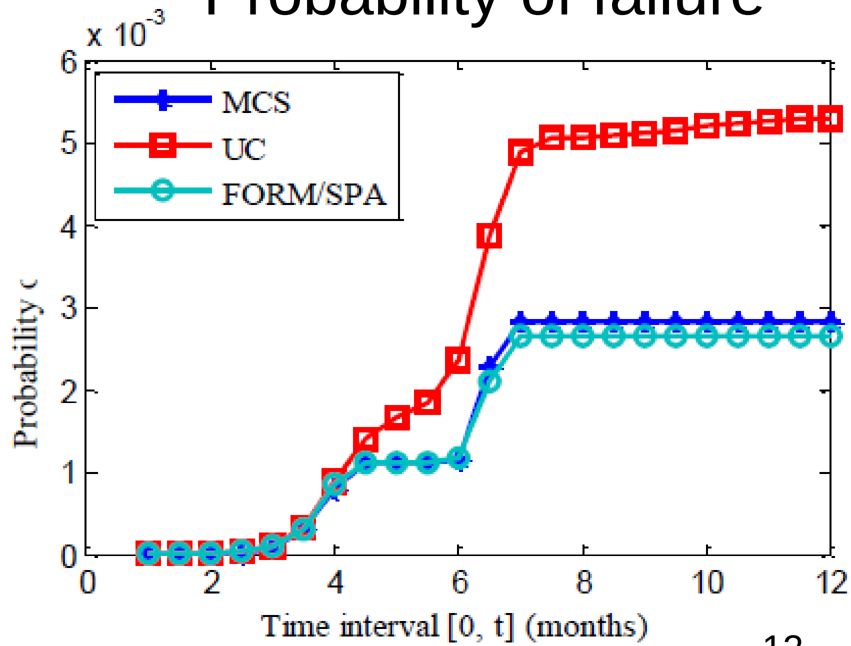
Results: Accuracy



Maximum river velocity

UC: Up-crossing rate method

Probability of failure



Results: Efficiency

Number of function calls

Time interval (months)	UC	Proposed	MCS
[0, 4]	2522	102	10^6
[0, 5]	6539	85	10^6
[0, 6]	6761	121	10^6
[0, 7]	11500	98	10^6
[0, 8]	6678	98	10^6
[0, 9]	19399	98	10^6

Conclusions

- The accuracy of the proposed method is good
- The proposed method is efficient
- Applicable for problems with non-stationary stochastic loading
- Limitations
  - $G = g(\mathbf{X}, Y(t))$
  - Y can be decomposed into Y_R and Y_S

Future Work

- We are working on more advanced methodologies
  - General problems $G=g(\mathbf{X}, Y(t), t)$
  - Time-dependent system reliability
- Design optimization with time-dependent uncertainties

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