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Probabilistic Inverse Simulation and Its Application in Vehicle Accident Reconstruction

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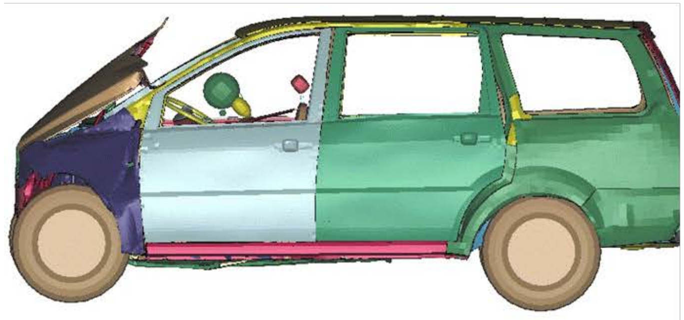
Outline

- Inverse simulation example: traffic accident reconstruction
- Generalized probabilistic inverse simulation
- Inverse simulation with the highest probability density
- Examples
- Conclusions
- Future work

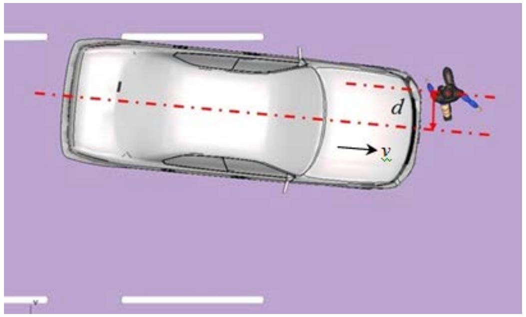
An Example for Inverse Simulation – Traffic Accident Reconstruction

- Direct simulation: Cause => Consequence
- Inverse simulation: Consequence => Cause
- Vehicle accident reconstruction involves inverse simulation
  - Given: accident consequences
  - Find: pre-accident events

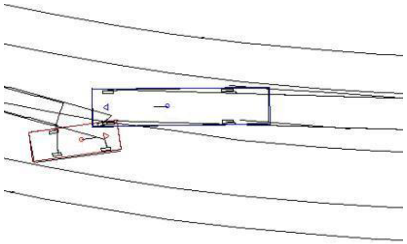


More Examples

Identify pre-impact velocity



Determine vehicle trajectory



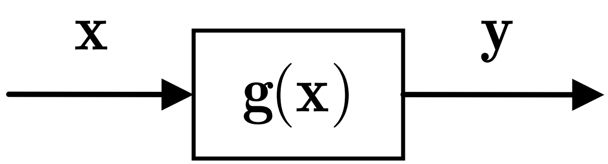
Identify cause of injury

Challenges in Traffic Accident Reconstruction

- Accident reconstruction simulation is computationally expensive
- Many uncertainties
- Input information is limited
- Traditional reconstruction may generate multiple solutions

Inverse Simulation Under Uncertainty

- Direct simulation: Given x find y



- Inverse simulation
  - Given: y
  - Find: x
- Vehicle Accident Reconstruction
  - x: vehicle velocity just before collision
  - y: accident consequences from the scene
  - Random variables exist, such as coefficient of friction

Probabilistic Inverse Simulation

Classify inputs into three groups

$$\mathbf{x} = (\mathbf{x}_{\text{unkn}}, \mathbf{x}_{\text{rand}}, \mathbf{x}_{\text{kn}})^{\mathrm{T}}$$

- $\mathbf{x}_{\text{unkn}}$, unknown deterministic variables
- $\mathbf{x}_{\text{rand}}$, unknown variables with known distributions
- $\mathbf{x}_{\text{kn}}$, variables that are known deterministically
- Simulation equations

$$\mathbf{y} = \mathbf{g}(\mathbf{x}) = \mathbf{g}(\mathbf{x}_{\text{unkn}}, \mathbf{x}_{\text{rand}})$$

The New Method

Maximize the joint probability density Function (PDF)

Max: Joint PDF

Subject to: Simulation Equations

Detailed Model

- Reliability analysis and optimization are employed
- Maximize the joint probability density
- A unique solution is identified
- The solution of $\mathbf{x}_{\mathrm{rand}}$ is $\mathbf{x}^*_{\mathrm{rand}}$, where the joint probability density is maximum.

$$\begin{cases} y_1 = g_1(\mathbf{x}_{\mathrm{unkn}}, \mathbf{x}_{\mathrm{rand}}) \\ y_2 = g_2(\mathbf{x}_{\mathrm{unkn}}, \mathbf{x}_{\mathrm{rand}}) \\ \dots \\ y_m = g_m(\mathbf{x}_{\mathrm{unkn}}, \mathbf{x}_{\mathrm{rand}}) \end{cases} \Rightarrow \begin{cases} \max\limits_{(\mathbf{x}_{\mathrm{unkn}}, \mathbf{x}_{\mathrm{rand}})} f(\mathbf{x}_{\mathrm{rand}}) \\ \text{subject to} \\ \mathbf{y} = \mathbf{g}(\mathbf{x}_{\mathrm{unkn}}, \mathbf{x}_{\mathrm{rand}}) \end{cases}$$

When Model Uncertainty Included

$$
\begin{cases}
\max\limits_{(\mathbf{x}_{\text{unkn}},\mathbf{x}_{\text{rand}})} f(\mathbf{x}_{\text{unc}},\mathbf{x}_{\text{rand}}) \\
\text{subject to} \\
(1-\varepsilon_1)y_1 \le g_1(\mathbf{x}_{\text{unc}},\mathbf{x}_{\text{rand}}) \le (1+\varepsilon_1)y_1 \\
(1-\varepsilon_2)y_2 \le g_2(\mathbf{x}_{\text{unc}},\mathbf{x}_{\text{rand}}) \le (1+\varepsilon_2)y_2 \\
\dots \\
(1-\varepsilon_m)y_m \le g_m(\mathbf{x}_{\text{unc}},\mathbf{x}_{\text{rand}}) \le (1+\varepsilon_m)y_m
\end{cases}
$$

- Model uncertainty is treated in an interval
- The bound is the percentage error

Implementation

Transform random variables into standard normal random variables **U**

$$
\begin{cases}
\min\limits_{(\mathbf{x}_{\text{unkn}},\mathbf{u})} \sum\limits_{i=1}^{n_{\text{rand}}} u_i^2 \\
\text{subject to} \\
(1-\varepsilon_1)y_1 \le g_1(\mathbf{x}_{\text{unkn}}, F^{-1}(\Phi(\mathbf{u}))^{\text{T}}) \le (1+\varepsilon_1)y_1 \\
(1-\varepsilon_2)y_2 \le g_2(\mathbf{x}_{\text{unkn}}, F^{-1}(\Phi(\mathbf{u}))^{\text{T}}) \le (1+\varepsilon_2)y_2 \\
\dots \\
(1-\varepsilon_m)y_m \le g_m(\mathbf{x}_{\text{unkn}}, F^{-1}(\Phi(\mathbf{u}))^{\text{T}}) \le (1+\varepsilon_m)y_m
\end{cases}
$$

Control variables are $\mathbf{x}_{\text{unkn}}$ and $\mathbf{u}$

Advantages

- All information available is used.
- Highest confidence is obtained.
- A unique solution is identified.

Examples – A Mathematical Example

$$\begin{cases} y_1 = g_1(\mathbf{x}_{\text{unkn}}, \mathbf{x}_{\text{rand}}) \\ \quad = x_{\text{unkn}} + x_{\text{rand},1} + x_{\text{rand},2} \\ y_2 = g_2(\mathbf{x}_{\text{unkn}}, \mathbf{x}_{\text{rand}}) \\ \quad = x_{\text{unkn}} + 2x_{\text{rand},1} + 3x_{\text{rand},2} \end{cases}$$

Table 1 Output variables and distributions of random input variables

Variable	y_1	y_2	$x_{\text{rand},1}$	$x_{\text{rand},2}$
Type	Det	Det	Normal	Normal
Mean	1	5	1	1
STD	0	0	0.5	0.5

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Examples – A Mathematical Example

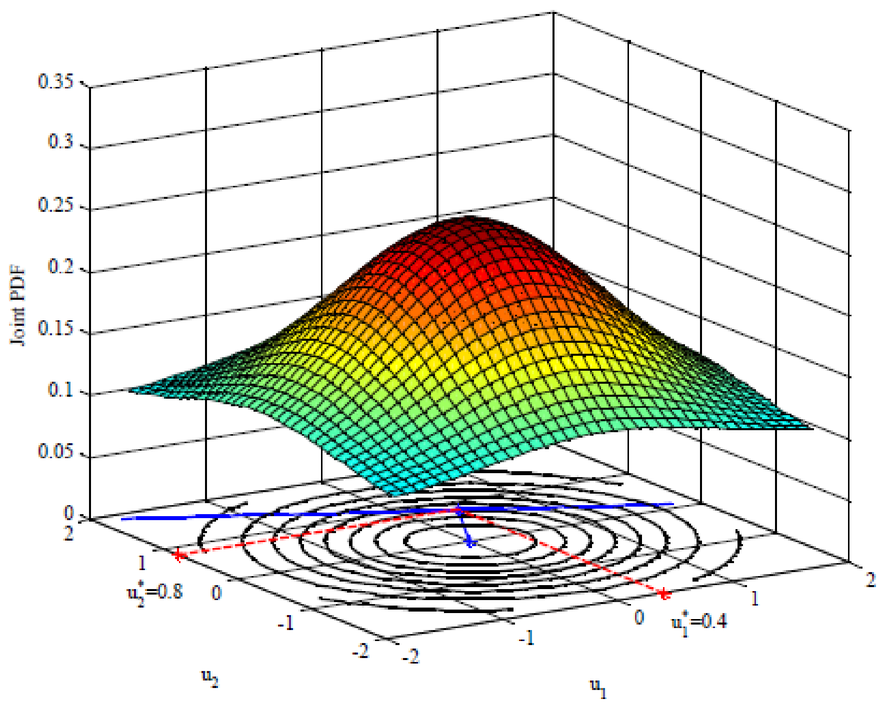


Figure 2. Joint PDF of u_1 and u_2

$$u_1^* = 0.4 \quad u_2^* = 0.8$$

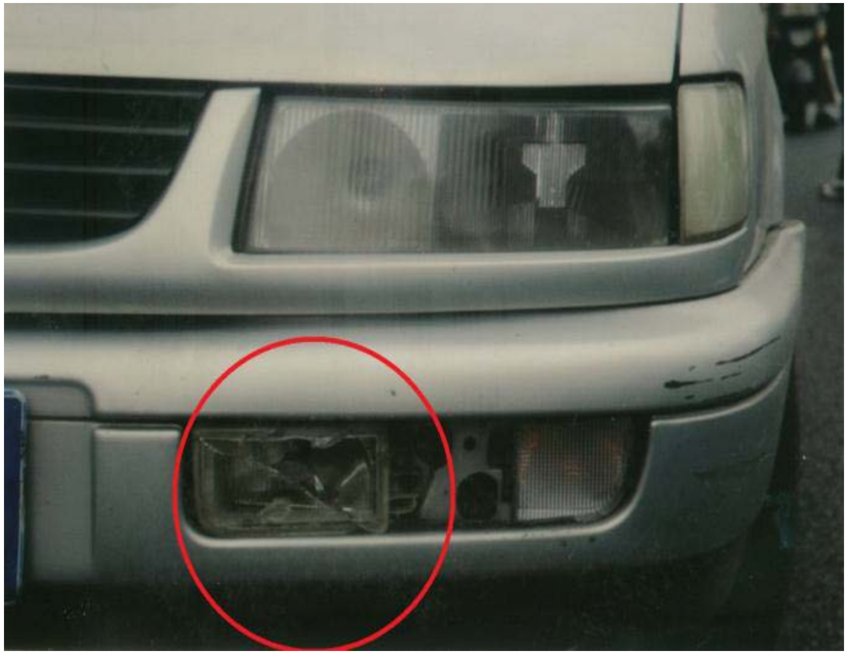
$$\begin{cases} y_1 = g_1(\mathbf{x}_{\text{unkn}}, \mathbf{x}_{\text{rand}}) \\ \quad = x_{\text{unkn}} + x_{\text{rand},1} + x_{\text{rand},2} \\ y_2 = g_2(\mathbf{x}_{\text{unkn}}, \mathbf{x}_{\text{rand}}) \\ \quad = x_{\text{unkn}} + 2x_{\text{rand},1} + 3x_{\text{rand},2} \end{cases}$$

$$\begin{cases} x_{\text{unkn}} + 0.5(u_1 + u_2) = -1 \\ x_{\text{unkn}} + u_1 + 1.5u_2 = 0 \end{cases}$$

$$u_1 + 2u_2 = 2$$

Application – Traffic Accident Reconstruction

- The vehicle speed at the moment of accident needs to be determined.
- Post-accident data were collected at the accident scene, such as the rest position of the victim.



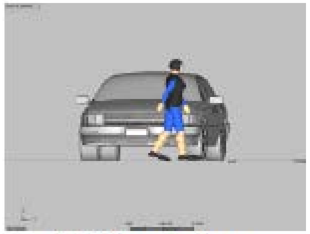
Task

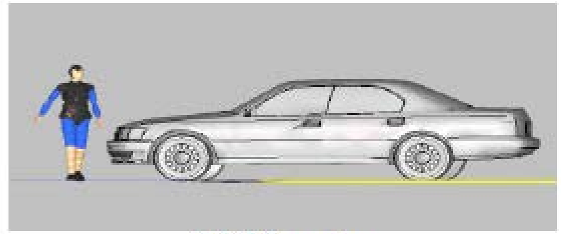
- **Given: *y*,** rest position of the victim according to blood marks
- **Find: $\mathbf{x}_{unkn}$,** vehicle speed at the moment of accident

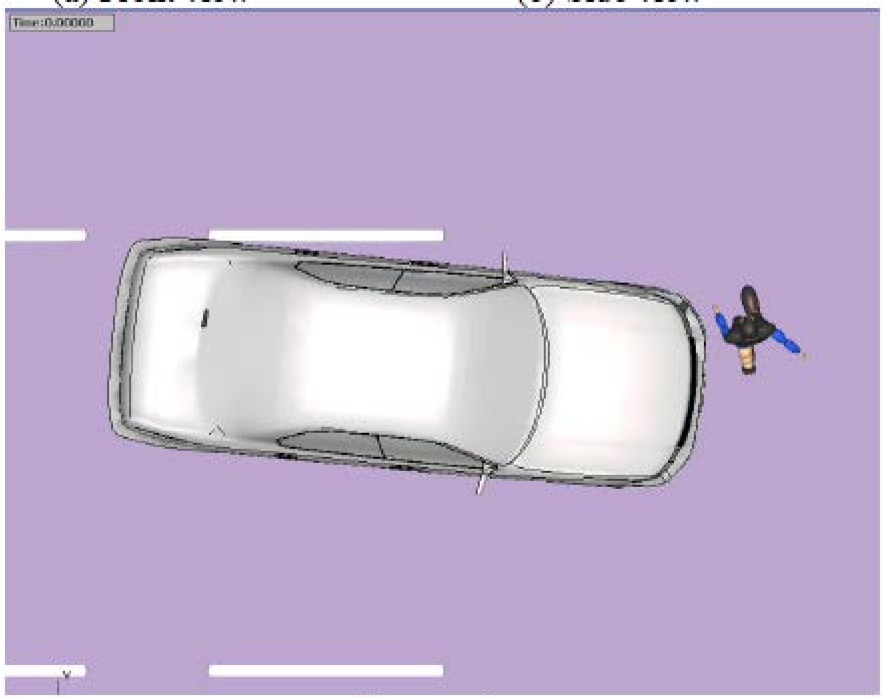
  $\mathbf{x}_{rand}$, coefficient of friction and relative distance between vehicle and victim

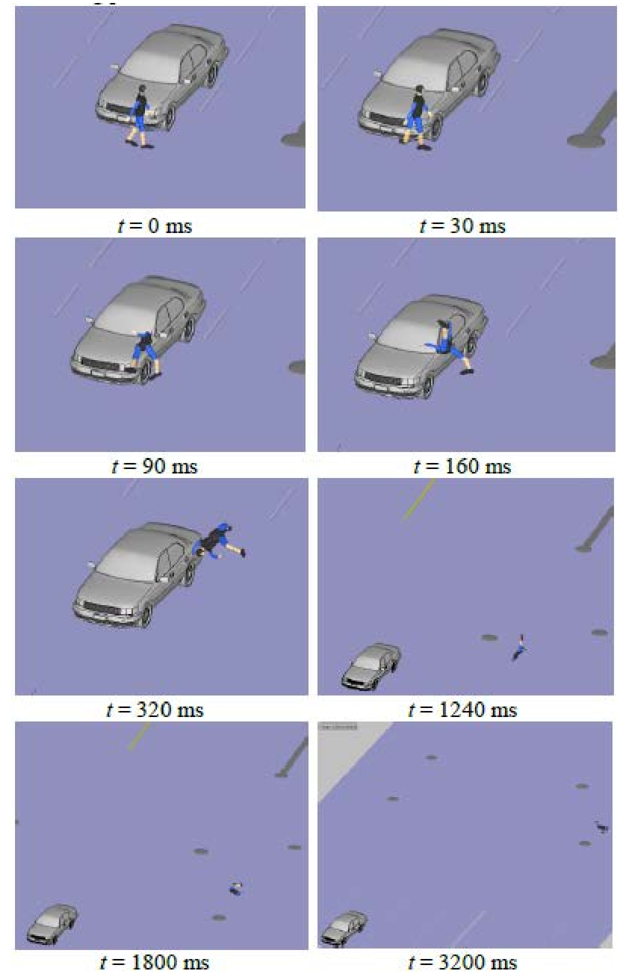
Variable	S_x(m)	S_y(m)	v (km/h)	d(m)	μ_k
Type	Det	Det	Det	Normal	Normal
Mean	9.59 m	17.02	[40, 100]	0.4 m	0.7
STD	0	0	0	0.2 m	0.1

Direct Crash Simulation


(a) Front view


(b) Side view


(c) Top view



Inverse Simulation Model

Probabilistic inverse simulation

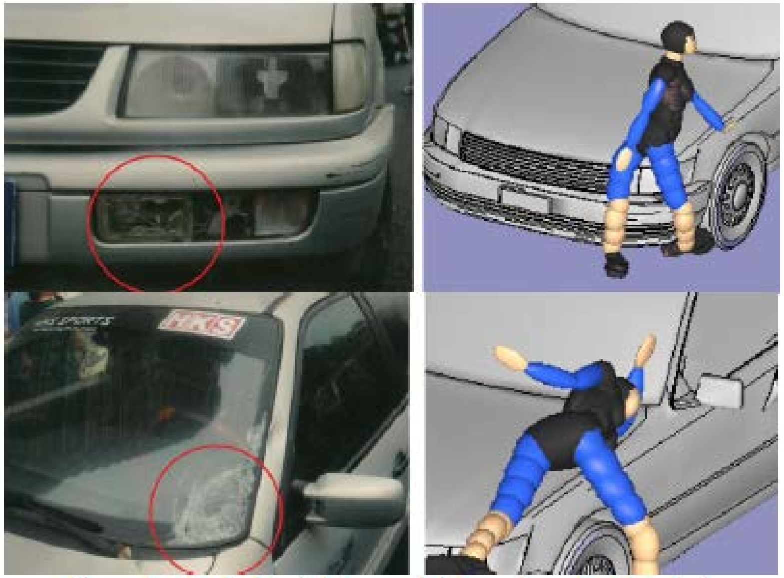
$$
\begin{cases}
\underset{v,\,u_1,\,u_2}{\text{Min}} \ \sum_{i=1}^{2} u_i^2 \\
\text{Subject to} \\
\xi_1 = u_1,\ \xi_2 = u_2 \\
\xi_3 = (2v - v_L - v_U) \big/ (v_U - v_L) \\
s_x^*(1-\varepsilon_x) \le \sum_{i=0}^{3} \sum_{j=0}^{3-i} \sum_{k=0}^{3-i-j} \chi_{(i,\,j,\,k)}^{x} L_i(\xi_1) H_j(\xi_2) H_k(\xi_3) \le s_x^*(1+\varepsilon_x) \\
s_y^*(1-\varepsilon_y) \le \sum_{i=0}^{3} \sum_{j=0}^{3-i} \sum_{k=0}^{3-i-j} \chi_{(i,\,j,\,k)}^{y} L_i(\xi_1) H_j(\xi_2) H_k(\xi_3) \le s_y^*(1+\varepsilon_y)
\end{cases}
$$

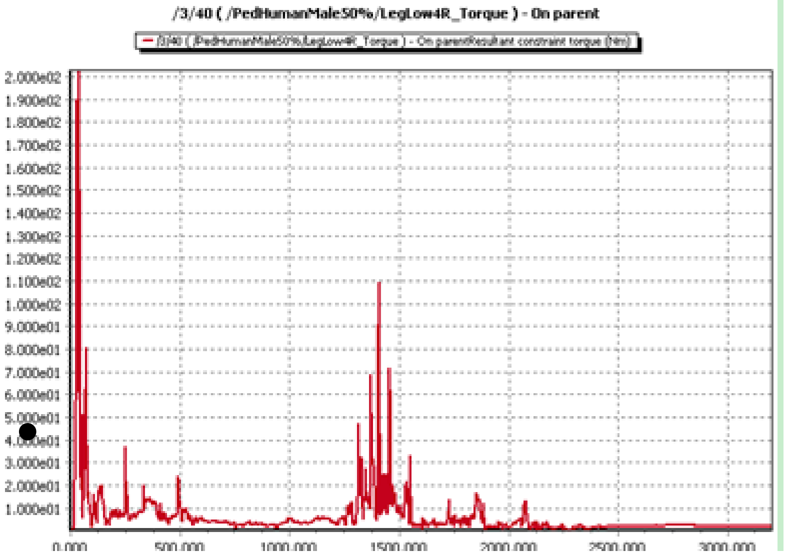
Polynomial Chaos Expansion (PCE)

- Surrogate models are constructed to replace the direct simulation model

Results

- Velocity estimated from video record = [67, 69] km/h
- Velocity from inverse simulation = 68.98 km/h





Conclusions

- Uncertainties in inverse simulation should be considered.
- Probabilistic analysis methods can be used to accommodate uncertainties.
- The proposed method involves reliability analysis and optimization.
- It maximizes the joint PDF.
- Examples demonstrate the effectiveness of the method.

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Future work

- Consider conditional probabilities
- Incorporate probabilistic model uncertainty

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