System Reliability Analysis with Dependent Component Failures during Early Design Stage

Yao Cheng and Xiaoping Du

Department of Mechanical and Aerospace Engineering

MISSOURI S&T

Founded 1870 | Rolla, Missouri

ASME SETTING THE STANDARD

November 13–19, 2015 • Houston, Texas

2015 IMECE

ONE GREAT LEARNING EXPERIENCE.

INTERNATIONAL MECHANICAL ENGINEERING CONGRESS & EXPOSITION

Outline

- Introduction
- Approaches
- Results
- Future work
- Conclusions

Introduction

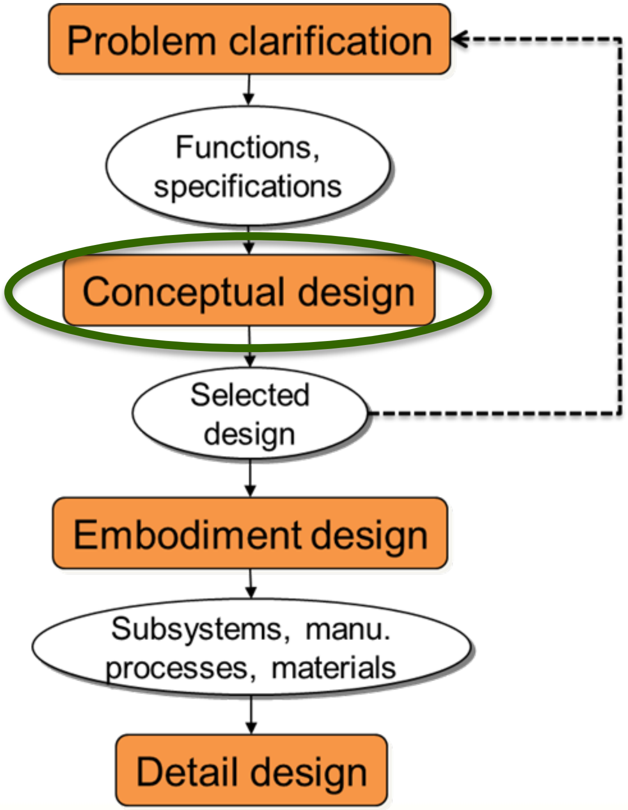
Objective

Predict system reliability quantitatively with dependent component failures in product early design stage

Background

What is reliability?

Reliability is the probability that a product performs its intended function under specified conditions during a specified period of time.



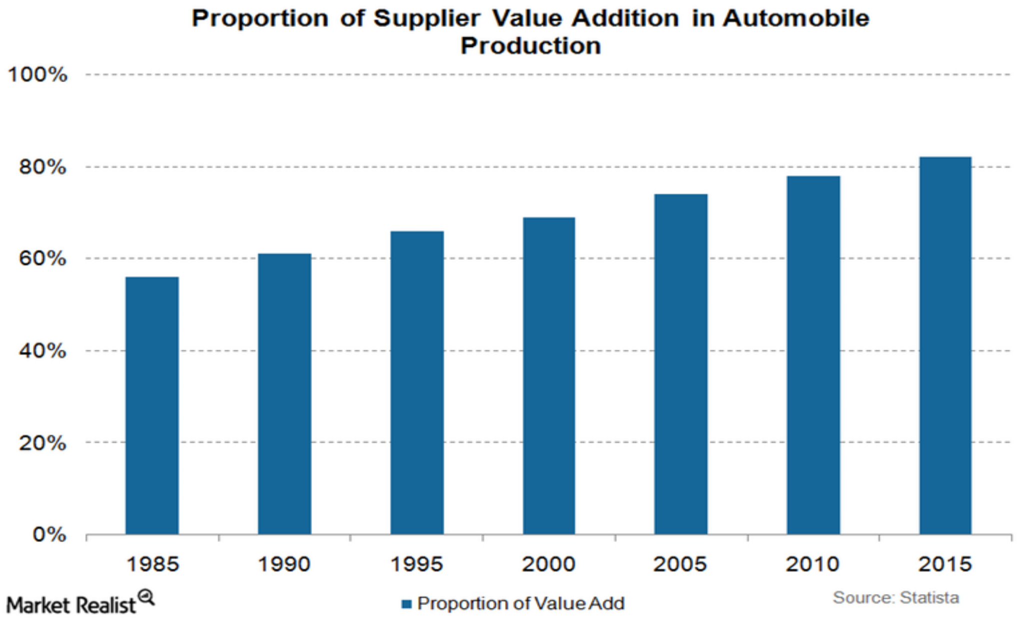
Why reliability prediction in early design stages?

- High reliability
- High robustness
- High safety
- High availability
- Low risk
- Low cost

Background cont.

Why consider dependent component failures?

- It is not necessary to design all the components for a new product.
- Components can be provided by outside suppliers.
- Example in auto suppliers' contribution (in terms of value)
    - 56% in 1985
    - 82% now



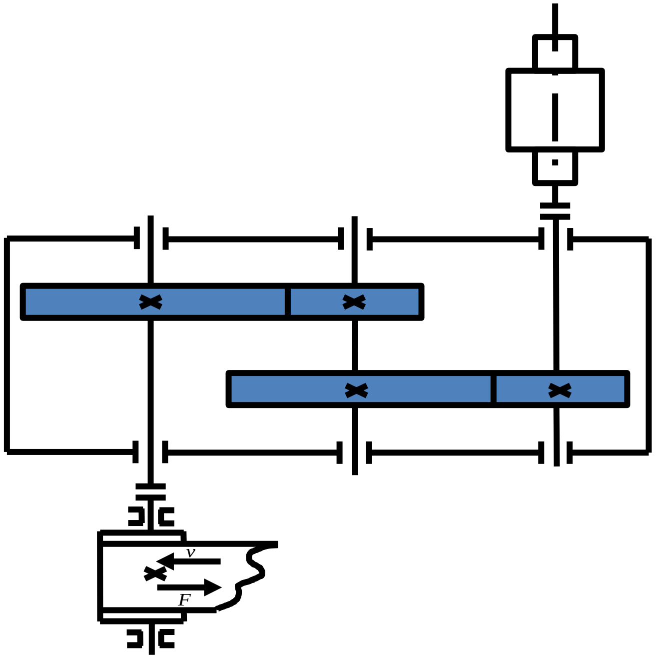
Kallstrom, H. (2015)

Background cont.

- System reliability from independent component assumption in series system

$$\prod_{i=1}^{n} R_i \leq R_s \leq \min\{R_i\}, i = 1, \dots, n$$

- Speed reducer example
  - If $R_i = 0.9999$
  - $0.9976 \leq R_s \leq 0.9999$
- The reliability bounds are too wide to make decisions; the lower bound is too conservative.
- We need to consider component dependency.



Challenge

Consider component dependence with limited information to system designers:

- Known component reliabilities
- Known distribution of the system load
- Unknown details of component design

Approaches

- Use physics-based approaches
- Consider the system load L shared by components; in the component design
  - Limit state function: $Y_i = S_{R,i} - w_i L$
  - $S_{R,i}$: generalized strength
  - w_i: constant, $L_i = w_i L$
- System reliability is then

$$R_s = \Pr(Y_1 > 0, Y_2 > 0, \ldots, Y_n > 0) = \Pr(\mathbf{Y} > 0)$$

- Use joint distribution to represent component dependence

Optimization Model

- Objective: min R_S or max R_S
- Design variables: distribution parameters

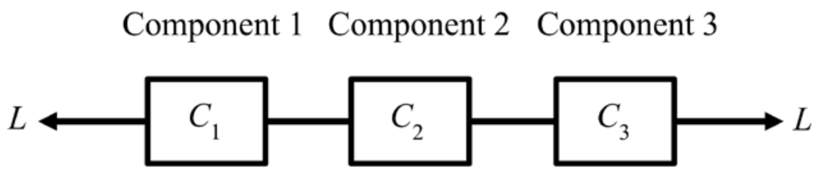
Find $\mathbf{d} = \{\text{unknown distribution parameters of } S_{R,i}\}$

- Constraints
  - Component reliabilities
  - Range of factors of safety
  - Other
- Solution: bounds $[R_{S,min}, R_{S,max}]$

$$
\begin{cases}
\min_{\mathbf{d}} \; R_S(\mathbf{d}; \mu_L, \sigma_L) \\
\text{subject to} \\
h_i(\mathbf{d}; \mu_L, \sigma_L) = \Phi\left(-\dfrac{w_i\mu_L - \mu_{S_{R,i}}}{\sqrt{\sigma_{S_{R,i}}^2 + (w_i\sigma_L)^2}} \right) = R_i, \; i = 1, 2, \ldots, n \\
g_i(\mathbf{d}; \mu_L, \sigma_L) = n_{s,i}^{\min} - \dfrac{\mu_{S_{R,i}}}{w_i\mu_L} \le 0, \\
g_{i+n}(\mathbf{d}; \mu_L, \sigma_L) = \dfrac{\mu_{S_{R,i}}}{w_i\mu_L} - n_{s,i}^{\max} \le 0, \\
g_{i+2n}(\mathbf{d}; \mu_L, \sigma_L) = c_i^{\min} - \dfrac{\sigma_{S_{R,i}}}{\mu_{S_{R,i}}} \le 0, \\
g_{i+3n}(\mathbf{d}; \mu_L, \sigma_L) = \dfrac{\sigma_{S_{R,i}}}{\mu_{S_{R,i}}} - c_i^{\max} \le 0,
\end{cases}
$$

Results

Three components sharing same load



Information to system designers

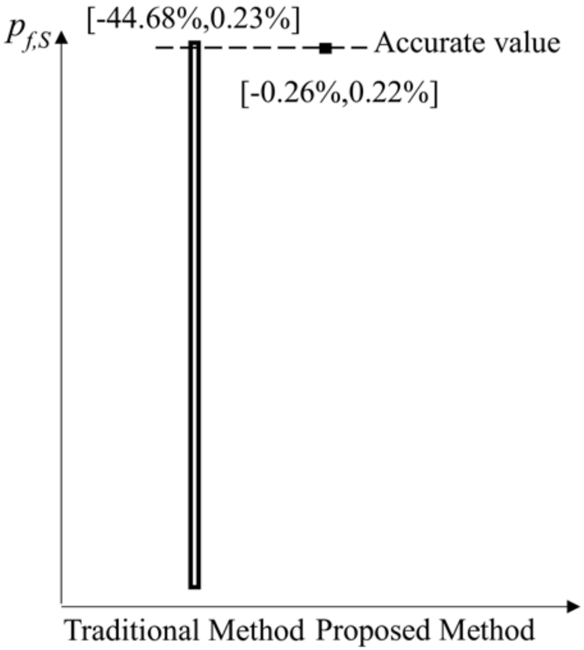
Known information	Value
Probability of component failure $p_{f,1}$	9.920×10^{-5}
Probability of component failure $p_{f,2}$	1.2696×10^{-4}
Probability of component failure $p_{f,3}$	3.87×10^{-6}
Distribution of system load L	$N(2000, 200^2)$ kN
Factor of safety for component 1 $n_{s,1}$	[1.5,2.5]
Factor of safety for component 2 $n_{s,2}$	[1.5,2.5]
Factor of safety for component 3 $n_{s,3}$	[1.5,2.5]
Coefficient of variation of resistance of component 1 c_1	[0.08, 0.20]
Coefficient of variation of resistance of component 2 c_2	[0.08, 0.20]
Coefficient of variation of resistance of component 3 c_3	[0.08, 0.20]

Results cont.

Methods	Bounds of $p_{f,s}$	Interval width
Traditional method	$[1.2696,\ 2.3002]\times10^{-4}$	1.0306×10^{-4}
Proposed method	$[2.2891,\ 2.30]\times10^{-4}$	0.0109×10^{-4}
Exact	2.2950×10^{-4}	

The bounds are much narrower

The bounds do contain exact value



Future Work

Extend this study in the following aspects:

- Reliability analysis for parallel and mix systems
- Develop the relationship between factor of safety and reliability
- Generate a general stress strength interference model in conceptual design stage
- Predict system reliability for a given period of time

Conclusions

- It is necessary to consider component dependent failures in system reliability analysis.
- It is possible to accurately predict system reliability with component dependence using physics-based methodologies.
- Our proposed method has a potential to be applied widely in the future.

Acknowledgement

National Science Foundation CMMI 1300870

Thank you!

Q & A