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Reliability Analysis for Multidisciplinary Systems Involving Stationary Stochastic Processes

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Outline

- Background
- MDRA with stationary SP
- Examples
- Conclusions
- Acknowledgement

Multidisciplinary Systems

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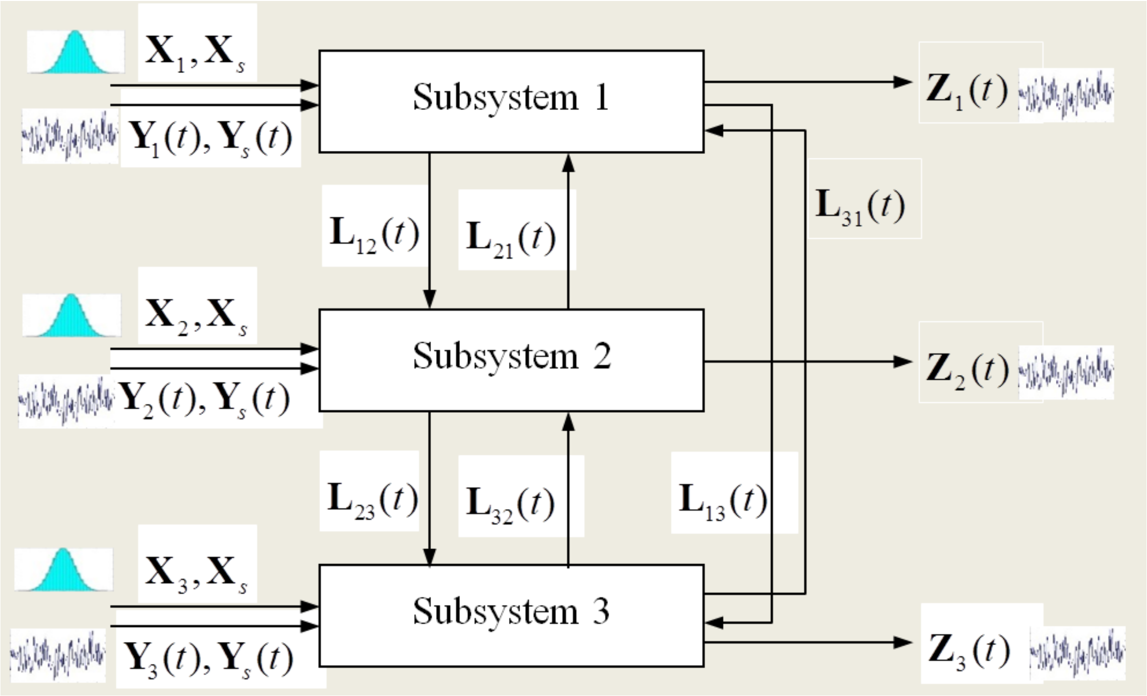
Multidisciplinary Systems



X: Random variables; **Y**: Stochastic processes

Problem Statement

- **Inputs:**

X: Random variables

Y: Stationary stochastic processes

- **Response of subsystem *i***

$$Z_i(t) = g_{Zi}(\mathbf{X}_s, \mathbf{X}_i, \mathbf{Y}_s(t), \mathbf{Y}_i(t), \mathbf{L}_{\bullet i}(t))$$

L: Linking variables

- **Time-dependent reliability over** $[t_0, t_s]$

$$R(t_0, t_s) = \Pr\{Z_i(t) = g_{Zi}(\mathbf{X}_s, \mathbf{X}_i, \mathbf{Y}_s(t), \mathbf{Y}_i(t), \mathbf{L}_{\bullet i}(t)) < e, \forall t \in [t_0, t_s]\}$$

Since the involvement of stochastic processes, the responses are time-dependent random variables, calculating the reliability is difficult.

Time-Dependent Reliability Methods

- **Upcrossing rate methods**

  -Asymptotic upcrossing rate of a Gaussian stochastic process (i.e. *Lindgren 1984, Breitung 1984, 1988*)

  - Vector out-crossing rate using parallel approach (i.e. *Hagen, 1992*)

  - The Rice's formula based method (i.e. Rice, 1944, *Sudret, Lemaire, 2004, Hu and Du, 2012*)

  - The joint-upcrossing rate method (i.e. *Hu and Du, 2013*)

- **Surrogate model methods**

  - Composite limit-state function method (i.e. *Mourelatos, 2011*)

  - Nested extreme value response method (i.e. *Wang and Wang, 2014*)

  - Mixed Efficient Global Optimization method (*i.e. Hu and Du, 2015*)

- **Sampling methods**

  - Importance sampling approach (i.e. *Singh and Mourelatos, 2011*)

  - Markov Chain Monte Carlo method (i.e. *Wang and Mourelatos, 2013*)

  - Sampling of extreme value distribution (i.e. *Hu and Du, 2013*)

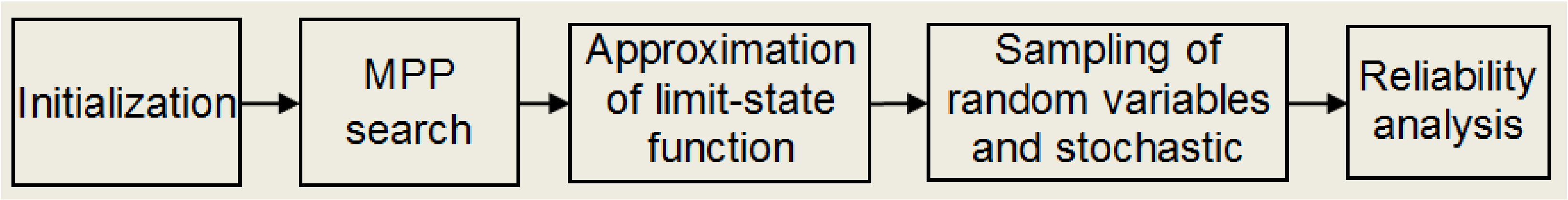
- **These methods are for components and may not be applicable for multidisciplinary systems.**

Proposed Method

- Approximate a response w.r.t. **X** by FORM and SORM at MPP
- Then the response is

  a linear stationary Gaussian process (FORM), or

  a quadratic stationary Gaussian process (SORM)
- Use MCS



Step 1 – MPP Search

Optimization[1]

$$\begin{cases} \min\limits_{(\mathbf{u},\mathbf{L})} \|\mathbf{u}\| \\ s.t. \\ \hat{g}_{Z_i}(\mathbf{u}_i, \mathbf{L}_{\bullet i}) > 0 \\ \mathbf{L}_{j\bullet}(t) = \widehat{g}_{\mathbf{L}_j}(\mathbf{u}_j, \mathbf{L}_{\bullet j}),\ j = 1, 2, \ldots, n \end{cases}$$

Failure constraint

Coupling between subsystems

[1] Du, X., Guo, J., and Beeram, H., 2008, "Sequential optimization and reliability assessment for multidisciplinary systems design," Structural and Multidisciplinary Optimization.

Step 2 – Approximation

FORM

$$\hat{g}_{Z_i}(\mathbf{U}) \approx \hat{g}_{Z_i}(\mathbf{u}_i^*) + \nabla\hat{g}_{Z_i}(\mathbf{u}_i^*)(\mathbf{U}_i - \mathbf{u}_i^*)^T$$

$$\Pr\left\{Z_i(t) > e\right\} = \Pr\left\{H(t) > \beta\right\}$$

SORM

$$\hat{g}_{Z_i}(\mathbf{U}) \approx \hat{g}_{Z_i}(\mathbf{u}_i^*) + \nabla\hat{g}_{Z_i}(\mathbf{u}_i^*)(\mathbf{U}_i - \mathbf{u}_i^*)^T + \frac{1}{2}(\mathbf{U}_i - \mathbf{u}_i^*)\nabla^2\hat{g}_{Z_i}(\mathbf{u}_i^*)(\mathbf{U}_i - \mathbf{u}_i^*)^T$$

Step 3 – Simulation

The Expansion Optimal Linear Estimation (EOLE) method[2].

$$U(t)=\sum_{i=1}^{p}\frac{V_i}{\sqrt{\eta_i}}\varphi_i^T\rho_U(t,t_i)$$

V_i are independent standard normal random variables; η_i and φ_i^T are the eigenvalues and eigenvectors of the matrix $\sum$, respectively.

$$\sum=\begin{pmatrix}\rho_U(t_1,t_1) & \rho_U(t_1,t_2) & \cdots & \rho_U(t_1,t_m)\\ \rho_U(t_2,t_1) & \rho_U(t_2,t_2) & \cdots & \rho_U(t_2,t_m)\\ \vdots & \vdots & \ddots & \vdots\\ \rho_U(t_m,t_1) & \rho_U(t_m,t_2) & \cdots & \rho_U(t_m,t_m)\end{pmatrix}_{m\times m}$$

$\rho_U(t_1,t_2)$ is the autocorrelation function of $U(t)$.

$$p_f(t_0,t_s)=\frac{N_f}{N}$$

[2] Li, C. C., Kiureghian, A. D., 1993, "Optimal discretization of random fields," Journal of Engineering Mechanics.

Examples

Two examples are solved using:

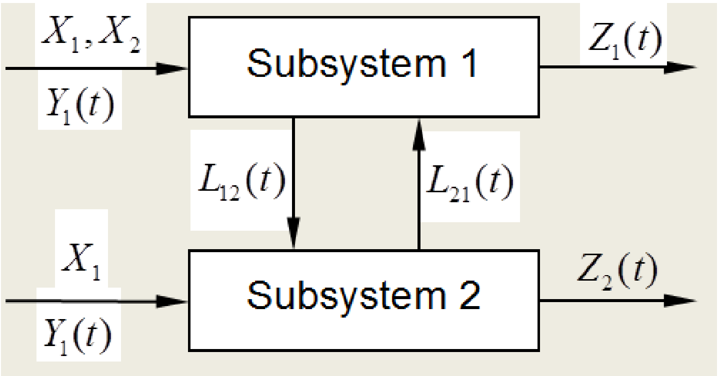
1. Proposed method based on FORM (FORM-MCS)

2. Proposed method based on SORM (SORM-MCS)

3. Upcrossing rate method (Upcrossing)

4. Direct MCS with the original limit-state function (MCS)

Example 1



Variable	Mean	Standard deviation	Distribution
x_1	1	0.1	Normal
x_2	1	0.1	Normal
$Y_1(t)$	1	0.1	Gaussian Process

For subsystem 1

$$L_{12}(t) = X_1^2 + X_2 + Y_1(t) - 0.2L_{21}(t)$$

$$Z_1(t) = X_2^2 + Y_1(t) + L_{12}(t) + L_{21}^2(t)$$

For subsystem 2

$$L_{21}(t) = \sqrt{L_{12}(t)} + X_1 + Y_1(t)$$

$$Z_2(t) = X_1^2 + Y_1(t) + L_{21}(t) + e^{-L_{12}(t)}$$

The autocorrelation coefficient function of $Y_1(t)$

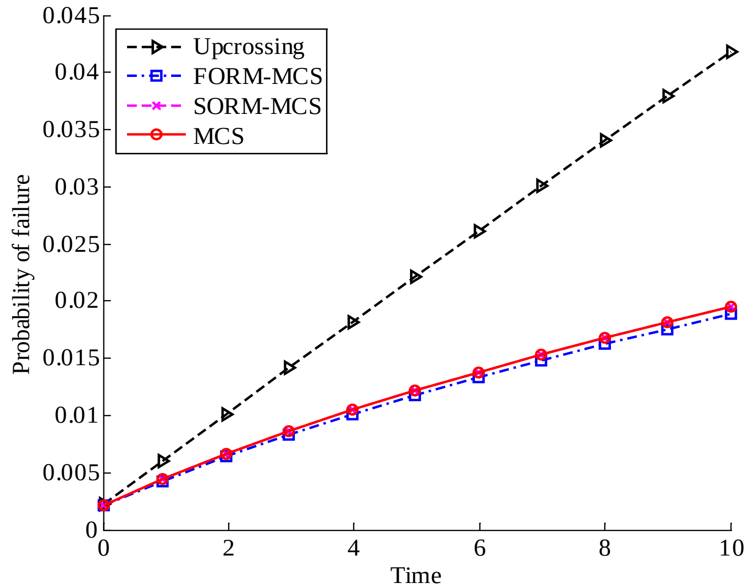
$$\rho_{Y_1}(t_1, t_2) = \exp\left\{-\left[\left(t_2 - t_1\right)/\lambda\right]^2\right\}$$

$\lambda = 0.9$ is a correlation length.

Limit state $e = 22$

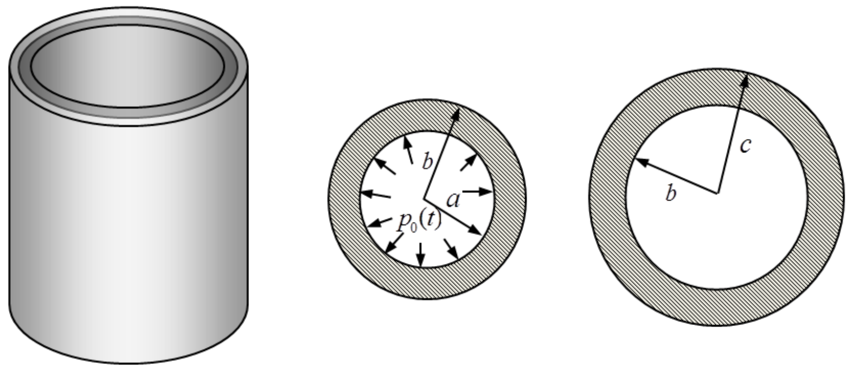
For MCS, Time interval $[0,10]$ is divided into 200 time instants and 10^6 samples are generated at each time instant.

Example 1

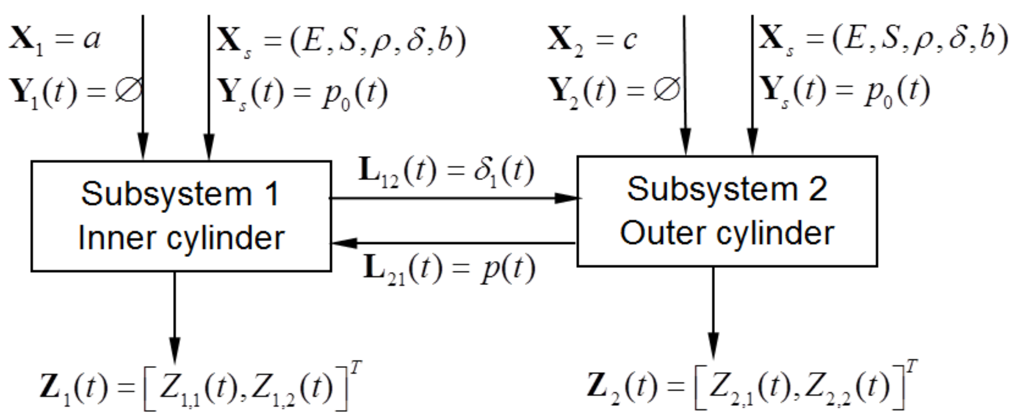


	p_f	Error (%)	Function calls
Upcrossing	0.041853	114.42	54
FORM-MCS	0.018857	3.39	54
SORM-MCS	0.019517	0.01	90
MCS	0.019519	N/A	2×10^{8}

Example 2

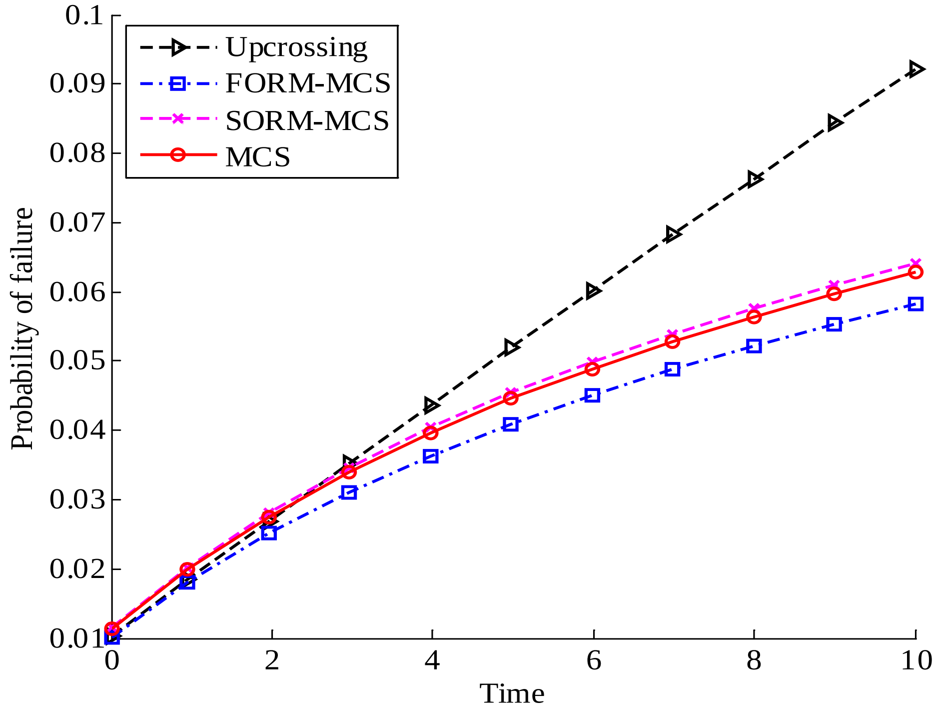


Compound cylinder system   Subsystem 1: Inner cylinder   Subsystem 2: outer cylinder



System structure of the compound cylinders

Example 2



	p_f	Error (%)	Function calls
Upcrossing	0.092251	46.99	596
FORM-MCS	0.058273	7.15	596
SORM-MCS	0.064065	2.08	1215
MCS	0.06276	N/A	2×10^8

Conclusions

- A reliability analysis method for time-dependent multidisciplinary system with stationary stochastic processes is developed.
- The results of the examples showed the efficiency and accuracy of the proposed method.

Future Work

- Explicit functions of time
- Non-stationary processes
- Higher efficiency

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