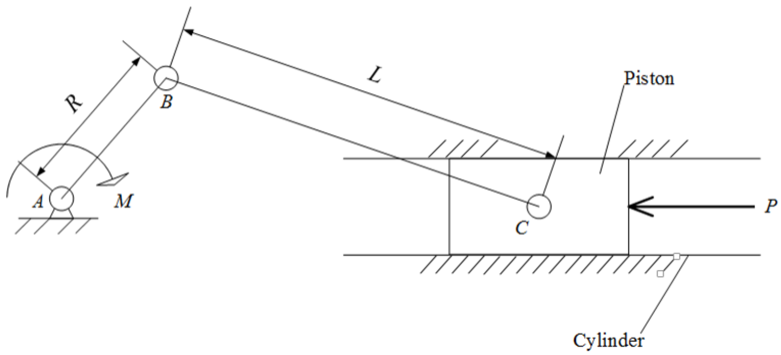
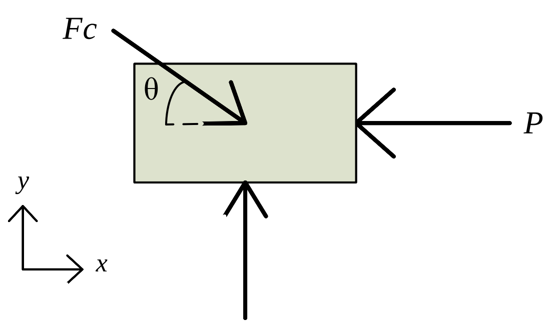
1-10. The piston in an engine is subject to a random force P that follows a normal distribution of $P \sim N\,(10, 2^2)$ kN. The connection rod BC has a diameter of d = 13 mm, and a length of L = 1.75 m. The length of the crank AB is R = 0.6 m. The normal stress allowed in the connection rod follows a normal distribution $S_a \sim N\,(135, 5^2)$ MPa. If P and S_a are independent, determine the probability of failure of the connecting rod.



Solution

Free Body Diagram



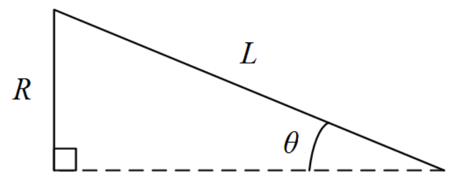
Referring to the free-body diagram

$$\sum F_x = ma_x;\; F_C \cos\theta - P = 0 \Rightarrow P = F_C \cos\theta$$

Finding the maximum allowable force in the connecting rod

$$F_{Callow} = S_a A \;\text{; where } A = \frac{\pi d^2}{4}$$

Note that the connecting rod receives the maximum force at the following instant, thus



$$\cos\theta = \frac{\sqrt{L^2 - R^2}}{L} = \sqrt{1 - \left(\frac{R}{L}\right)^2}$$

Solving for the allowable force P

$$P_a = F_{Callow}\cos\theta = S_a A\cos\theta = S_a \frac{\pi d^2}{4}\sqrt{1-\left(\frac{R}{L}\right)^2}$$

The probability of failure is $p_f = \Pr\left(P_a < P\right)$. Let $Y = P_a - P$, then

$$Y = P_a - P = S_a\left(\frac{\pi d^2}{4}\sqrt{1-\left(\frac{R}{L}\right)^2}\right) - P$$

Then, p_f is written as $p_f = \Pr\left(Y < 0\right)$. Since normal random variables S_a and P are independent, Y also follows a normal distribution, $Y \sim N\left(\mu_Y, \sigma_Y^2\right)$. Thus,

$$\mu_Y = \mu_{S_a}\left(\frac{\pi d^2}{4}\sqrt{1-\left(\frac{R}{L}\right)^2}\right) - \mu_P = (135)\times\left(\frac{\pi\times(13)^2}{4}\right)\times\sqrt{1-\left(\frac{0.6}{1.75}\right)^2} - \left(10\times10^3\right)$$

= 6833 N = 6.833 kN

$$\sigma_Y = \sqrt{\sigma_{S_a}{}^2 \times\left(\frac{\pi d^2}{4}\sqrt{1-\left(\frac{R}{L}\right)^2}\right)^2 + \sigma_P{}^2} = \sqrt{(5)^2\times\left(\frac{\pi\times(13)^2}{4}\sqrt{1-\left(\frac{0.6}{1.75}\right)^2}\right)^2 + (2\times10^3)^2}$$

= 2095 N = 2.095 kN

Therefore

$$p_f = \Pr(Y<0) = \Pr\left(\frac{Y-\mu_Y}{\sigma_Y} < \frac{-\mu_Y}{\sigma_Y}\right) = \Phi\left(-\frac{\mu_Y}{\sigma_Y}\right) = 1-\Phi\left(\frac{\mu_Y}{\sigma_Y}\right) = 1-\Phi\left(\frac{6.833}{2.095}\right)$$

$$= 1 - \Phi\left(3.2616\right) = 1 - 0.99945 = 5.5 \times 10^{-4}$$ **Ans.**