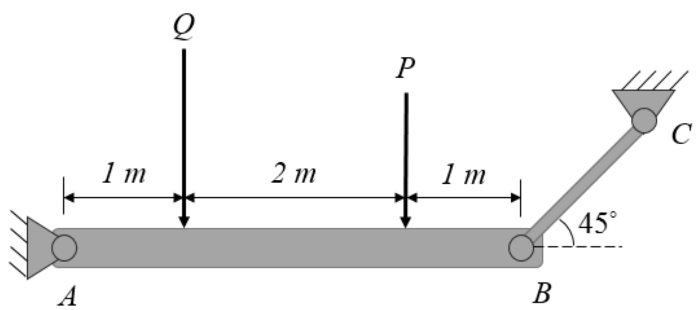
1-11. The loads on the beam follow the distributions of $P \sim N(12, 3^2)$ kN and $Q \sim N(36, 6^2)$ kN. The pins at A and B are in double shear, and both have a diameter of d = 15 mm. What is the distribution of the shear stress in pin B? Assume P and Q are independent.



Solution

Solve for F_B

$$\Sigma M_A = -Q(1) - P(3) + F_B \sin 45^\circ (4) = 0$$
$$\Rightarrow F_B = \frac{Q + 3P}{4\sin 45^\circ}$$

Pin B is in double shear, thus

$$V_B = \frac{F_B}{2} = \frac{Q + 3P}{8\sin 45^\circ}$$

Solve for area

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.015)^2 = 1.767 \times 10^{-4} \text{ m}^2$$

Solve for τ_B

$$\tau_B = \frac{V_B}{A} = \frac{Q + 3P}{8A\sin 45^\circ}$$

Because normal random variables Q and P are independent, τ_B also follows a normal distribution, $\tau_B \sim N\left(\mu_{\tau_B}, \sigma_{\tau_B}^2\right)$.

$$\mu_{\tau_B} = \frac{\mu_Q + 3\mu_P}{8A\sin 45^\circ} = \frac{36\left(10^3\right) + 3(12)\left(10^3\right)}{8(1.77)\left(10^{-4}\right)\sin 45^\circ} = 72.0 \text{ MPa}$$

$$\sigma_{\tau_B} = \sqrt{\left(\frac{1}{8A\sin 45^\circ}\right)^2 Q^2 + \left(\frac{3}{8A\sin 45^\circ}\right)^2 P^2}$$

$$= \sqrt{\left(\frac{1}{8(1.77)(10^{-4})\sin 45^\circ}\right)^2 \left(6(10^3)\right)^2 + \left(\frac{3}{8(1.77)(10^{-4})\sin 45^\circ}\right)^2 \left(3(10^3)\right)^2} = 10.8 \text{ MPa}$$

Then

$$\tau_B \sim N(72.0, 10.8^2) \text{ MPa}$$ **Ans.**