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An Envelope Approach to Time-Dependent Reliability Analysis for Mechanisms

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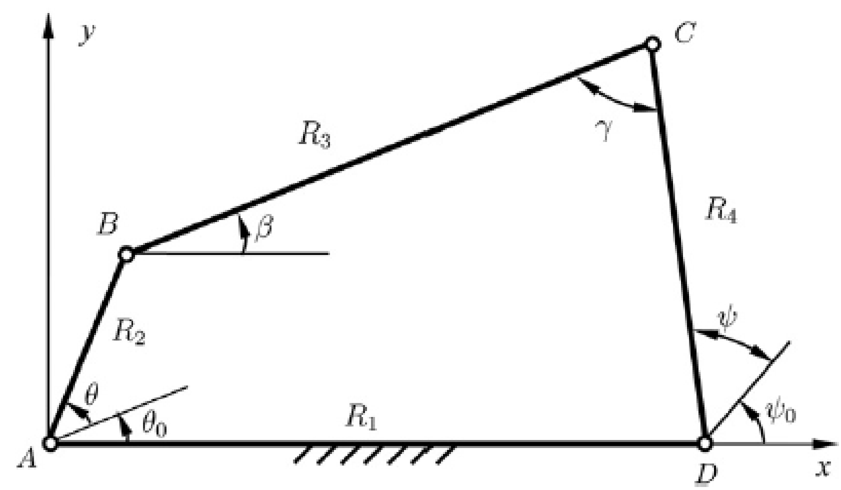
Introduction

- Motion error
- Why time-dependent reliability
- The envelope method
- Example
- Conclusions

Mechanisms and Motion Error

- Mechanisms transmit desired motion and/or force
- Motion error



$$g(\mathbf{X}, \theta) = \psi(\mathbf{X}, \theta) - \psi_d(\theta)$$

Dimensions — time — Motion output — Required motion output

Motion Reliability

- Motion error $g(\mathbf{X}, \theta)$
  - Structural error: caused by types of mechanisms, synthesis methods, etc.
  - Random (mechanical) error: caused by randomness in **X**.
- We wish $|g(\mathbf{X}, \theta)| \leq \varepsilon$, but it may not hold absolutely.
- So we define motion reliability

$$R = \Pr\{|g(\mathbf{X}, \theta)| \leq \varepsilon\}$$

What's Done for Mechanism Reliability

- Point reliability at a specific instant

$$R(\theta) = \Pr\{|g(\mathbf{X}, \theta)| \leq \varepsilon\}$$

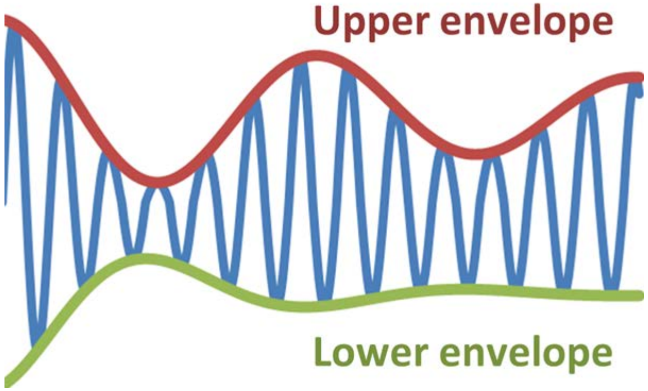
- Probability $\Pr\{|g(\mathbf{X}, \theta)| \leq \varepsilon\}$ at θ
- Probabilistic analysis and synthesis
  - FOSM
  - MCS
  - FORM

Unsolved Problems

- Probability that a mechanism achieves its intended motion over $[0, \theta]$?
  $$R(\theta) = \Pr\{|g(\mathbf{X}, \tau)| \leq \varepsilon, \forall \tau \in [0, \theta]\}$$
- It is time-dependent reliability.
- Rice's formula can be used; may be inaccurate.
- MCS can be used, but costly.

The Envelope Approach

- $R(0, \theta) = \Pr\{|g(\boldsymbol{X}, \tau)| \leq \varepsilon, \forall \tau \in [0, \theta]\}$
- Find an envelope function $G(\mathbf{X})$ of $g(\mathbf{X}, \tau)$
- Then $R(0, \theta) = \Pr\{|G(\boldsymbol{X})| \leq \varepsilon\}$, which is time independent.



Source:
http://en.wikipedia.org/wiki/File:Signal_envelopes.png

A Little Math

- $G^{+}(\mathbf{X})$ is determined by

$$\begin{cases} g(\boldsymbol{X},\tau)=\varepsilon \\ \dot{g}(\boldsymbol{X},\tau)=0 \end{cases}, \tau\in[0,\theta]$$

- $G^{-}(\mathbf{X})$ is determined by

$$\begin{cases} g(\boldsymbol{X},\tau)=-\varepsilon \\ \dot{g}(\boldsymbol{X},\tau)=0 \end{cases}, \tau\in[0,\theta]$$

- It is possible to find $G^{+}(\mathbf{X})$ and $G^{-}(\mathbf{X})$
  - $g(\boldsymbol{X},\tau)$: displacement
  - $\dot{g}(\boldsymbol{X},\tau)$: velocity

Procedure

- Linearize $g(\mathbf{X}, \tau)$ at means $\boldsymbol{\mu}_X$
- The approximation is accurate due to small tolerances.
- It is still nonlinear w.r.t time τ.
- $g(\mathbf{X}, \tau) \approx L(\mathbf{X}, \tau) = c_0(\tau) + \sum c_i(\tau) X_i$
- Transform $\mathbf{X}$ into $\mathbf{U}$ (standard normal)

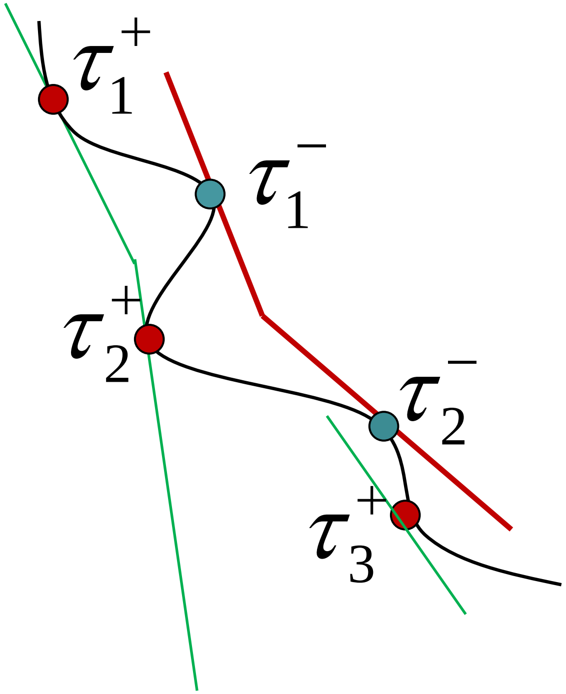
$$L = b_0(\tau) + \sum b_i(\tau) U_i$$

- Obtain τ_i by solving (for G^+)

$$\dot{b}_0(\tau) + \left(\varepsilon - b_0\right)\frac{\dot{\mathbf{b}}(\tau)\cdot\mathbf{b}(\tau)}{\mathbf{b}(\tau)\cdot\mathbf{b}(\tau)} = 0$$

Procedure

- Then $g(\mathbf{X}, \tau)$ is linearized at τ_i (piecewise linearization)
- Then reliability is estimated by multivariate normal CDF at τ_i.
- But the covariance matrix may not be positive definite (as τ_3^+ is not needed.)
- So eliminate τ_3^+.

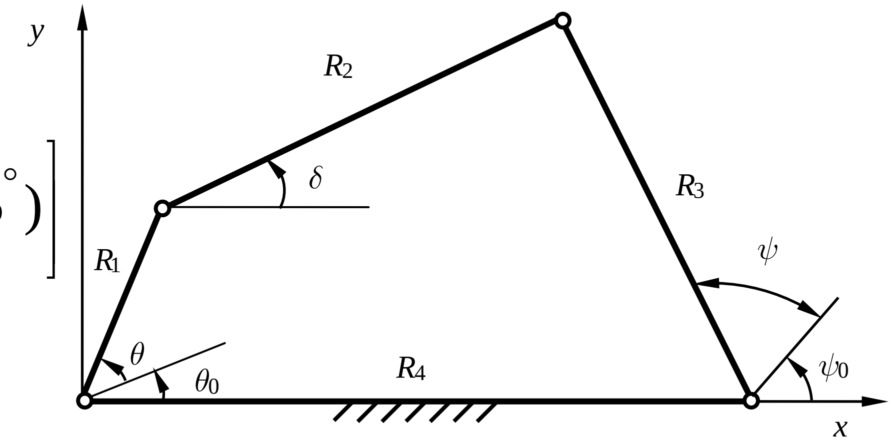


Example: 4-Bar Function Generating Mechanisms

- Required motion

$$\psi_d(\theta) = 76^\circ + 60^\circ \sin\left[\frac{3}{4}(\theta - 95.5^\circ)\right]$$

$$\theta \in [95.5^\circ, 215.5^\circ]$$



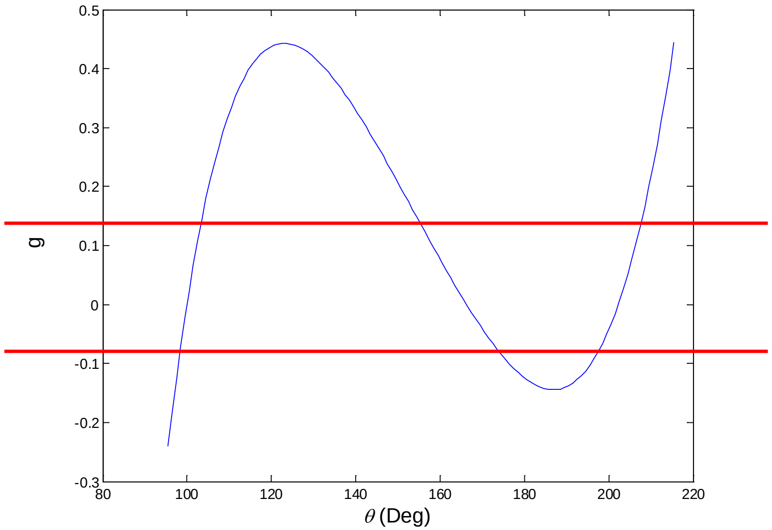
Variables	Mean (mm)	Standard deviation (mm)	Distribution
R_1	$\mu_1 = 53.0$	$\sigma_1 = 0.1$	Normal
R_2	$\mu_2 = 122.0$	$\sigma_2 = 0.1$	Normal
R_3	$\mu_3 = 66.5$	$\sigma_3 = 0.1$	Normal
R_4	$\mu_4 = 100.0$	$\sigma_4 = 0.1$	Normal

Results

Average motion error

$$\tau_i = 95.5^\circ, 122.982^\circ, 186.8522^\circ, 215.5^\circ$$

$$\mathbf{\Sigma} = \begin{pmatrix} 0.5373 & -0.4787 & 0.4466 & -0.4193 \\ -0.4787 & 0.4823 & -0.5235 & 0.4933 \\ 0.4466 & -0.5235 & 0.6576 & -0.6262 \\ -0.4193 & 0.4933 & -0.6262 & 0.6028 \end{pmatrix} \times 10^{-5}$$



Rank = 3; not positively definite

Delete 186.8522° where the point probability of failure is the least.

$$\mathbf{\Sigma}' = \begin{pmatrix} 0.6028 & 0.4933 & -0.4193 \\ 0.4933 & 0.4823 & -0.4787 \\ -0.4193 & -0.4787 & 0.5373 \end{pmatrix} \times 10^{-5}$$

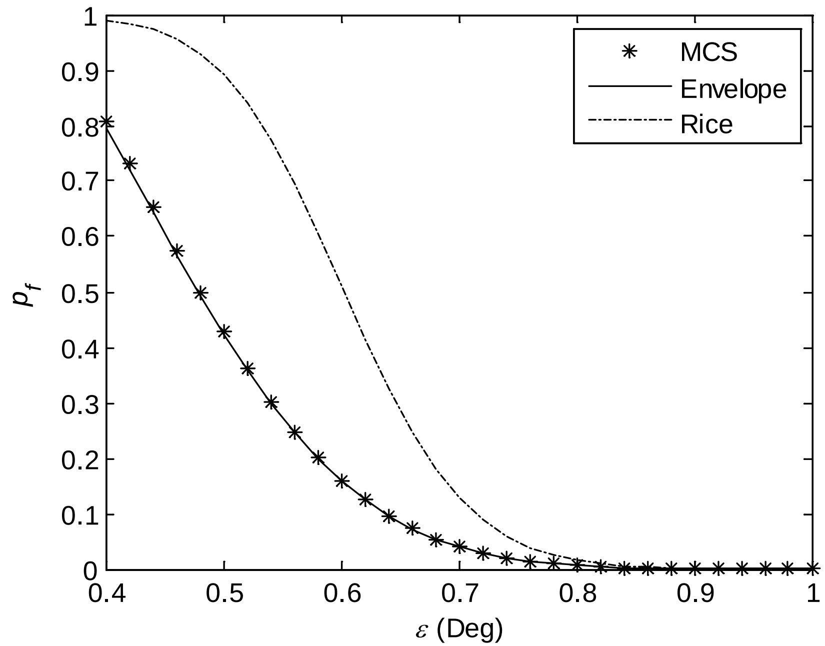
Positive definite now

Then calculate joint CDF at $\tau_i = 95.5^\circ, 122.982^\circ, 215.5^\circ$

Results

Probability of failure $p_f = 1 - R(95.5°, 215.5°)$

ε	Rice	Envelope	MCS
0.4°	9.9043×10^{-1}	7.9536×10^{-1}	8.0842×10^{-1}
0.5°	8.9227×10^{-1}	4.2287×10^{-1}	4.2760×10^{-1}
0.6°	5.0934×10^{-1}	1.5858×10^{-1}	1.5975×10^{-1}
0.7°	1.2874×10^{-1}	3.9466×10^{-2}	3.9769×10^{-2}
0.8°	1.5408×10^{-2}	6.2935×10^{-3}	6.3632×10^{-3}
0.9°	1.0173×10^{-3}	6.3494×10^{-4}	6.4310×10^{-4}



Conclusions

- The method is good for mechanism reliability, where
  - The main error dominates
  - Then number of upcrossings and downcrossings are small
- Produces analytical solution
  - No optimization
  - No simulations

Future Work

- Time-dependent probabilistic mechanism synthesis

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