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Extreme Value Metamodeling for System Reliability with Time-Dependent Functions

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Outline

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  - Time-dependent system reliability
  - Kriging model
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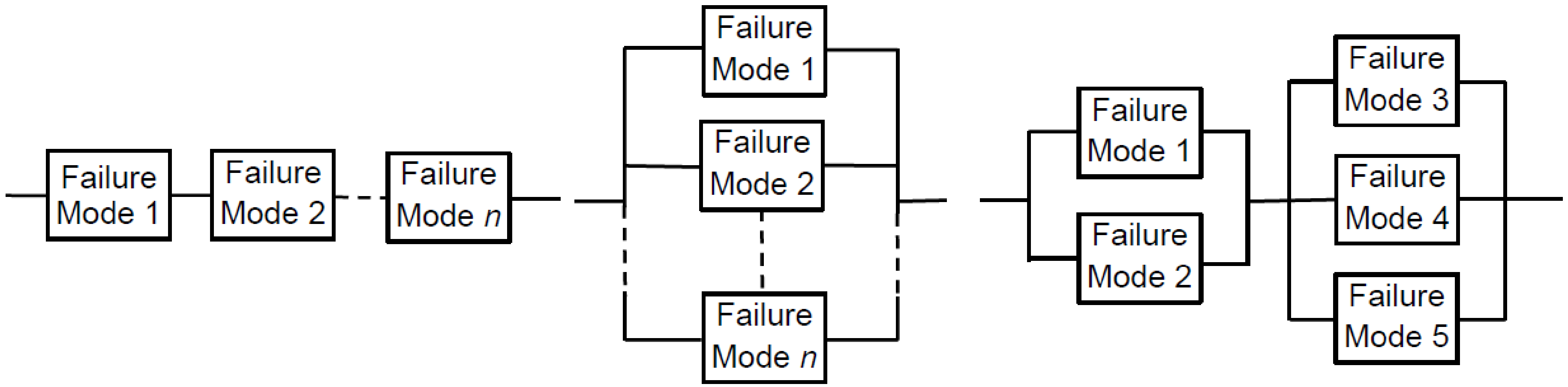
Objective

- **Objective**

Develop a new time-dependent system reliability method



A series system A parallel system Two parallel subsystems in series

Time-Dependent System Reliability

The limit-state function of failure mode *i*

$$Y_i = g_i(\mathbf{X}, t),\ t \in [t_0, t_s]$$

$Y_i > 0$ leads to a failure.

Probability of failure

$$p_f^i(t_0, t_s) = \Pr\left\{g_i(\mathbf{X}, t) > 0,\ \exists\ t \in [t_0, t_s]\right\}$$

For a series system, the system probability of failure

$$p_f^S(t_0, t_s) = \Pr\{g_1(\mathbf{X}, t) > 0 \bigcup g_2(\mathbf{X}, t) > 0 \bigcup \cdots \bigcup g_n(\mathbf{X}, t) > 0,\ \exists\ t \in [t_0, t_s]\}$$

Extreme Values

$$p_f^S(t_0, t_s) = \Pr\{\max_{i=1,2,\cdots,n} g_i(\mathbf{X}, t) > 0,\ \exists\ t \in [t_0, t_s]\}$$

$$= \Pr\{\max_{t\in[t_0,t_s]} \max_{i=1,2,\cdots,n} g_i(\mathbf{X}, t) > 0\}$$

Let extreme values be

$$Y_i^{\max} = g_i^{max}(\mathbf{X}) = \max_{t\in[t_0,t_s]} g_i(\mathbf{X}, t)$$

Therefore

$$p_f^S(t_0, t_s) = \Pr\left\{\max_{i=1,2,\cdots,n}\left(Y_i^{\max}\right) > 0\right\}$$

We create surrogate models for $Y_i^{\max}$

Surrogate Modeling-Kriging Model

- Kriging prediction and variance for $g(\mathbf{x})$

$$\hat{y} = \hat{g}(\mathbf{x}) \sim N(\mu_g(\mathbf{x}), \sigma_g^2(\mathbf{x}))$$

- Our problem: build $\hat{Y}_i^{\max}$ for $g_i(\mathbf{X}, t), (i = 1, 2, \cdots, n)$

- Solution: mixed Efficient Global Optimization[1] (mEGO)

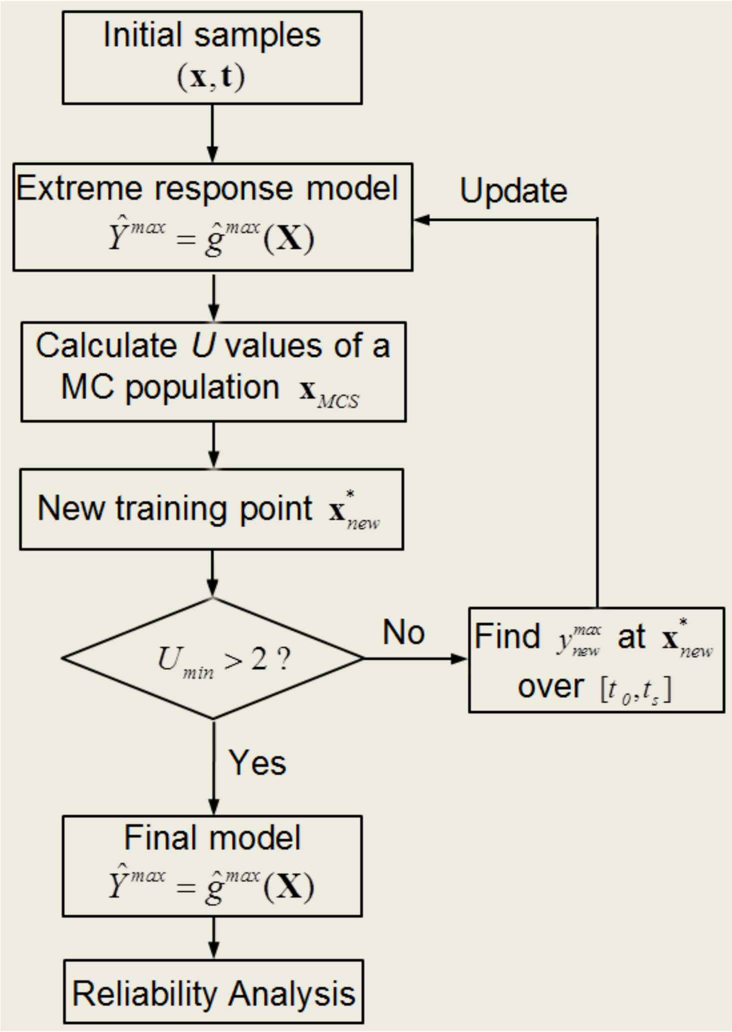
mEGO method has two major advantages:

1) Sample variables **X** and *t* simultaneously.
2) Use AK-MCS[2] to improve efficiency.

[1] Hu, Z., and Du, X., 2015, "Mixed Efficient Global Optimization for Time-Dependent Reliability Analysis," Journal of Mechanical Design.

[2] Echard, B., etc., 2011, “AK-MCS: An active learning reliability method combining Kriging and Monte Carlo Simulation,” Structural Safety.

mEGO Procedures



- For each point of $\mathbf{x}_{MCS}$, find the global maximum response over $[t_0, t_s]$ using *EI.*
- $\mathbf{x}^{*}_{new}$ is a point from $\mathbf{x}_{MCS}$ with minimum *U* value.

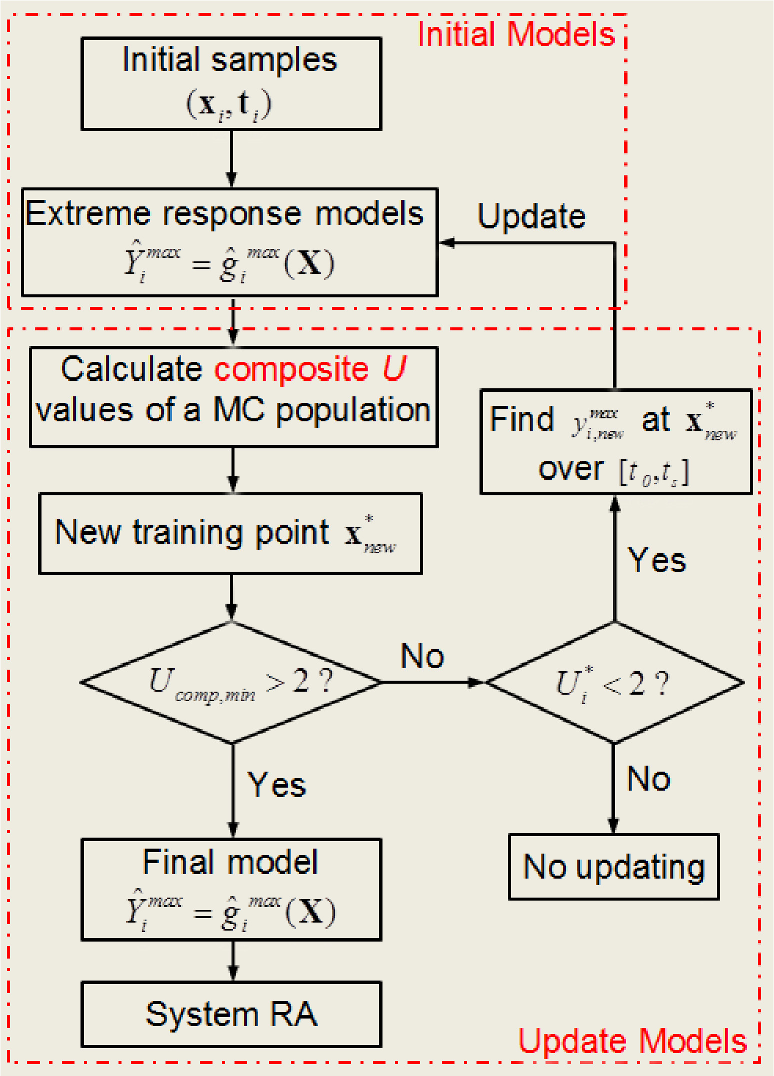
U-function defined by AK-MCS

$$U_{\hat{g}^{\max}(\mathbf{X})} = \frac{\left|\mu_{\hat{g}^{max}(\mathbf{X})}\right|}{\sigma_{\hat{g}^{max}(\mathbf{X})}}$$

Indicate the accuracy of Kriging model at the limit-state.

mEGO is only for component

Proposed Method (mSEGO)



For a series system, the composite prediction $\boldsymbol{\mu}^*$ [3] is

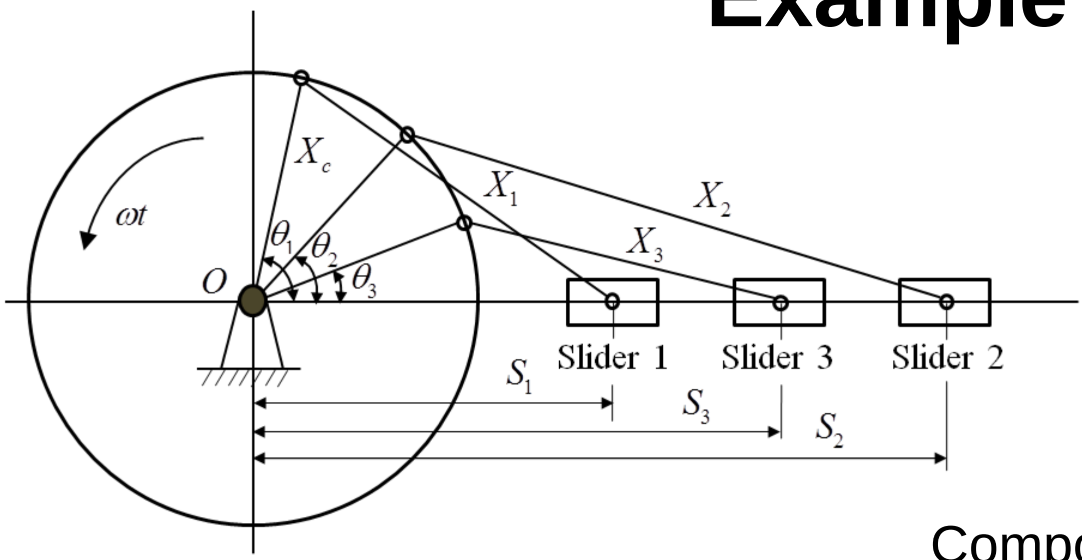
$$\mu^*(\mathbf{x}) = \max(\hat{g}_i^{max}(\mathbf{x}))$$

High efficiency

- Sample **X** and t simultaneously
- Check component contributions

[3] Bichon,B.J. et al., 2011, "Efficient surrogate models for reliability analysis of systems with multiple failure modes".

Example



Variable	Mean (mm)	Standard deviation (mm)	Distribution
X_c	100	0.5	Normal
X_1	150	0.75	Normal
X_2	250	1.25	Normal
X_3	200	1.0	Normal

$$\theta_1 = \omega t, \theta_2 = \omega t - \frac{\pi}{6}, \theta_3 = \omega t - \frac{\pi}{3}.$$

Motion outputs

$$S_i = X_c \cos\theta_i + \sqrt{X_i^2 - (X_c \sin\theta_i)^2}$$

Required motion outputs

$$S_{R_i} = \mu_c \cos\theta_i + \sqrt{\mu_i^2 - (\mu_c \sin\theta_i)^2}$$

The motion errors

$$\Delta S_i = \left| S_{R_i} - S_i \right|$$

Component probabilities of failure

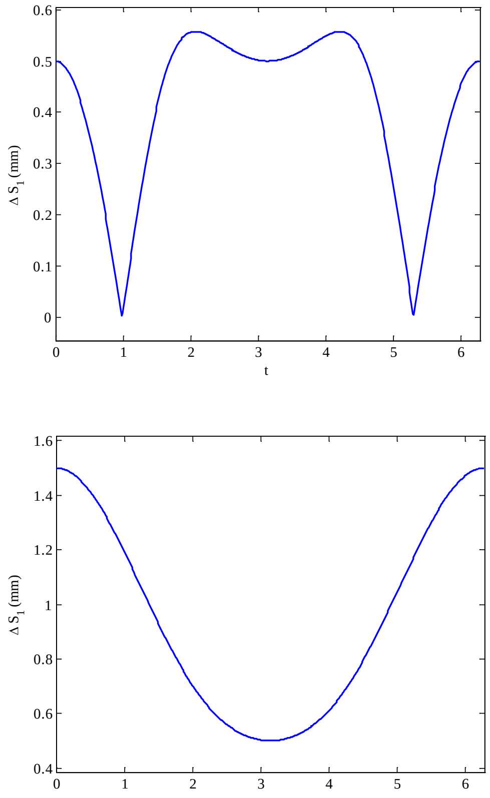
$$Y_i = g_i(\mathbf{X}, t) = \left| \begin{array}{l} (X_c - \mu_c)\cos\theta_i + \\ \sqrt{X_i^2 - (X_c \sin\theta_i)^2} - \sqrt{\mu_i^2 - (\mu_c \sin\theta_i)^2} \end{array} \right| - \varepsilon_i$$

Allowable motion errors: $\varepsilon_i = 4.8, 5.5, 5.2\text{mm}.$

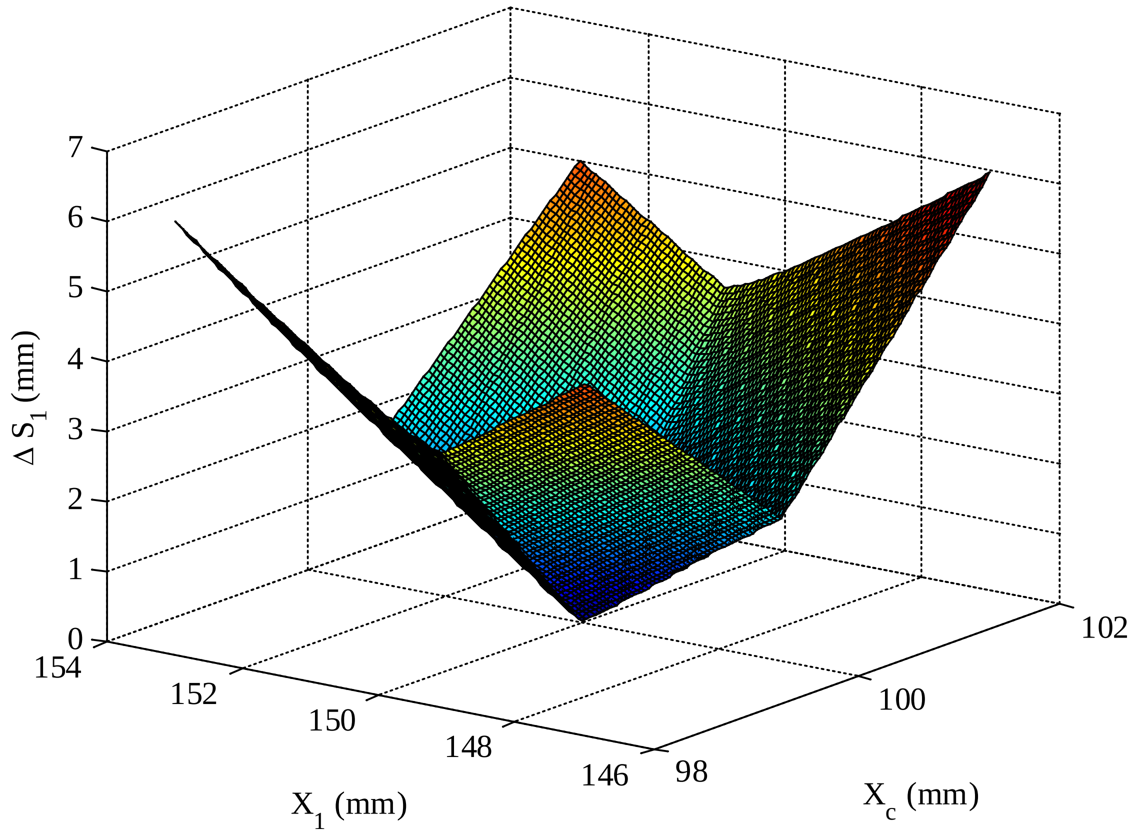
System probability of failure

$$p_f^S = \Pr\left\{ Y_1^{max} > 0 \bigcup Y_2^{max} > 0 \bigcup Y_3^{max} > 0 \right\}$$

Example



Motion error of mechanism 1 at point (100.5, 150.0) and (100.5, 151.0) mm



Extreme motion error of mechanism 1

Results

Time interval [0, 2π] second is divided into 360 time instants. For MCS, 10^6 samples are generated at each time instant.

Method	p_f	Error (%)	Function calls
MCS	1.76E-4	N/A	$(3.6, 3.6, 3.6)\times 10^8$
mSEGO	1.71E-4	2.95%	(268, 365, 261)

Conclusions

mSEGO method works well for the following systems:

- Limit-state functions are explicit functions of time .
- No stochastic processes in the input variables.
- Components of system can be in series, parallel, or their combination.

Future Work

- Share the training points and their responses among the components.
- Use adaptive convergence criterion for the *EI* for the extreme responses.

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