A Mixed Efficient Global Optimization (m-EGO) Based Time-Dependent Reliability Analysis Method

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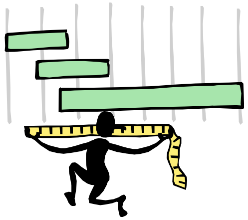
Outline

- Problem statement
- Mixed Efficient Global Optimization (m-EGO)
- Time-Dependent Reliability Analysis with m-EGO
- Numerical Examples
- Conclusion and future works

Problem statement

Uncertainties

- ❑ Variation in dimensions, material properties, and other parameters ⟸ Does not change with time



- ❑ Stochastic loadings including wind, river, wave loadings ⟸ Vary with time

Effects on the design

- ❑ System response varies with time (i.e. time-dependent characteristics)
- ❑ The longer the time interval, the lower the reliability
- ❑ Time-dependent reliability analysis methods need to be employed

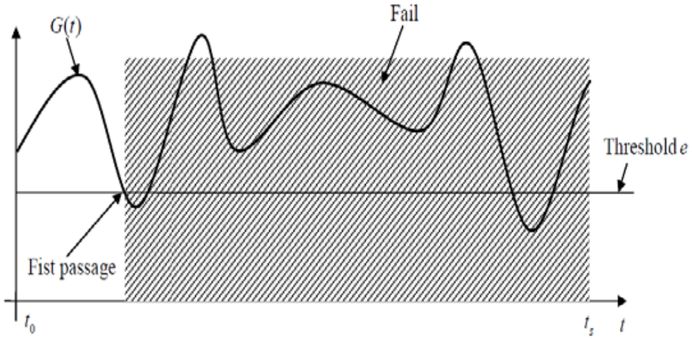
Limit-State Function: $G = g(\mathbf{X}) \Longrightarrow G(t) = g(\mathbf{X}, \mathbf{Y}(t), t)$

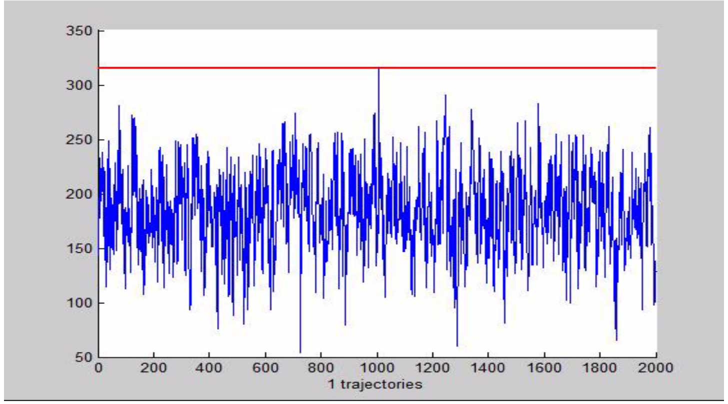
Problem statement

Time-dependent reliability: *the probability that the system can still work after a certain time period*

$$p_f = \Pr\{G = g(\mathbf{X}, \mathbf{Y}(\tau), \tau) < e, \exists \tau \in [0, T]\}$$

First-passage failure:



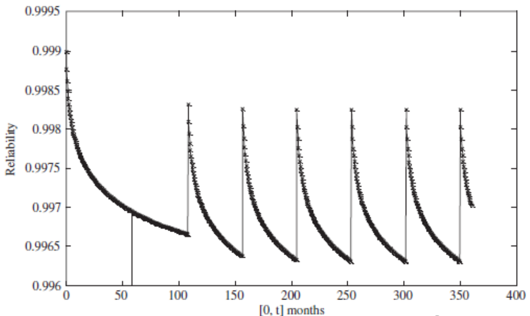


Challenges:

- Statistical characteristics of the response change with time
- More computationally expensive than the traditional reliability analysis

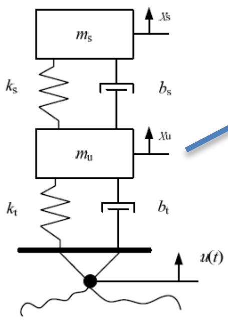
Significances

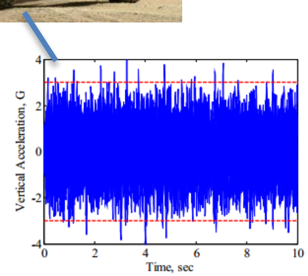
- Directly related to lifecycle cost optimization and maintenance
- Essential for guaranteeing the reliability of a system
- Basis for designing high reliability into a product



Problem statement

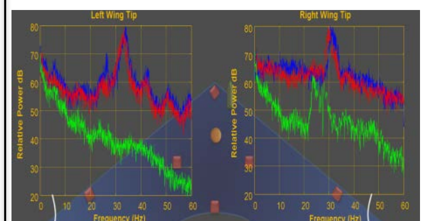
Examples

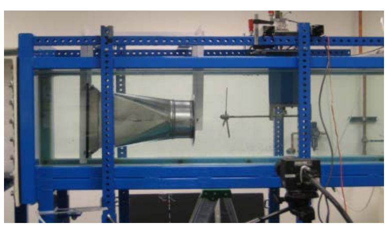


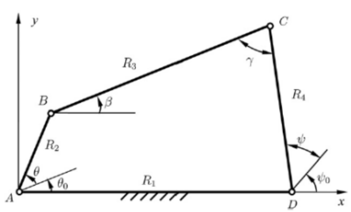


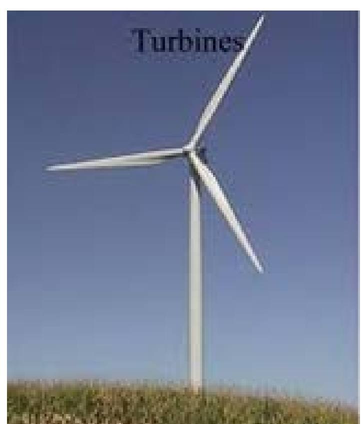
Vehicles (Mourelatos, 2011)

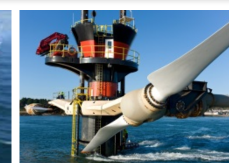
Aircrafts











Offshore structures

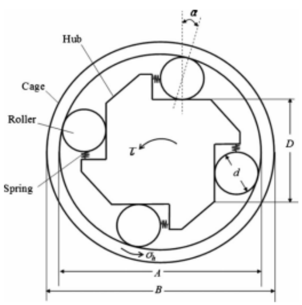
Problem statement

Focus of this work

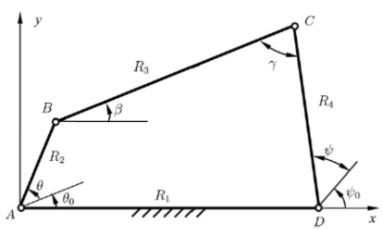
For a special group of problem:

$$p_f = \Pr\{G = g(\mathbf{X}, \tau) > e, \exists \tau \in [0, T]\}$$
$$= \Pr\{G_{\max}(\mathbf{X}) > e\}$$

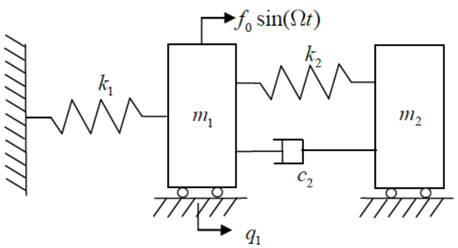
Examples:



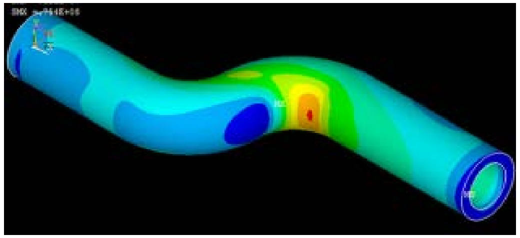
(Mourelatos, 2010)



(Zhang and Du, 2011)



(Zang and Friswell, 2005)



(Wang and Wang, 2012)

Challenges:

- Global optimization for given values of ***X***
- Surrogate model of global extreme value response

Problem statement

State of the Art

- **Upcrossing rate method**

  -Asymptotic upcrossing rate of a Gaussian stochastic process (i.e. *Lindgren 1984, Breitung 1984, 1988*)

  - Vector out-crossing rate using parallel approach (i.e. *Hagen, 1992*)

  - The Rice's formula based method (i.e. Rice, 1944, *Sudret, Lemaire, 2004, Zhang and Du, 2011*)

  - The joint-upcrossing rate method (i.e. *Hu and Du, 2013*)

- **Surrogate model method**

  - Composite limit-state function method (i.e. *Mourelatos, 2010*)

  - Nested extreme value response method (i.e. *Wang and Wang, 2012*)

- **Sampling method**

  - Importance sampling approach (i.e. *Singh and Mourelatos, 2011*)

  - Markov Chain Monte Carlo method (i.e. *Wang and Mourelatos, 2013*)

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A Mixed EGO-Based Method

Overview

Existing Methods	New Method
• Repeatedly use EGO to get extreme values of responses • Sample on ***X*** • Given ***X***, sample on *t* • Not efficient	• Proposed a mixed EGO method to identify extreme values • Sample on ***X*** and *t* simultaneously • More efficient

➢ EGO -- Efficient global optimization (i.e. Jones, 1998) is an efficient sampling based method for global optimization

A Mixed EGO-Based Method

Efficient Global Optimization

For a given **X=x**: $G_{\max}(\mathbf{x}) = \max\limits_{\tau \in [0,T]} \{ g(\mathbf{x}, \tau) \}$

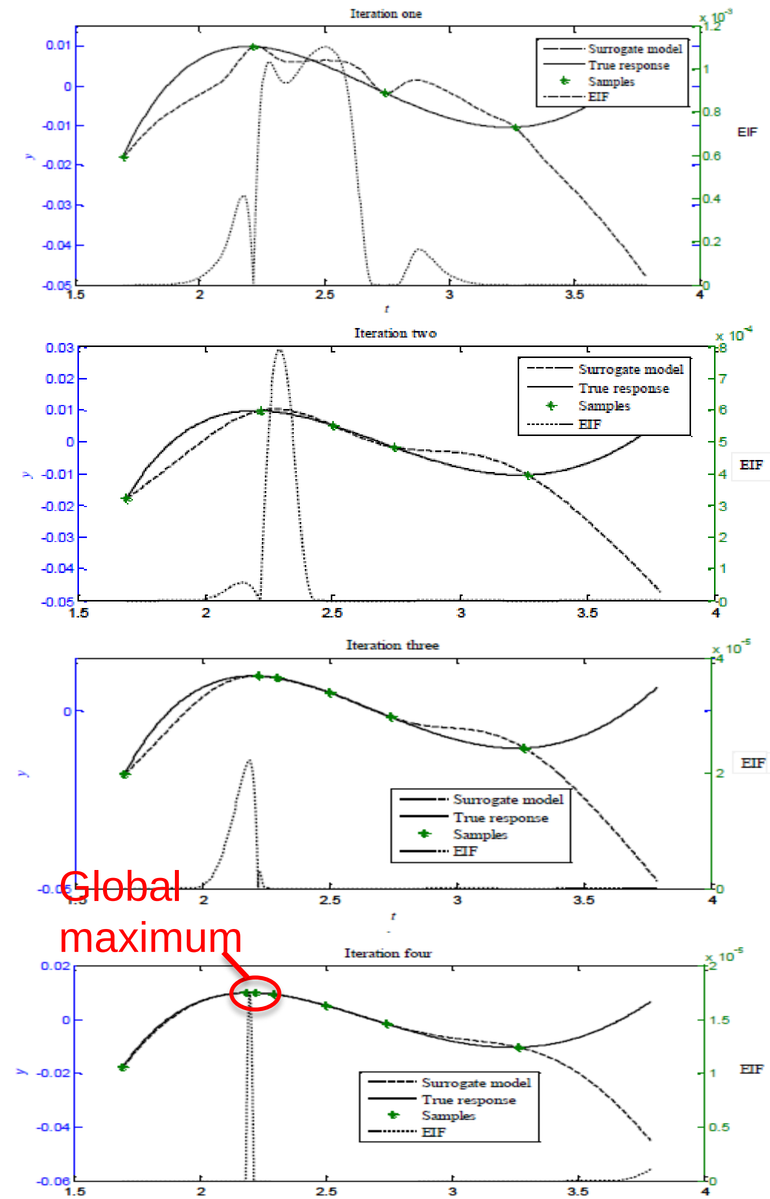
Algorithm 1 Efficient Global Optimization (EGO)

1. Generate initial samples $\mathbf{x}^s = [\mathbf{x}^{(1)}; \mathbf{x}^{(2)}; \cdots; \mathbf{x}^{(k)}]$
2. Compute $\mathbf{y}^s = [g(\mathbf{x}^{(1)}), g(\mathbf{x}^{(2)}), \cdots, g(\mathbf{x}^{(k)})]$; set $m = 1$
3. **While** $\{ m = 1 \}$ **or** $\{ \max\limits_{\mathbf{x} \in \mathbf{X}} \mathrm{EI}(\mathbf{x}) < \varepsilon_{EI} \}$ **do**
4. Construct a Kriging model $\hat{y} = \hat{g}(\mathbf{X})$ using $\{\mathbf{x}^s, \mathbf{y}^s\}$
5. Find $y^* = \max\limits_{i=1,2,\cdots,k+m-1} \{ g(\mathbf{x}^{(i)}) \}$
6. Search for $\mathbf{x}^{(k+m)} = \arg\max\limits_{\mathbf{x} \in \mathbf{X}} \mathrm{EI}(\mathbf{x})$, where $\mathrm{EI}(\mathbf{x})$ is computed by Eq. (4)
7. Scale $\max\limits_{\mathbf{x} \in \mathbf{X}} \mathrm{EI}(\mathbf{x}) = \max\limits_{\mathbf{x} \in \mathbf{X}} \mathrm{EI}(\mathbf{x}) / |\beta(1)|$, where $\beta(1)$ is the first element of the trend coefficients $\boldsymbol{\beta}$ given in Eq. (1)
8. Compute $g(\mathbf{x}^{(k+m)})$; update $\mathbf{y}^s = [\mathbf{y}^s, g(\mathbf{x}^{(k+m)})]$ and $\mathbf{x}^s = [\mathbf{x}^s; \mathbf{x}^{(k+m)}]$
9. $m = m + 1$
10. **End While**

$$\mathrm{EI}(\mathbf{x}) = (\mu(\mathbf{x}) - y^*)\Phi\left(\frac{\mu(\mathbf{x}) - y^*}{\sigma(\mathbf{x})}\right) + \sigma(\mathbf{x})\phi\left(\frac{\mu(\mathbf{x}) - y^*}{\sigma(\mathbf{x})}\right)$$

Current best solution

Mean and standard deviation of prediction



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A Mixed EGO-Based Method

Independent EGO

Algorithm 2 Independent EGO method

1 Generate initial training points $\mathbf{x}^s = [\mathbf{x}^{(1)}; \mathbf{x}^{(2)}; \cdots; \mathbf{x}^{(k)}]$

2 Solve for $\mathbf{y}^s_{\max} = [g_{\max}(\mathbf{x}^{(1)}), g_{\max}(\mathbf{x}^{(2)}), \cdots, g_{\max}(\mathbf{x}^{(k)})]$, where $g_{\max}(\mathbf{x}^{(i)}) = \max_{t \in [t_0, t_s]} \{g(\mathbf{x}^{(i)}, t)\}$, using EGO; set $m = 1$

3 **While** $\{m = 1\}$ **or** $\{\max_{\mathbf{x} \in \mathbf{X}} \mathrm{MSE}(\mathbf{x}) < \varepsilon_{MSE}\}$ **do**

4 Construct a Kriging model $Y_{\max} = \hat{g}_{\max}(\mathbf{X})$ using $\{\mathbf{x}^s, \mathbf{y}^s_{\max}\}$

5 Find $\mathbf{x}^{(k+m)} = \arg\max_{\mathbf{x} \in \mathbf{X}} \{\mathrm{MSE}(\mathbf{x})\}$

6 Search for $g_{\max}(\mathbf{x}^{(k+m)}) = \max_{t \in [t_0, t_s]} \{g(\mathbf{x}^{(k+m)}, t)\}$ using EGO

7 Update $\mathbf{x}^s = [\mathbf{x}^s; \mathbf{x}^{(k+m)}]$ and $\mathbf{y}^s_{\max} = [\mathbf{y}^s_{\max}, g_{\max}(\mathbf{x}^{(k+m)})]$

8 $m = m + 1$

9 **End While**

10 Reliability analysis using $Y_{\max} = \hat{g}_{\max}(\mathbf{X})$

Similar method: the nested extreme method (*Wang and Wang, 2012*)

- EGO needs to be performed independently and repeatedly for each training point of **X**
- Mean Square Error (MSE) is used for convergence study of the surrogate model

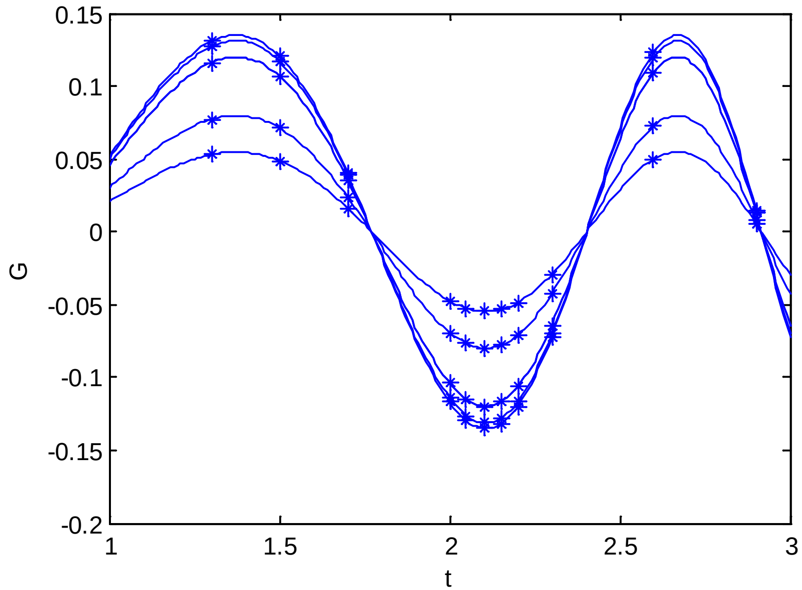
Can be improved from the following two aspects:

- Reduce the number of function evaluations required by global optimizations
- Change the update criterion for training points

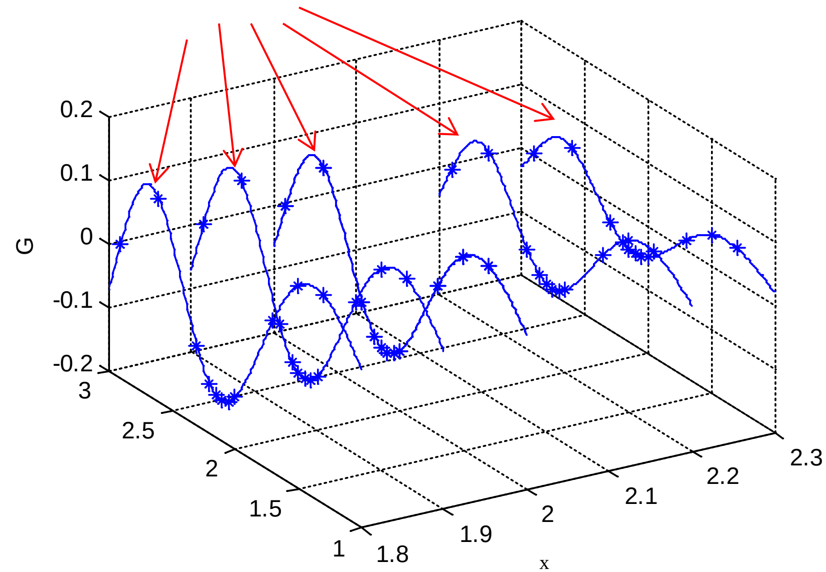
A Mixed EGO-Based Method

Mixed EGO

From the same model $G=g(X, t)$



Multiple independent EGOs under different values of *x*



Multiple independent EGOs

- Independently constructing surrogate models $g(x_1, t)$, $g(x_2, t)$, …, $g(x_n, t)$ ignored the correlations between these surrogate models
- The ignored information can reduce the number of function evaluations required

A Mixed EGO-Based Method

Mixed EGO

Algorithm 3 Mixed EGO model for initial $Y_{max} = \hat{g}_{max}(\mathbf{X})$

1 At initial training points, compute $\mathbf{y}^s = [y^{(i)}]_{i=1,\ldots,k} = [g(\mathbf{x}^{(i)},\ t^{(i)})]_{i=1,\ldots,k}$

2 Set $\mathbf{x}_t^s = \mathbf{x}^s$, $m = 1$, and the initial current best solution vector $\mathbf{y}_{max}^s = \mathbf{y}^s$

3 **While** $\{m = 1\}$ **or** $\{I_{max} < \varepsilon_{EI}\}$ **do**

4 Construct Kriging model $Y = \hat{g}(\mathbf{X},\ t)$ using $\{[\mathbf{x}_t^s, \mathbf{t}^s], \mathbf{y}^s\}$

5 Find a point with maximum EI: $[\mathbf{x}^{(i_{EI})}, t^{EI}] = \arg\max\{\max_{i=1,2,\ldots,k}\ \max_{t\in[t_0,t_s]}\{\mathrm{EI}(\mathbf{x}^{(i)}, t)\}\}$, where $i_{EI} \in [1,\ldots,k]$ and $\mathrm{EI}(\mathbf{x}^{(i)}, t)$ is computed based on $Y = \hat{g}(\mathbf{X},\ t)$; calculate $I_{max} = \mathrm{EI}(\mathbf{x}^{(i_{EI})}, t^{EI}) / |\beta_{(\mathbf{x},t)}(1)|$.

6 Compute $y^{EI} = g(\mathbf{x}^{(i_{EI})}, t^{EI})$

7 Update current best solution

$$y_{max}^s(i_{EI}) = \begin{cases} y^{EI} & \text{if } y^{EI} > y_{max}^s(i_{EI}) \\ y_{max}^s(i_{EI}) & \text{otherwise} \end{cases}$$

8 Update data points $\mathbf{x}_t^s = [\mathbf{x}_t^s; \mathbf{x}^{(i_{EI})}]$, $\mathbf{t}^s = [\mathbf{t}^s;\ t^{EI}]$, and $\mathbf{y}^s = [\mathbf{y}^s, y^{EI}]$

9 $m = m + 1$

10 **End While**

11 Record $\mathbf{y}_{max}^s$, $[\mathbf{x}_t^s, \mathbf{t}^s]$, and $\mathbf{y}^s$

12 Construct $Y_{max} = \hat{g}_{max}(\mathbf{X})$ using $\{\mathbf{x}^s, \mathbf{y}_{max}^s\}$

- Construct surrogate model for $g(\boldsymbol{X}, t)$ instead of $g(x_1, t), g(x_2, t), \ldots, g(x_n, t)$ independently
- Sampling for variables $\boldsymbol{X}$ and t simultaneously
- Modify the updating criterion of original EGO

$$\mathrm{EI}(t) = (\mu(t) - y^*)\Phi\left(\frac{\mu(t) - y^*}{\sigma(t)}\right) + \sigma(t)\phi\left(\frac{\mu(t) - y^*}{\sigma(t)}\right)$$

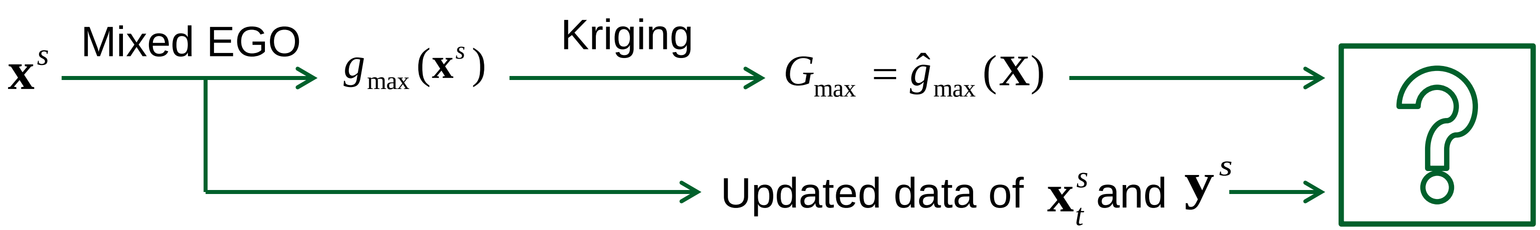
⇩

$$EI(\mathbf{x}^{(i)}, t) = (\mu(\mathbf{x}^{(i)}, t) - y_i^*)\Phi\left(\frac{\mu(\mathbf{x}^{(i)}, t) - y_i^*}{\sigma(\mathbf{x}^{(i)}, t)}\right) + \sigma(\mathbf{x}^{(i)}, t)\phi\left(\frac{\mu(\mathbf{x}^{(i)}, t) - y_i^*}{\sigma(\mathbf{x}^{(i)}, t)}\right)$$

A Mixed EGO-Based Method

Reliability analysis with mixed EGO

Initial Samples: $\mathbf{x}_t^s = [\mathbf{x}^s, \mathbf{t}^s] = \begin{bmatrix} x_1^{(1)} & x_2^{(1)} & \cdots & x_n^{(1)}, & t^{(1)} \\ x_1^{(2)} & x_2^{(2)} & \cdots & x_n^{(2)}, & t^{(2)} \\ \vdots & \vdots & \ddots & & \vdots \\ x_1^{(k)} & x_2^{(k)} & \cdots & x_n^{(k)}, & t^{(k)} \end{bmatrix}$ $\quad \mathbf{y}^s = [y^{(i)}]_{i=1,\ldots,k} = [g(\mathbf{x}^{(i)},\ t^{(i)})]_{i=1,\ldots,k}$



Two Purposes:

- Generate more training points near the limit state
- Use available data of $\mathbf{x}_t^s$ and $\mathbf{y}^s$ to reduce the number of function evaluations required to identify the $g_{\max}(\mathbf{x}^{new})$ corresponding to the new training point $\mathbf{x}^{new}$

A Mixed EGO-Based Method

For Purpose 1: Generate new training points

- **Kriging model based method**

  - Efficient Global Reliability Analysis (EGRA) method proposed by Bichon, Mahadevan, and *et.al.* (i.e. Bichon, Mahadevan, and et.al., 2008)

  - AK-MCS method developed after the EGRA method (i.e. Echard. Gayton, and Lemaire, 2011)

  - Dubourg and Sudret integrated the importance sampling approach with the AK-MCS method (i.e. Dubourg and Sudret, 2013)

- **Support vector machine based method**

  - Generate explicit decision functions using SVM (i.e. *Basudhar and Missoum, 2008*)

  - Further improved in 2010 (i.e. *Basudhar and Missoum, 2010*)

Note: In this work, the EGRA method is employed, but it is not limited to the EGRA method. Other methods, such as AK-MCS, the SVM-based method can be used as well.

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A Mixed EGO-Based Method

Efficient Global Reliability Analysis (EGRA) (i.e. Bichon 2008)

$$\mathrm{EI}(t) = (\mu(t) - y^*)\Phi\left(\frac{\mu(t) - y^*}{\sigma(t)}\right) + \sigma(t)\phi\left(\frac{\mu(t) - y^*}{\sigma(t)}\right)$$

$$\begin{aligned}\mathrm{EF}(\mathbf{x}) = {} & (\mu_g(\mathbf{x}) - e)\left[2\Phi\left(\frac{e - \mu_g(\mathbf{x})}{\sigma_g(\mathbf{x})}\right) - \Phi\left(\frac{e^- - \mu_g(\mathbf{x})}{\sigma_g(\mathbf{x})}\right) - \Phi\left(\frac{e^+ - \mu_g(\mathbf{x})}{\sigma_g(\mathbf{x})}\right)\right] \\ & - \sigma_g(\mathbf{x})\left[2\phi\left(\frac{e - \mu_g(\mathbf{x})}{\sigma_g(\mathbf{x})}\right) - \phi\left(\frac{e^- - \mu_g(\mathbf{x})}{\sigma_g(\mathbf{x})}\right) - \phi\left(\frac{e^+ - \mu_g(\mathbf{x})}{\sigma_g(\mathbf{x})}\right)\right] \\ & + \delta\left[\Phi\left(\frac{e^+ - \mu_g(\mathbf{x})}{\sigma_g(\mathbf{x})}\right) - \Phi\left(\frac{e^- - \mu_g(\mathbf{x})}{\sigma_g(\mathbf{x})}\right)\right]\end{aligned}$$

- Identify the point that is close to the limit state as the new training point
- Use the similar principle as the mixed EGO to identify the extreme value corresponding to the new training point

A Mixed EGO-Based Method

For Purpose 2: Use of available data

Algorithm 4 Sampling update

1: Set $p=1$

2: **While** $\{p=1\}$ **or** $\{\max_{\mathbf{x}\in\mathbf{X}} EF(\mathbf{x}) < \varepsilon_{EF}\}$ **do**

3: Construct a Kriging model of $Y_{\max} = \hat{g}_{\max}(\mathbf{X})$ using $\{\mathbf{x}^s, \mathbf{y}^s_{\max}\}$

4: Find a point with maximum EF: $\mathbf{x}^{(k+p)} = \arg\max_{\mathbf{x}\in\mathbf{X}}\{\mathrm{EF}(\mathbf{x})\}$

5: Generate a new random t_r that follows uniform distribution on $[t_0, t_s]$

6: Compute $y^{\mathrm{EF}} = g(\mathbf{x}^{(k+p)}, t_r)$; Update $\mathbf{x}^s_t = [\mathbf{x}^s_t; \mathbf{x}^{(k+p)}]$ $\mathbf{t}^s = [\mathbf{t}^s;\ t_r]$ and $\mathbf{y}^s = [\mathbf{y}^s, y^{\mathrm{EF}}]$

7: Set $y^s_{\max}(k+p) = y^{\mathrm{EF}}$ and $q=1$

8: **While** $\{q=1\}$ or $\{\max_{t\in[t_0, t_s]} \mathrm{EI}(\mathbf{x}^{(k+p)}, t) < \varepsilon_{EI}\}$ **do**

9: Construct an $n+1$ dimensional Kriging model $Y = \hat{g}(\mathbf{X}, t)$ using $\{[\mathbf{x}^s_t, \mathbf{t}^s], \mathbf{y}^s\}$

10: Find a point with maximum EI: $t^{\mathrm{EI}} = \max_{t\in[t_0, t_s]}\{\mathrm{EI}(\mathbf{x}^{(k+p)}, t)\}$, where $\mathrm{EI}(\mathbf{x}^{(k+p)}, t)$ is computed based on $Y = \hat{g}(\mathbf{X}, t)$

11: Scale $\mathrm{EI}(\mathbf{x}^{(k+p)}, t^{\mathrm{EI}}) = \mathrm{EI}(\mathbf{x}^{(k+p)}, t^{\mathrm{EI}}) / \left|\beta_{(\mathbf{x}, t)}(1)\right|$, where $\beta_{(\mathbf{x}, t)}(1)$ is the first element of the trend coefficients of $Y = \hat{g}(\mathbf{X}, t)$ model

12: Compute $y^{\mathrm{EI}} = g(\mathbf{x}^{(k+p)}, t^{\mathrm{EI}})$

13: Update current best solution

$$y^s_{\max}(k+p) = \begin{cases} y^{\mathrm{EI}}, & \text{if } y^{\mathrm{EI}} > y^s_{\max}(k+p) \\ y^s_{\max}(k+p), & \text{otherwise} \end{cases}$$

14: Update data points $\mathbf{x}^s_t = [\mathbf{x}^s_t; \mathbf{x}^{(k+p)}]$, $\mathbf{t}^s = [\mathbf{t}^s;\ t^{\mathrm{EI}}]$, $\mathbf{y}^s = [\mathbf{y}^s, y^{\mathrm{EI}}]$

15: $q = q+1$

16: **End While**

17: Record $\mathbf{y}^s_{\max}$, $\mathbf{x}^s = [\mathbf{x}^s; \mathbf{x}^{(k+p)}]$, $\mathbf{x}^s_t$, $\mathbf{t}^s$, and $\mathbf{y}^s$

18: $p = p+1$

19: **End While**

EGRA (i.e. Bichon, 2008) [steps 2–6]

Used the available data from mixed EGO for initial surrogate model

Modified EGO for the new training point [steps 8–14]

Updated data set for next global optimization

- Integrated the proposed mixed EGO method with available EGRA method
- Used available data to reduce the number of function evaluations required to identify the extreme value corresponding to the new training point
- Updated the data set iteratively

A Mixed EGO-Based Method

Summary

Table 1 Major Procedure of EGORA

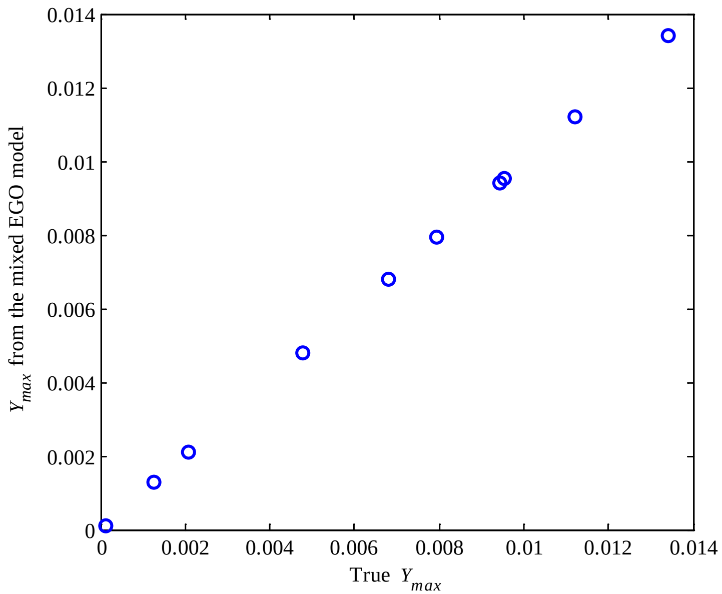
Step 1: Initial sampling	
1.	Generate initial samples $\mathbf{x}^s$ and $\mathbf{t}^s$
Step 2: Build initial extreme response model (Algorithm 3)	
2.	Build time-dependent surrogate model $Y = \hat{g}(\mathbf{X}, t)$
3.	Solve for the maximum responses $Y_{\max}$ at $\mathbf{x}^s$ based on $Y = \hat{g}(\mathbf{X}, t)$
4.	Build initial extreme response model $Y_{\max} = \hat{g}_{\max}(\mathbf{X})$
Step 3: Update extreme response model (Algorithm 4)	
5.	Adding new samples of $\mathbf{X}$ though updating and using $Y = \hat{g}(\mathbf{X}, t)$
6.	Obtain final model $Y_{\max} = \hat{g}_{\max}(\mathbf{X})$
Step 4: Reliability analysis	
7.	Monte Carlo simulation based on $Y_{\max} = \hat{g}_{\max}(\mathbf{X})$.

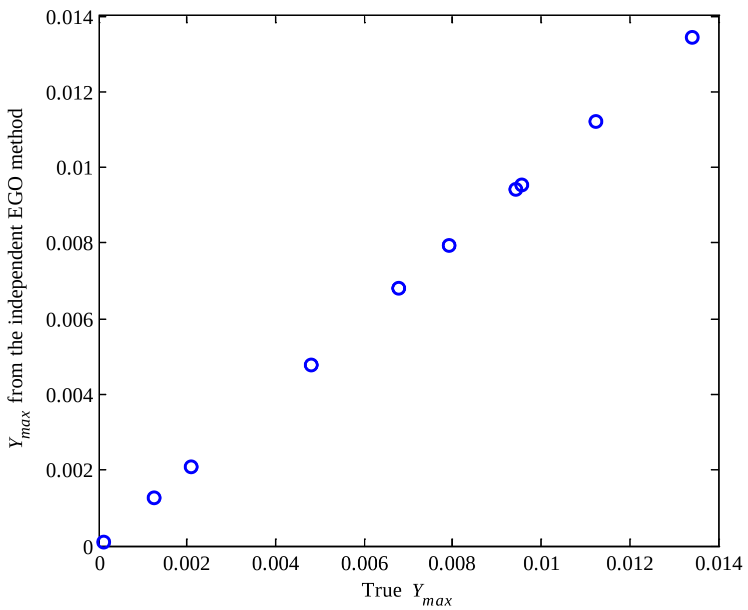
- Mixed EGO reduces the functioncall required for the global optimization
- EGRA reduces the number of training points needed for surrogate model
- Principle of mixed EGO method further improves the efficiency of global optimization with new training points

Examples

Example 1

$$y(X,t)=\frac{1}{X^2+4}\sin(2.5\mathrm{X})\cos(t+0.4)^2 \Rightarrow X \sim N(10, 0.5^2) \quad t\in[1, 2.5]$$





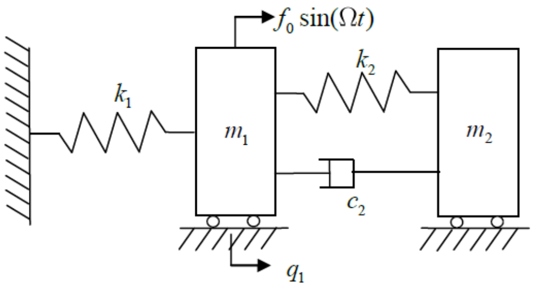
- Same samples of ***X*** and same convergence criterion

$\varepsilon = 1\times10^{-5}$

Number of samples of X	NOF	
	Independent EGO	Mixed EGO
10	85	49
15	127	59
18	153	66
20	170	69

Examples

A Vibration Problem



Variable	Mean	STD	Distribution
k_1 (N/m)	3×10^6	2×10^4	Normal
m_1 (kg)	1.6×10^4	2×10^2	Normal
k_2 (N/m)	8.5×10^4	0	Deterministic
m_2 (kg)	480	0	Deterministic
c_2 (Ns/m)	300	0	Deterministic

Amplitude of vibration of mass *m1*

$$q_{1\max} = f_0 \left(\frac{c_2^2\Omega^2 + (k_2 - m_2\Omega^2)^2}{c_2^2\Omega^2(k_1 - m_1\Omega^2 - m_2\Omega^2)^2 + (k_2 m_2\Omega^2 - (k_1 - m_1\Omega^2)(k_2 - m_2\Omega^2))^2} \right)^{1/2}$$

Nondimensionalized

$$Y = g(\mathbf{X}, \Omega) = k_1 \left(K_1 / \left(K_2 + K_3^2 \right) \right)^{1/2}$$

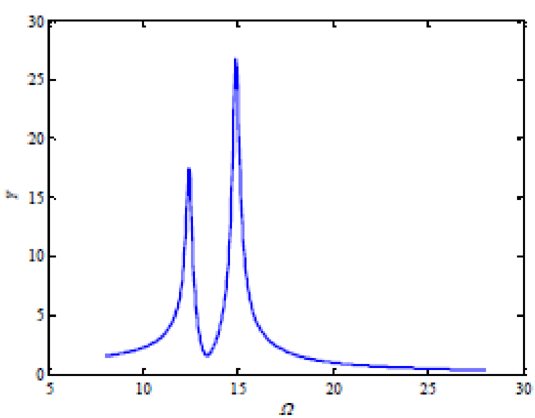
$$K_1 = \left(c_2^2\Omega^2 + (k_2 - m_2\Omega^2)^2 \right)$$

$$K_2 = c_2^2\Omega^2(k_1 - m_1\Omega^2 - m_2\Omega^2)^2$$

$$K_3 = \left(k_2 m_2\Omega^2 - (k_1 - m_1\Omega^2)(k_2 - m_2\Omega^2) \right)$$

Probability of failure over a certain excitation frequency:

$$p_f(8, 28) = \Pr\{ g(\mathbf{X}, \Omega) > 31, \exists \Omega \in [8, 28] \}$$



Number of samples of **X**	NOF	
	Nested	Mixed EGO
30	579	156
80	1521	482
110	2142	513
140	2663	588

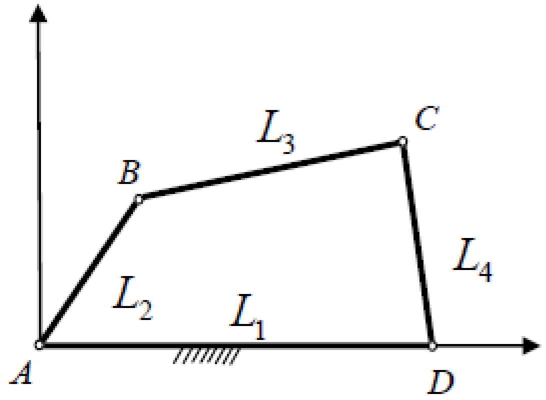
Test the efficiency of the mixed EGO

Method	NOF	p_f $(\times10^{-5})$	Error (%)
Rice	34235	0	100
Independent EGO	2663	3.9	20
EGORA	704	3.25	0
MCS	1×10^9	3.25	N/A

Test the efficiency of the mixed EGO+EGRA

Examples

A Mechanism Example



Variable	Mean	STD	Distribution
L_1 (mm)	100	0.05	Normal
L_2 (mm)	55.5	0.05	Normal
L_3 (mm)	144.1	0.05	Normal
L_4 (mm)	72.5	0.05	Normal

$$\varepsilon(\mathbf{X}, t) = 2\arctan\left(\frac{-E \pm \sqrt{E^2 + D^2 - F^2}}{F - D}\right) - (60^o + 60^o \sin[0.75(t - 97^o)])$$

⇨ Differences between designed function output and the actual function output over a certain design region:

Probability of failure over the design region:

$$p_f(t_0, t_s) = \Pr\{\varepsilon(\mathbf{X}, \tau) > 0.75, \exists \tau \in [97^\circ, 217^\circ]\}$$

Method	NOF	p_f $(\times 10^{-1})$	Error (%)
Rice	21677	1.986	10.86
Independent EGO	181	2.231	1.3
EGORA	123	2.231	1.3
MCS	5×10^8	2.228	N/A

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Conclusions and Future Works

- Proposed a mixed-EGO method for global optimizations in the time-dependent reliability analysis
- Integrated the proposed mixed-EGO method with the EGRA method, which further improves the efficiency of reliability analysis
- Demonstrated the effectiveness of the proposed method using numerical examples

Future Works

- Time-dependent reliability analysis based design optimization is one of the future works
- Test the proposed method with high-dimensional problems is also one of the future works

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Thank You