1-1. The force acting on the truss is $P \sim N(50, 4^2)$ kN as shown in Fig. 1.1.1. If the allowable normal stress of each of the bar is $S_a \sim N(260, 15^2)$ MPa, what are the probabilities of failure of these three bars? Assume that P and S_a are independent.

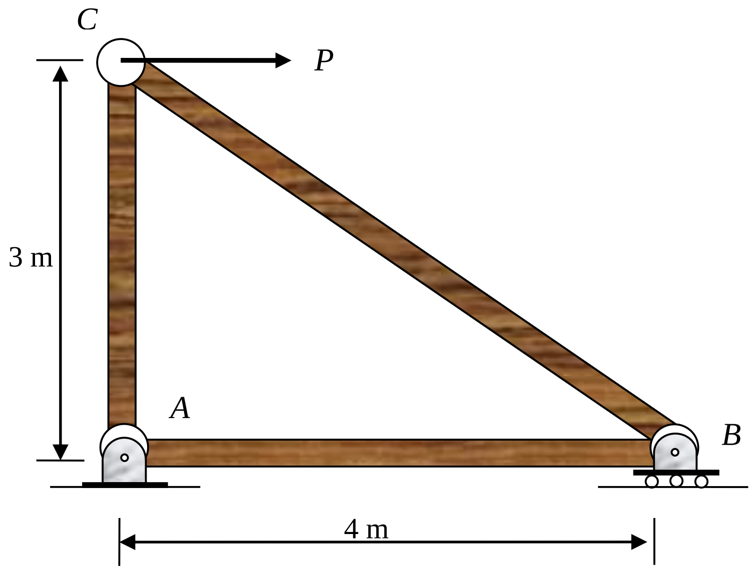

Fig. 1.1.1

Solution

Internal Loadings: The force developed in each member of the truss can be determined by using the method of joints. First, consider the equilibrium of joint C, Fig. 1.1.2,

$$\underset{\rightarrow}{+}\Sigma F_x = 0; \qquad P - F_{BC}\left(\frac{4}{5}\right) = 0; \qquad F_{BC} = 1.25P$$

$$+\uparrow \Sigma F_y = 0; \quad F_{BC}\left(\frac{3}{5}\right) - F_{AC} = 0; \quad F_{AC} = 0.75P$$

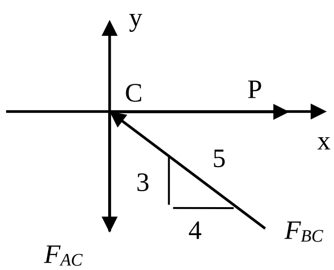

Fig. 1.1.2

Subsequently, the equilibrium of joint B, Fig. 1.1.3, is considered

$$\underset{\rightarrow}{+}\Sigma F_x = 0; \qquad F_{BC}\left(\frac{4}{5}\right) - F_{AB} = 0; \quad F_{AB} = P$$

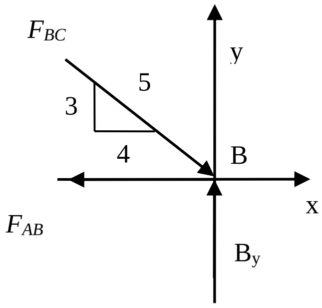

Fig. 1.1.3

The cross-sectional area of each of the bars is

$$A = \frac{\pi}{4}(0.02^2) = 3.142 \times 10^{-4}\ \text{m}^2.$$

For bar BC, the probability of failure p_f is

$$p_f = \Pr(S_{BC} > S_a) = \Pr(Y = S_a - S_{BC} < 0) = \Pr\left(Y = S_a - \frac{F_{BC}}{A} < 0\right) = \Pr\left(Y = S_a - \frac{1.25P}{A} < 0\right) \quad (1)$$

Since $P \sim N(50, 4^2)\,\text{kN}$, $S_a \sim N(260, 15^2)\,\text{MPa}$, and P and S_a are independent, Y also follows a normal distribution, $Y \sim N(\mu_Y, \sigma_Y^2)$.

$$\mu_Y = \mu_{S_a} - \left(\frac{1.25}{A}\right)\mu_P = 260 - \left(\frac{1.25}{0.3142}\right)50 = 61.08\,\text{MPa}$$

$$\sigma_Y = \sqrt{\sigma_{S_a}^2 + \left(\frac{1.25}{A}\right)^2 \sigma_P^2} = \sqrt{15^2 + \left(\frac{1.25}{0.3142}\right)^2 (4)^2} = 21.86\,\text{MPa}$$

Equation (1) can be written as

$$p_f = \Pr(Y < 0) = \Pr\left(\frac{Y - \mu_Y}{\sigma_Y} < \frac{-\mu_Y}{\sigma_Y}\right) = \Phi\left(\frac{-\mu_Y}{\sigma_Y}\right) = \Phi(-2.794) = 0.0026 \qquad \textbf{Ans.}$$

Similarly, for bar AC, the probability of failure p_f is

$$p_f = \Pr(S_{AC} > S_a) = \Pr(Y = S_a - S_{AC} < 0) = \Pr\left(Y = S_a - \frac{F_{AC}}{A} < 0\right) = \Pr\left(Y = S_a - \frac{0.75P}{A} < 0\right) \quad (2)$$

$$\mu_Y = \mu_{S_a} - \left(\frac{0.75}{A}\right)\mu_P = 260 - \left(\frac{0.75}{0.3142}\right)50 = 140.65\,\text{MPa}$$

$$\sigma_Y = \sqrt{\sigma_{S_a}^2 + \left(\frac{0.75}{A}\right)^2 \sigma_P^2} = \sqrt{15^2 + \left(\frac{0.75}{0.3142}\right)^2 (4)^2} = 17.78\,\text{MPa}$$

Equation (2) can be written as

$$p_f = \Pr(Y < 0) = \Pr\left(\frac{Y - \mu_Y}{\sigma_Y} < \frac{-\mu_Y}{\sigma_Y}\right) = \Phi\left(\frac{-\mu_Y}{\sigma_Y}\right) = \Phi(-7.911) = 1.286\times 10^{-15} \qquad \textbf{Ans.}$$

For bar AB, the probability of failure p_f is

$$p_f = \Pr\,(S_{AB} > S_a) = \Pr\,(Y = S_a - S_{AB} < 0) = \Pr\left(Y = S_a - \frac{F_{AB}}{A} < 0\right) = \Pr\left(Y = S_a - \frac{P}{A} < 0\right) \qquad (3)$$

$$\mu_Y = \mu_{S_a} - \left(\frac{1}{A}\right)\mu_P = 260 - \left(\frac{1}{0.3142}\right)50 = 100.86\text{ MPa}$$

$$\sigma_Y = \sqrt{\sigma_{S_a}^2 + \left(\frac{1}{A}\right)^2 \sigma_P^2} = \sqrt{15^2 + \left(\frac{1}{0.3142}\right)^2 (4)^2} = 19.67\text{ MPa}$$

Equation (3) can be written as

$$p_f = \Pr\left(Y < 0\right) = \Pr\left(\frac{Y - \mu_Y}{\sigma_Y} < \frac{-\mu_Y}{\sigma_Y}\right) = \Phi\left(\frac{-\mu_Y}{\sigma_Y}\right) = \Phi(-5.128) = 1.473 \times 10^{-7} \qquad \textbf{Ans.}$$