A Physics-Based Method for System Reliability Prediction in Early Design Stage

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Acknowledgements

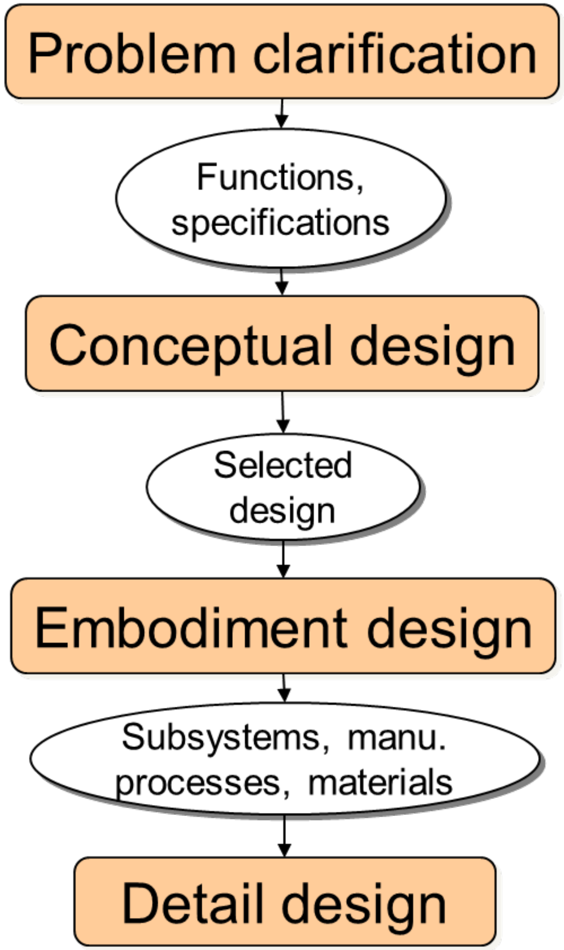
- The National Science Foundation through grants CMMI 1300870 and CMMI 1234855.

Outline

- Background
- Review of system reliability
- Proposed method
- Numerical example
- Conclusions and future work

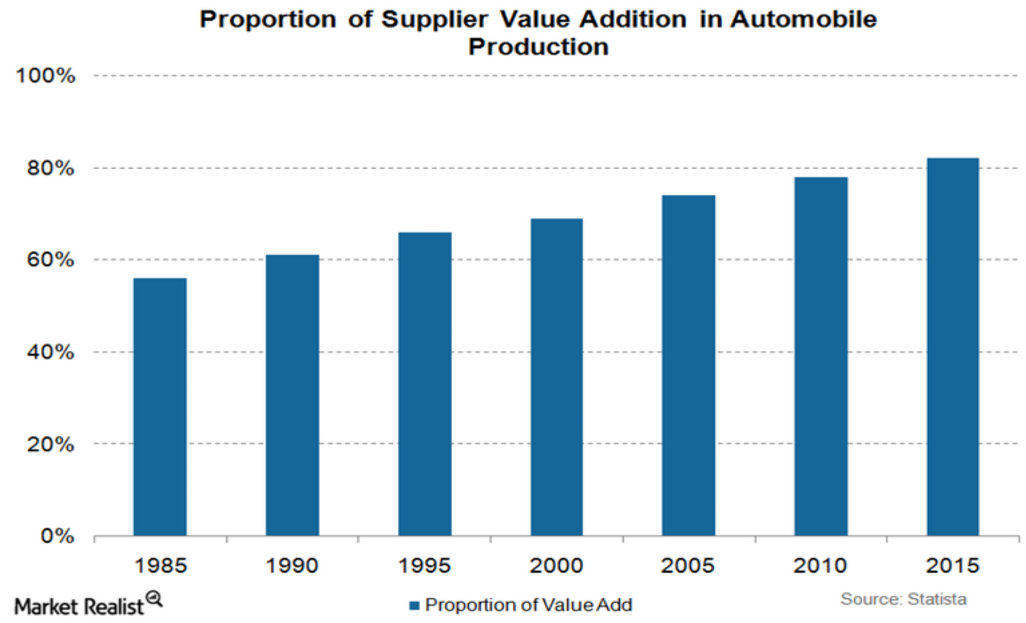
Introduction: Reliability in Early Design

- Conceptual design generates, evaluates, and selects design concepts.
- One important criterion is product (system) reliability.
- Predicting system reliability is hard in early design stage.



Systems with Outsourced Components

- It is now a common practice to have components from suppliers.
- Auto suppliers' contribution (in terms of value)
  - 56% in 1985
  - 82% now



Kallstrom, H. (2015)

Challenges

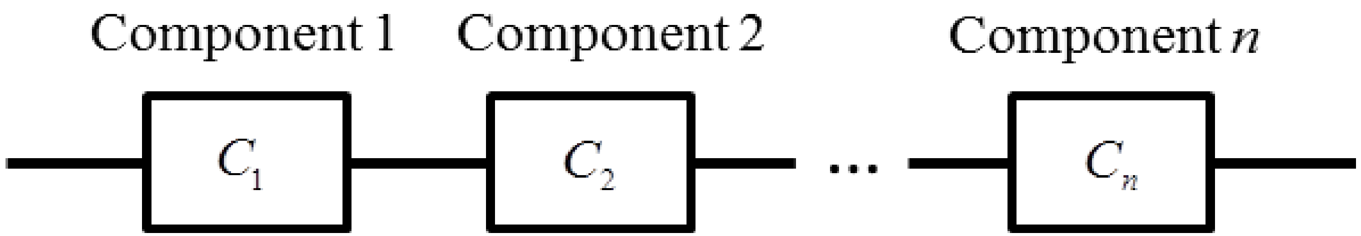
- To accurately predict system reliability, system designers need to know details about the component designs.
- But the component details are proprietary to the component designers.

Objective of This Research

- Explore the feasibility of accurate system reliability prediction in early design stage.
  - without revealing proprietary component design details
    - Materials
    - Concrete structures
    - Dimensions
  - using a physics-based approach
  - focusing on static problems (no time involved)

Review of System Reliability

- Series systems



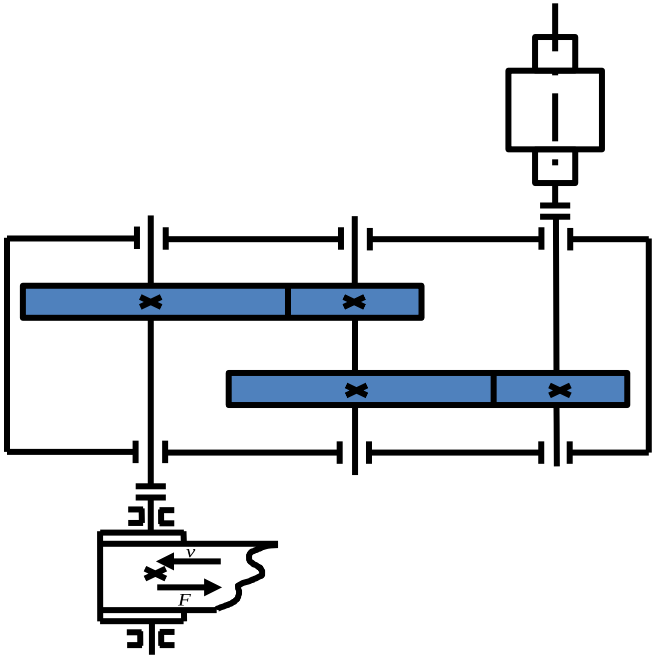
- Assumption of independent component states

$$R_s = \prod_{i=1}^{n} R_i \,, \qquad R\text{: Reliability}$$

- In mechanical applications, component states are likely dependent, leading to large errors.

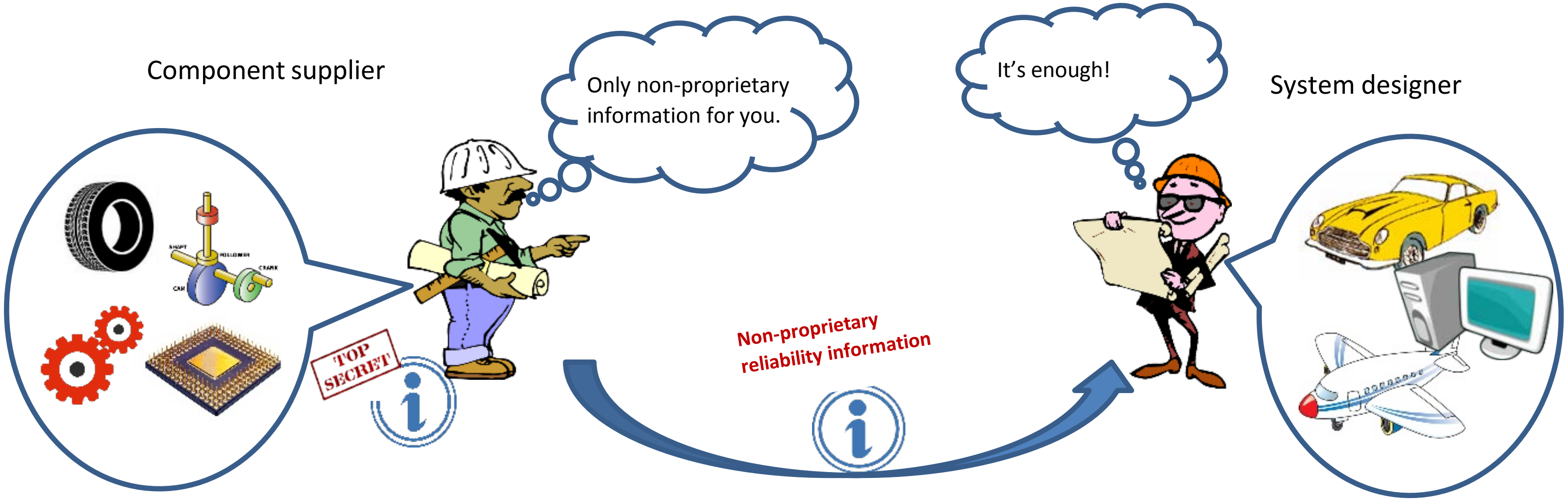
System Reliability Bounds

- $\prod_{i=1}^{n} R_i \leq R_S \leq \min\{R_i\}$
- Speed reducer example
  - If $R_i = 0.9999$
  - Then $0.9976 \leq R_s \leq 0.9999$
- Shortcomings:
  - Wide bounds
  - Conservative lower bound
  - Hard to make decisions



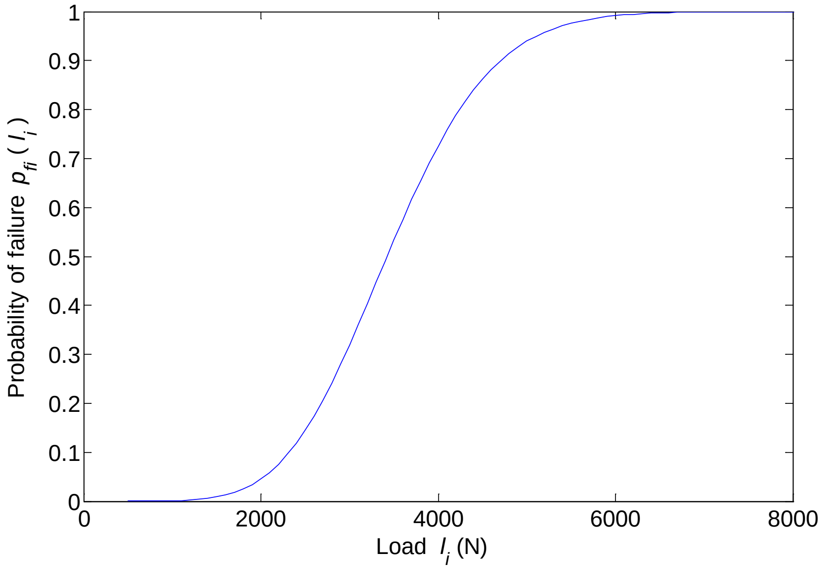
Proposed Method

- At the system design level, accurately predict system reliability for systems whose components share the same system load
  - Use a physics-based method
  - Request more information about component reliabilities without revealing proprietary component details



Component Reliabilities Requested by System Designers

- Reliability function of the *i*-th component $R_i(l_i)$ ($i = 1,2,\cdots,n$) w.r.t. component load l_i
- Or probability of failure $p_{fi}(l_i) = 1 - R_i(l_i)$
- $R_i(l_i)$ can be obtained at different load levels from
  - Testing
  - Field data
  - Physics-based approach (structural reliability analysis)
- Note that a component may have multiple failure modes.



Information Available to System Designers

- Component probabilities of failure $p_{fi}(l_i)$
- Distribution of the system load L
- Relationship between system load and component loads $L_i = w_i L$
- Now L_i is a random variable.

System Reliability Analysis

- Construct component limit-state functions

$$Y_i = S_i - w_i L$$

  - S_i is the generalized strength of component *i.*
  - S_i is usually a function of component details (material properties, dimensions, etc.) (We will see this in the example.)
  - No component details appear explicitly.
- When $Y_i = S_i - w_i L < 0$, component *i* fails.

System Reliability Analysis

- System load L and generalized component strengths S_i are independent.
- We can prove that the cumulative distribution function (CDF) of S_i is the component probability of failure function, namely,

$$F_{Si}(s) = p_{fi}(s), F = \mathrm{CDF}$$

- Then the joint CDF of all random variables $\mathbf{Z} = (S_1, S_2, \cdots, S_n, L)$ is known.

$$F_{\mathbf{Z}}(\boldsymbol{z}) = F_L(l) \prod_{i=1}^{n} F_{S_i}(s_i)$$

- The joint probability density function (PDF) of all random variables $f_{\mathbf{Z}}(\boldsymbol{z})$ is also known.

System Reliability Analysis

- System probability of failure

$$p_{fs} = \Pr\left\{\bigcup_{i=1}^{n} Y_i = S_i - w_i L < 0\right\}$$

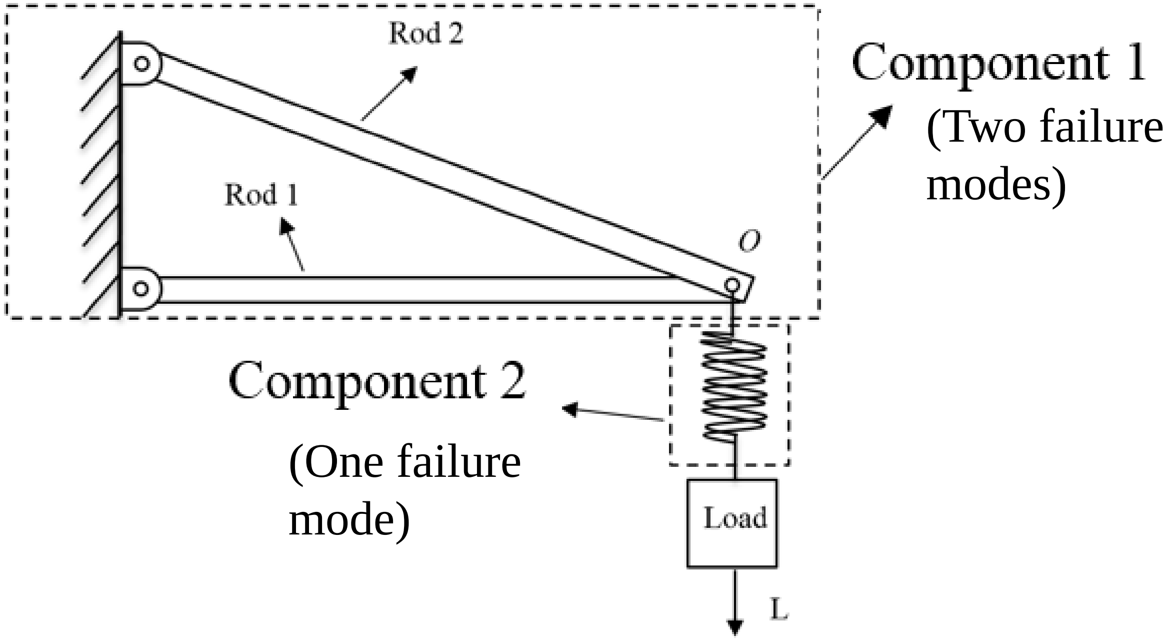
$$p_{fs} = \int_{s_i - w_i l < 0} f_{\mathbf{Z}}(\mathbf{z}) d\mathbf{z}$$

$$\mathbf{z} = (s_1, s_2, \cdots, s_n, l)$$

- We can use Monte Carlo simulation, First/Second Order Reliability Method (FORM/SORM), Saddlepoint approximations, etc.

Example

- A system consists of two components that are independently designed and manufactured by two suppliers.
- System designers request component reliability functions.

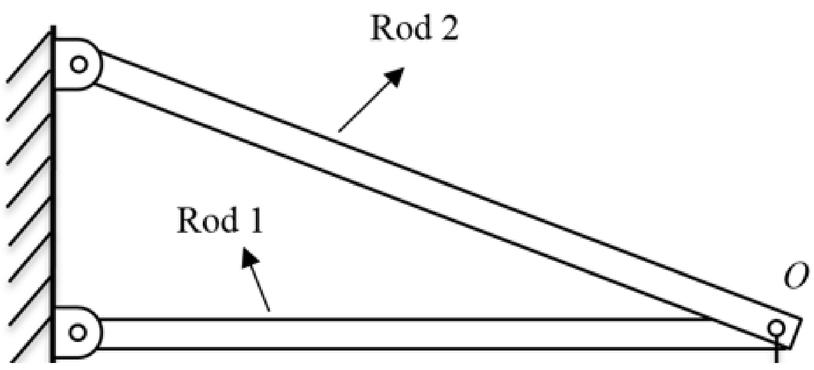


Component Reliability Analysis Performed by Supplier One

- Supplier one uses physics-based approach.
- There are two failure modes and then two limit-state functions.

$$Y_{11} = S_{y1} - \frac{4l_1 a_2}{\sqrt{a_2^2 - a_1^2}(\pi d_1^2)}$$

$$Y_{12} = S_{y2} - \frac{4l_1 a_1}{\sqrt{a_2^2 - a_1^2}(\pi d_2^2)}$$



Component 1

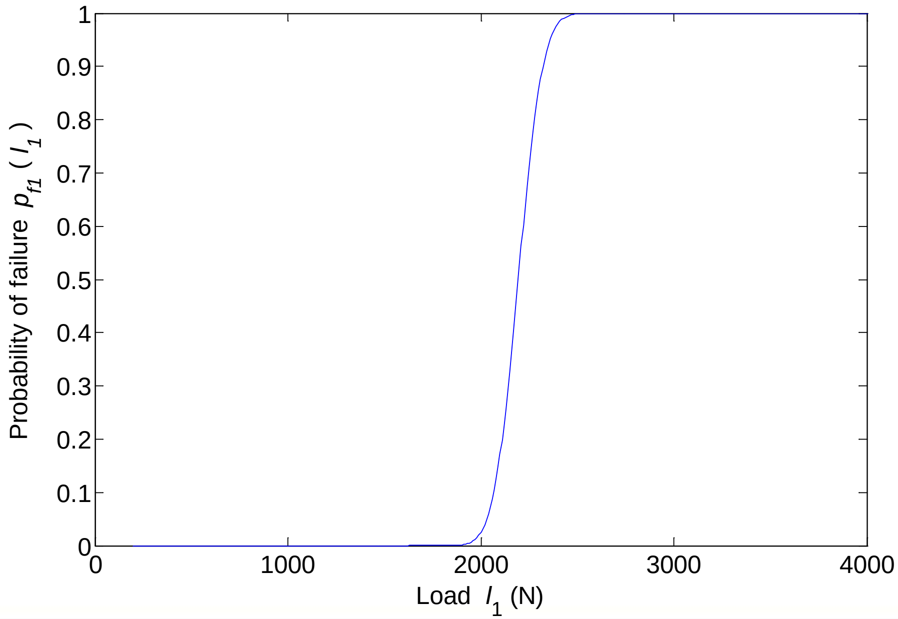
- S_{y1} is the yield strength, RV
- S_{y2} is the yield strength, RV
- l_1 is the load
- a_1, a_2, d_1, and d_2 are dimensions, RVs
- RV: random variable

Component Reliability Analysis Performed by Supplier One

- By changing l_1, component designers calculate component reliability with

$$p_{f1}(l_1) = \Pr\{Y_{11} < 0 \cup Y_{12} < 0\}$$

$$= \Pr\left\{S_{y1} - \frac{4l_1a_2}{\sqrt{a_2^2 - a_1^2}(\pi d_1^2)} < 0 \cup S_{y2} - \frac{4l_1a_1}{\sqrt{a_2^2 - a_1^2}(\pi d_2^2)} < 0\right\}$$



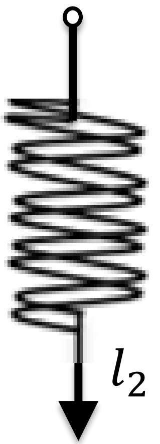
Component Reliability Analysis Performed by Supplier Two

- Limit-state function for component Two

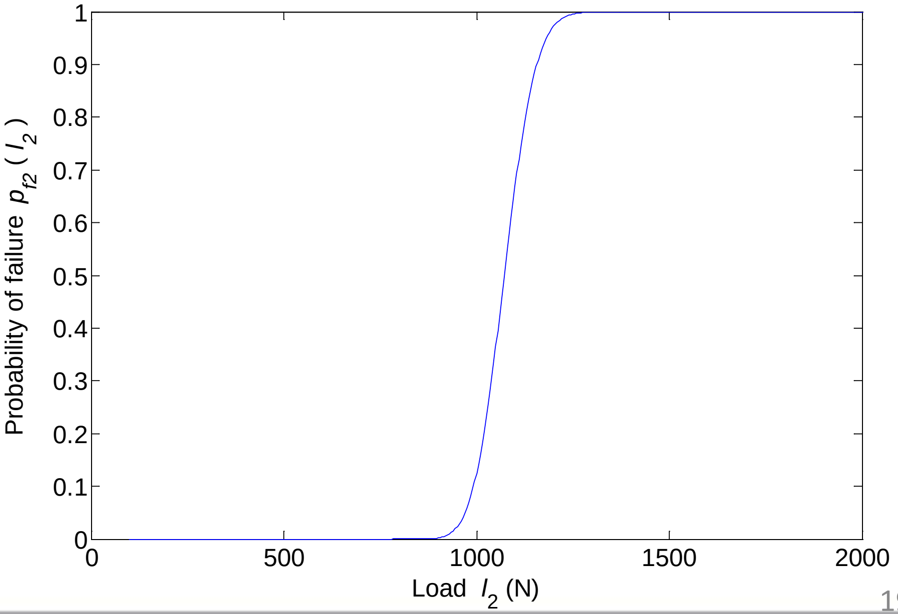
$$Y_{21} = \tau - \frac{4l_2 D\left(\frac{4(D-d)}{\sqrt{4D-4d}} + \frac{0.615d}{D}\right)}{\pi d^3}$$

  - τ is the allowable shear stress

$$p_{f2} = \Pr\{Y_{21} < 0\}$$



Component 2



System Reliability Analysis

- Now system designers have the following information
  - Distribution of system load $L \sim N(1200, 250^2)$ N
  - Probability of failure function of component 1 $p_{f1}(l_1)$
  - Probability of failure function of component 2 $p_{f2}(l_2)$
- They then construct limit-state functions
  - $Y_1 = S_1 - w_1 L$
  - $Y_2 = S_2 - w_2 L$
  - From force analysis, $w_1 = 1$, and $w_2 = 0.5$
  - CDFs: $F_{S1}(s) = p_{f1}(s)$, and $F_{S2}(s) = p_{f2}(s)$
- They then calculate the probability of system failure $\Pr\{Y_1 < 0 \cup Y_2 < 0\}$

Summary of component and system analyses

Component Designers (suppliers)	System Designers
Limit-state functions	Reconstructed limit-state functions
Component 1 $$Y_{11} = S_{y1} - \frac{4l_1 a_2}{\sqrt{a_2^2 - a_1^2}(\pi d_1^2)}$$ $$Y_{12} = S_{y2} - \frac{4l_1 a_1}{\sqrt{a_2^2 - a_1^2}(\pi d_2^2)}$$ Component 2 $$Y_{21} = \tau - \frac{4l_2 D\left(\frac{4(D-d)}{\sqrt{4D-4d}} + \frac{0.615d}{D}\right)}{\pi d^3}$$	Component 1 $$Y_1 = S_1 - w_1 L$$ Component 2 $$Y_2 = S_2 - w_2 L$$

No component details are required, and component dependencies due to the sharing load are automatically considered.

Result

	Independent assumption	Proposed Method	True value
p_{fs}	4.591×10^{-4}	4.139×10^{-4}	4.059×10^{-4}
Error (%)	13.10	1.97	–

- Independent component assumption: large error
- Proposed method: small error
- True value: result obtained as if all the original limit-state functions are known.

Conclusions

- It is possible to accurately predict system reliability of a new product during the early design stage.
- Component limit-state functions could be reconstructed using component reliability functions with respect to component load.
- The dependency of components could be considered automatically.
- Component reliability functions can be generated in different ways.

Future Work

- Extend the proposed method into parallel systems and mix systems.
- Consider more complex situations where more system loads are applied.
- Extend the method to time-dependent reliability problems.

CDF of Strength of Component *i*

- $p_{fi}(l_i) = \Pr\{S_i < l_i\}$
- $p_{fi}(s_i) = \Pr\{S_i < s_i\} = F_{si}(s_i) = \text{CDF of } S_i$