System Reliability Analysis for New Products Using Existing Components

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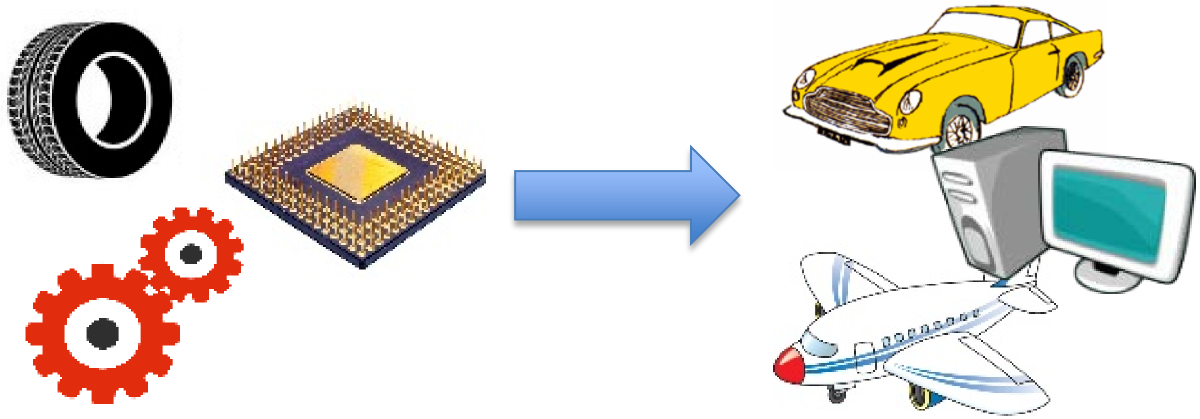
Outline

- Introduction
- System reliability analysis with independent component assumption
- System reliability analysis with dependent components sharing a common load
- Examples
- Conclusions

Introduction

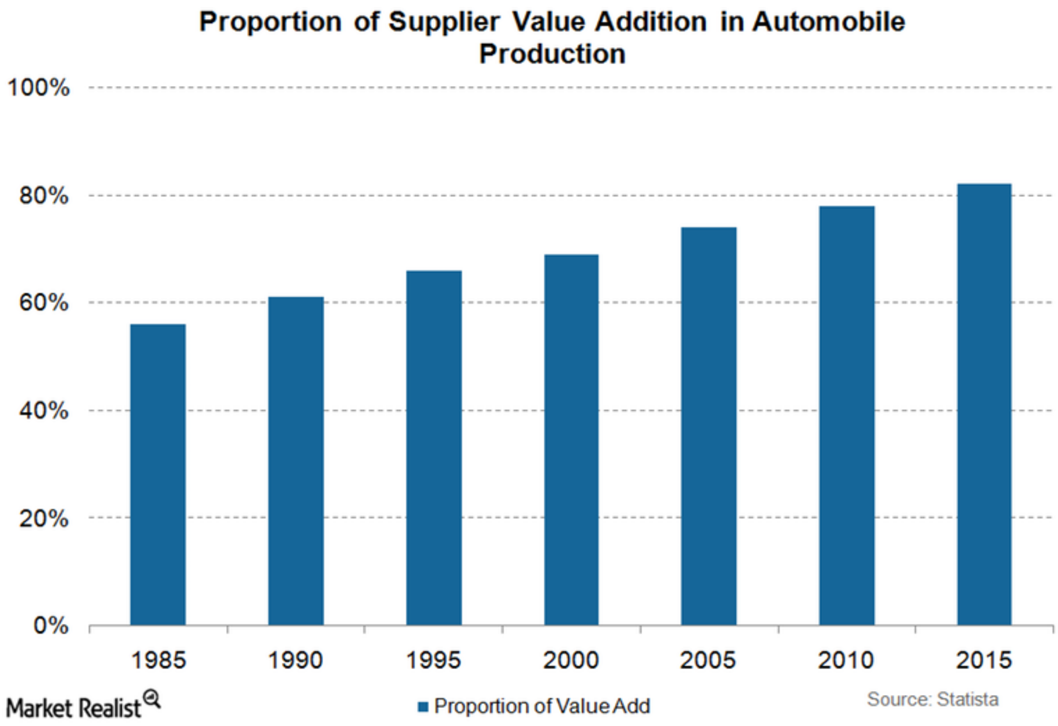
- It is not necessary to design all components for a new product
- Existing standard components can be used.
- They may be from suppliers.



Auto Industry Example

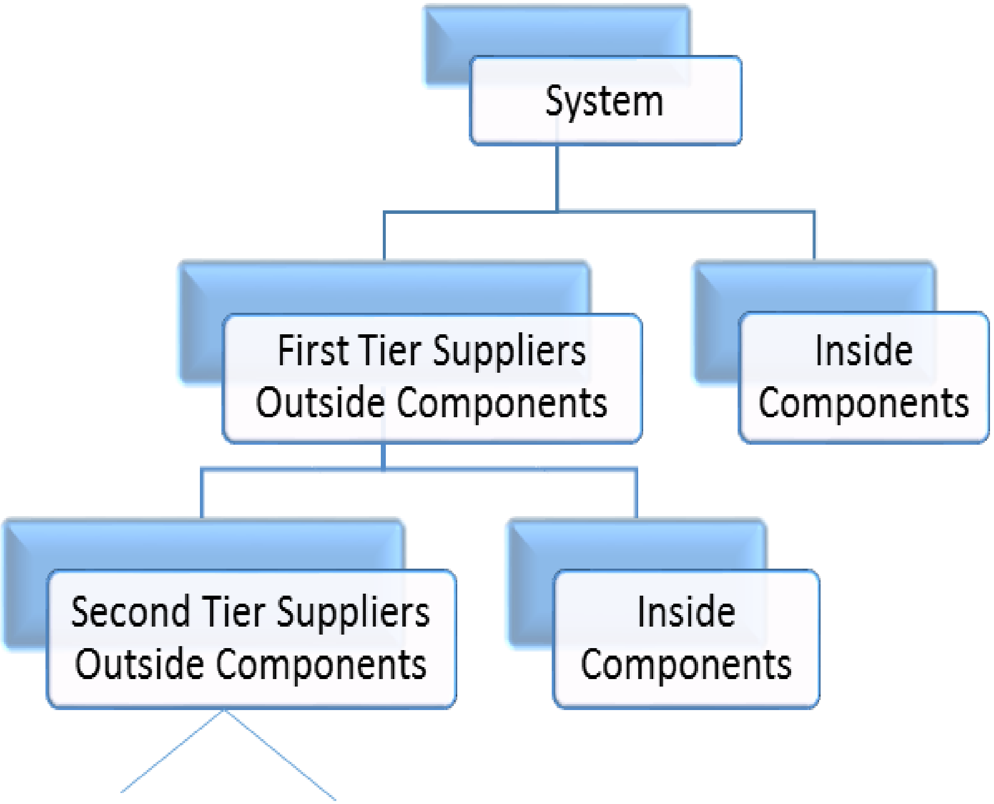
Auto suppliers' contribution (in terms of value)

- 56% in 1985
- 82% now



H. Kallstrom, 2015, *Suppliers' power is increasing in the automobile industry*

System Design

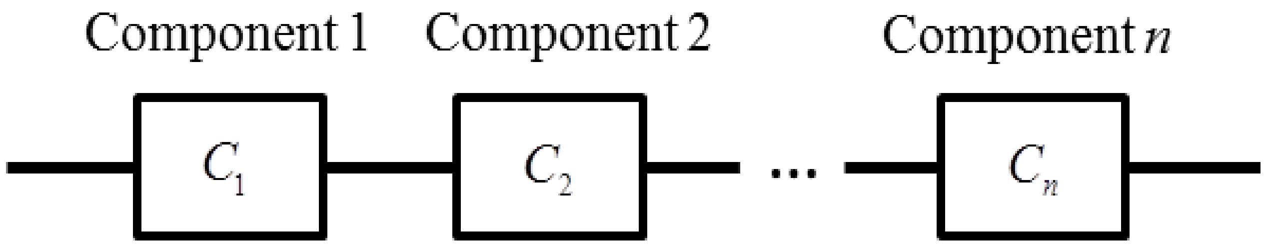


Challenges

- Predict system reliability early in the design stage.
- Information is limited.
- Component design details are proprietary to component designers.
- Different operation conditions may be for component design and system design.

Review of System Reliability

- Series systems



- Dependent component assumption

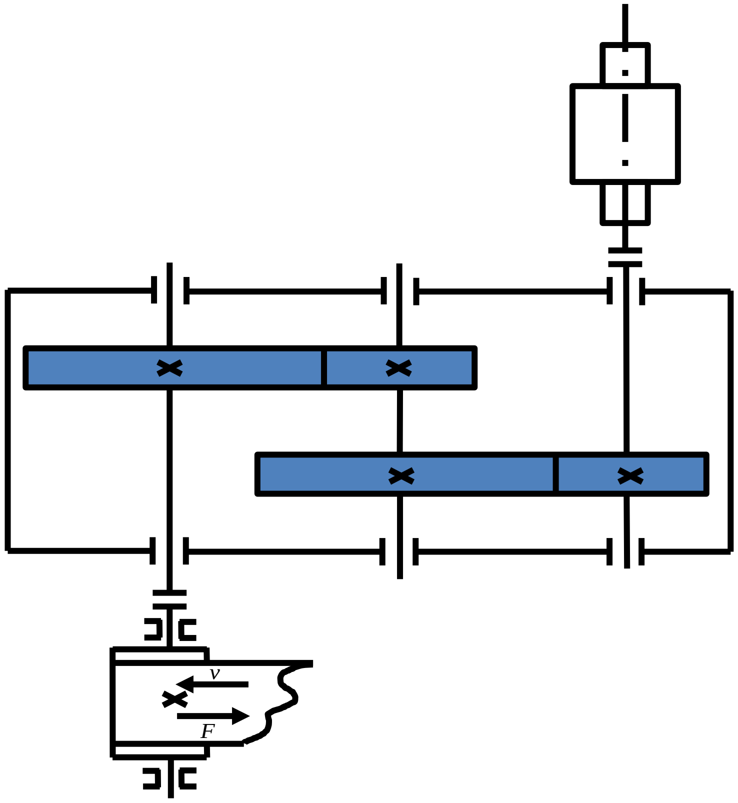
$$R_S = \prod_{i=1}^{n} R_i$$

- Reliability bounds

$$\prod_{i=1}^{n} R_i \leq R_S \leq \min\{R_i\}$$

Drawbacks

- For mechanical systems, the reliability bounds are too wide to make decisions.
- The lower bound is too conservative.
- Example
  - If $R_i = 0.9999$
  - $0.9976 \leq R_s \leq 0.9999$
- We need to consider component dependency.



Research Issue

- System designers know only
  - Component reliabilities
  - Distribution of the system load
- They do not know details of component design.
- What is the system reliability?
  - Without using the independent component reliability assumption

Account for Component Dependencies

- Consider the system load shared by components
- Use physics-based approaches, or limit-state functions defined by

$$Y_{ij} = g_{ij}(\mathbf{X}_i)$$

  for the *j*-th failure mode of the *i*-th component

- The probability of failure is

$$p_{fij} = \Pr\{Y_{ij} = g_{ij}(\mathbf{X}_i) < 0\}$$

- The system probability of failure is then

$$p_{fS} = \Pr\{\cup\, Y_{ij} = g_{ij}(\mathbf{X}_i) < 0\}$$

Account for Component Dependencies

- The system probability of failure is

$$p_{fS} = \Pr\{\cup\, Y_{ij} = g_{ij}(\mathbf{X}_i) \ < 0\}$$

- $\mathbf{X}_i$ contains component load L_i
- L_i is a function of the system load L.
- Then Y_{ij} are statistically dependent.
- With dependent limit-state functions, system reliability analysis will be more accurate.

Strategy One
Narrow System Reliability Bounds

- Assume in the component design
  - Rewrite $Y_{ij} = g_{ij}(\mathbf{X}_i)$ as $Y_{ij} = S_{ij} - L_i$
  - S_{ij}: generalized strength
  - $L_i = w_{ij}L, w_{ij}$ =constant
- Component probabilities of failure p_{fij} are known to the system designers.
- The range of factors of safety $n_{Sij} = \frac{\mu_{Sij}}{\mu_{Li}}$ can be estimated.
- The distribution of the system load L is known.

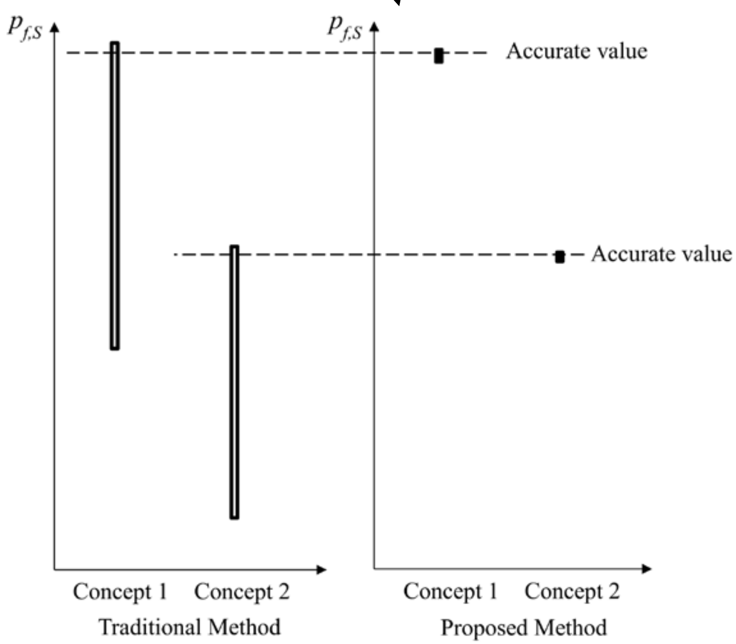
Optimization Model

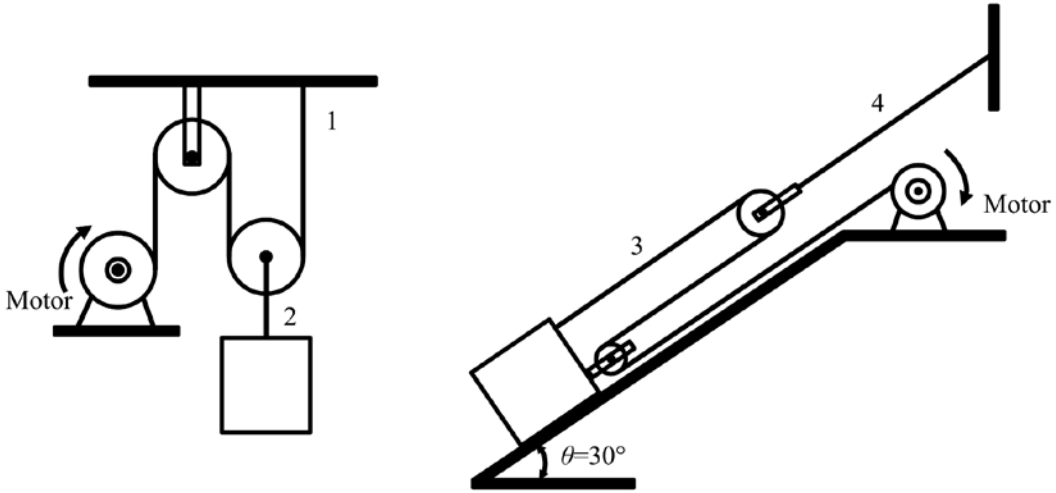
- Find $\mathbf{d} = \{\text{unknown distribution parameters of } S_{ij}\}$
- $\min p_{fS}$ or $\max p_{fS}$
- subject to constraints
  - Range of factors of safety
  - Component probabilities of failure
  - Other
- Solution: bounds $[p_{fS,min}, p_{fS,max}]$

Example

Difficult to select design concepts

Easy to select design concepts

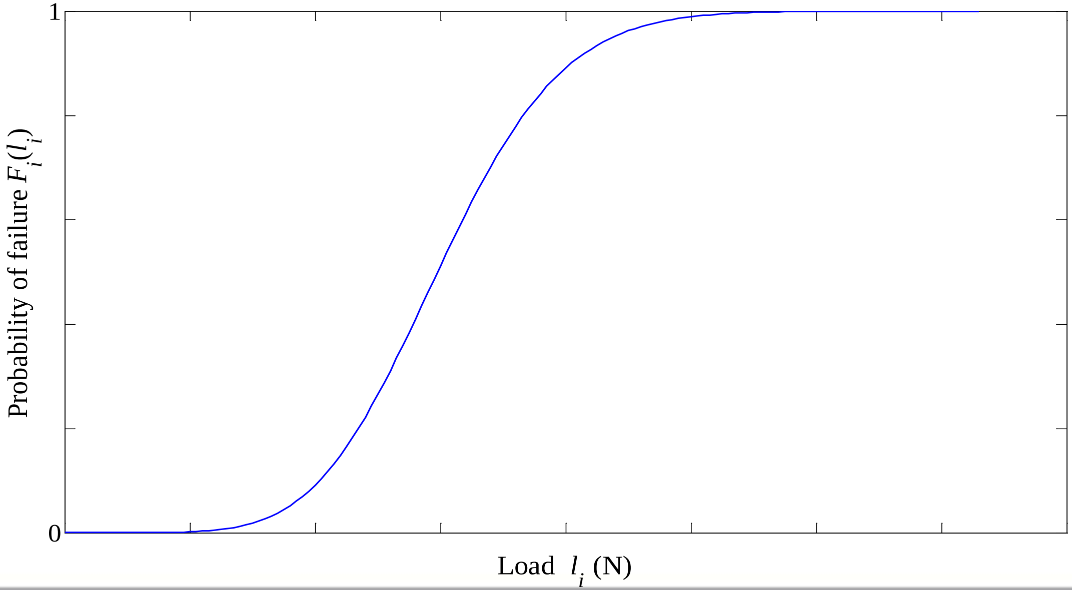




(a) Design concept 1 (b) Design concept 2

Strategy Two
Reconstruct Component Limit-State Functions

- Component designers
  - Provide component probability of failure functions w.r.t. component load $p_{fij}(L_i)$
  - They may use $Y_{ij} = g_{ij}(\mathbf{X}_i)$ or any other methods



Strategy Two

- System design designers
  - Reconstruct equivalent component limit-state functions

$$Y_i = S_i - w_i L$$

  - Such that

$$Y_i < 0 \Leftrightarrow Y_{i1} < 0 \cup Y_{i2} < 0 \cup \cdots$$

  - Then evaluate the system probability of failure

$$p_{fS} = \Pr\left\{\bigcup_{i=1}^{n} Y_i < 0\right\} = \Pr\left\{\bigcup_{i=1}^{n} S_i < w_i L\right\}$$

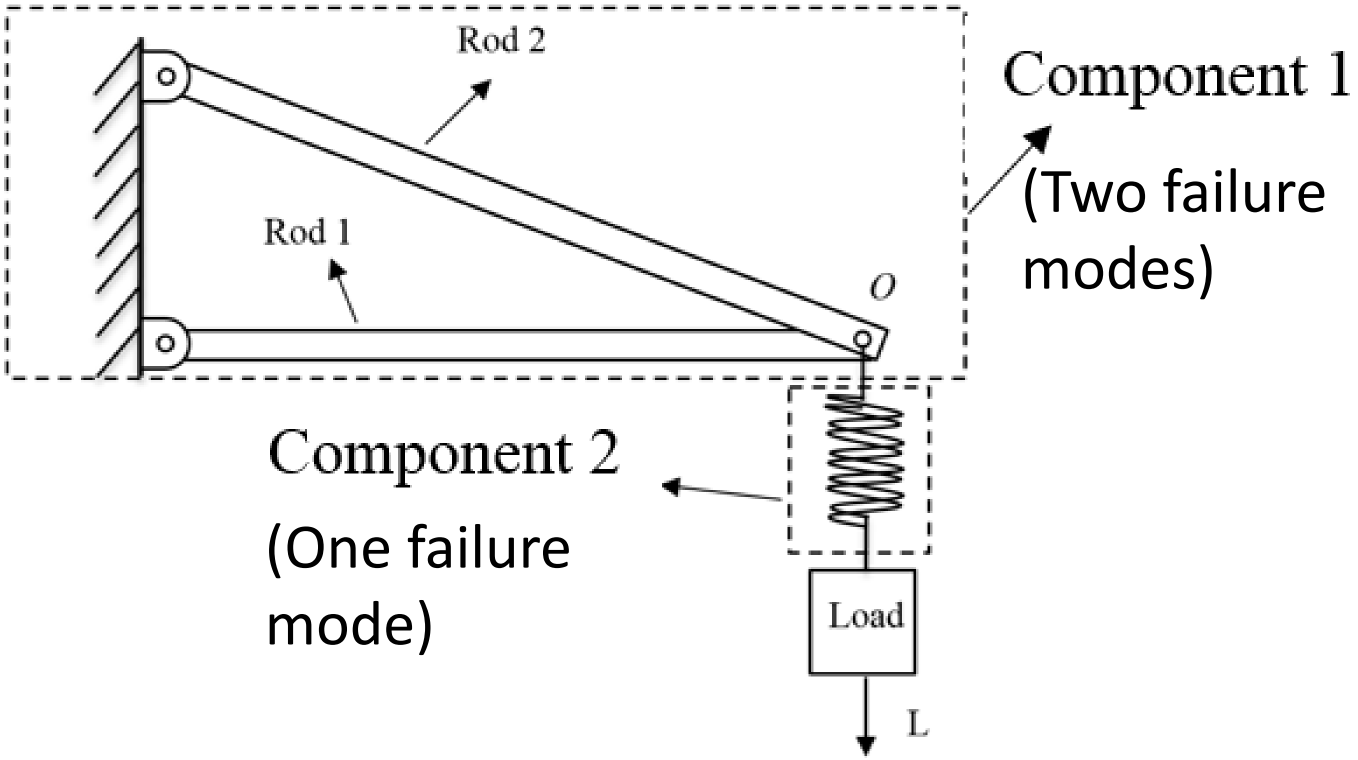
Strategy Two

- The system probability of failure

$$p_{fS} = \Pr\left\{\bigcup_{i=1}^{n} S_i < w_i L\right\}$$

- What's known to the system designers
  - The distribution of L
  - S_i $(i = 1,2,\ldots)$ are independent
  - The distributions of S_i are the component probability of failure functions w.r.t. load $p_{fij}(L_i)$
- Then it's ready to estimate p_{fS}.

Example: A system with two components

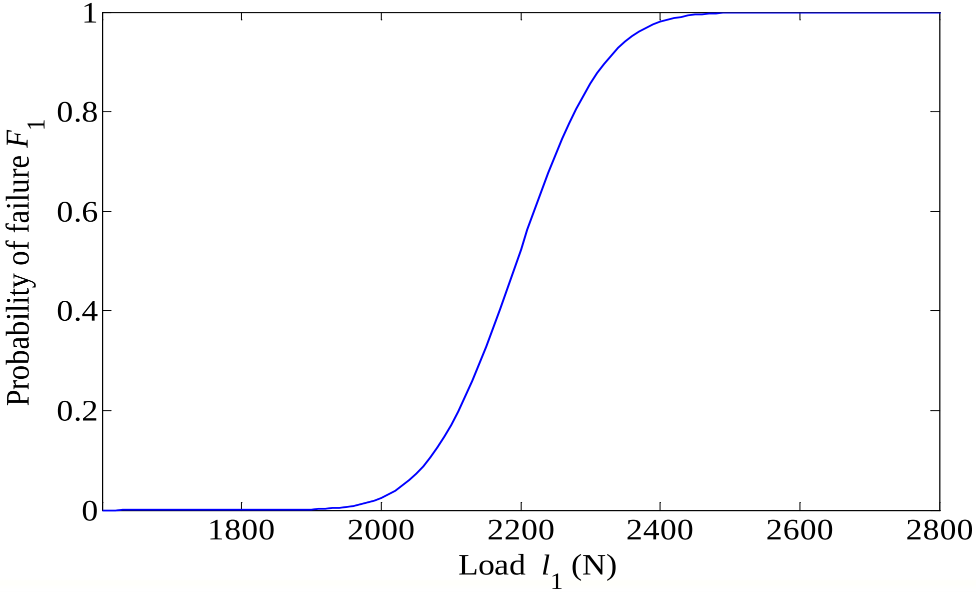


Component Reliability Analysis by Company 1

Limit-state functions for two failure modes

$$Y_{11} = S_{11} - \frac{4L_1 a_2}{\sqrt{a_2^2 - a_1^2}(\pi d_1^2)} < 0$$

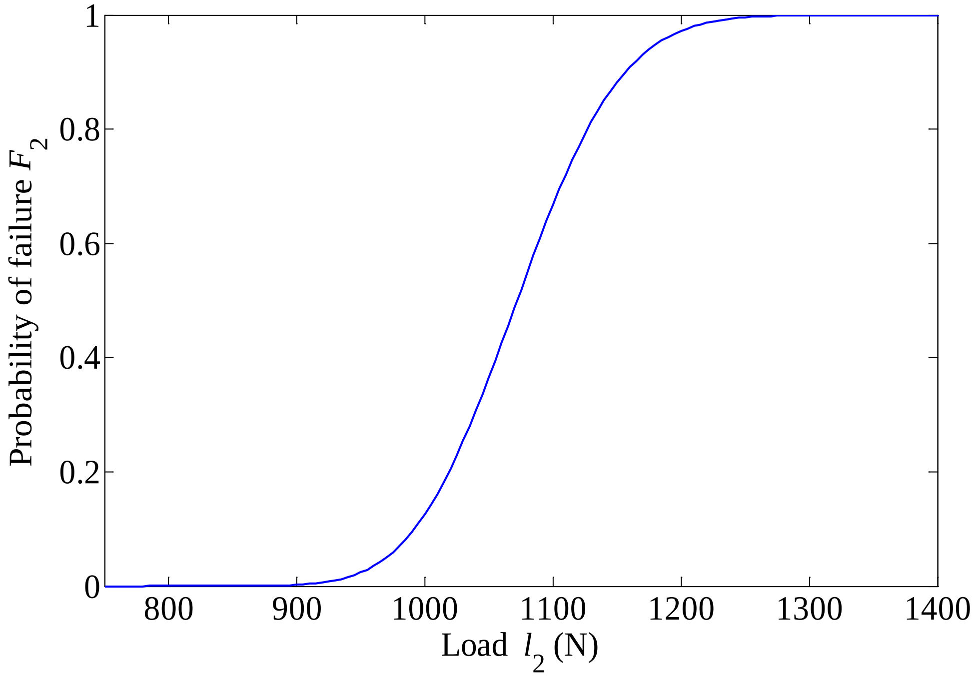
$$Y_{12} = S_{12} - \frac{4L_1 a_1}{\sqrt{a_2^2 - a_1^2}(\pi d_2^2)} < 0$$



Component Reliability Analysis by Company 2

Limit-state function for one failure mode

$$Y_2 = S_2 - \frac{4L_2 D\left(\frac{4(D-d)}{\sqrt{4D-4d}} + \frac{0.615d}{D}\right)}{\pi d^3} < 0$$



System Reliability Analysis by System Designers

- Reconstruct two equivalent component limit-state functions for Y_1 and Y_2
- Calculate $p_{fS} = \Pr\{Y_1 < 0 \cup Y_2 < 0\}$
- Errors are w.r.t. p_{fs} (true value) as if everything was known.

	Independent assumption	Proposed Method	True value
p_{fS}	4.591×10^{-4}	4.139×10^{-4}	4.059×10^{-4}
Error (%)	13.10	1.97	-

Conclusions

- It is possible to accurately predict system reliability of a new product during the early design stage.
- Component limit-state functions could be reconstructed using component reliability functions with respect to component load.
- The dependency of components could be considered automatically.
- The proposed methods are more accurate than the method using the assumption of independent components.

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