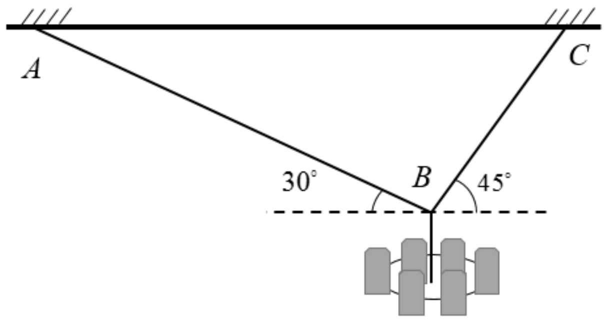
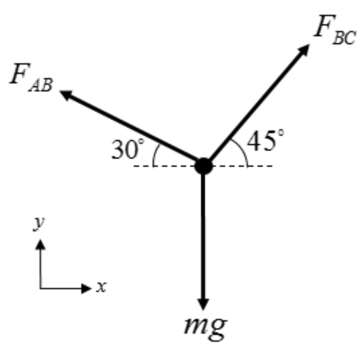
1-13. The chandelier is hung from the ceiling using two rods. Rod *AB* has a diameter of d_{AB} = 4 mm, and rod *BC* has a diameter of d_{BC} = 6 mm. The allowable stress in either rod follows the distribution $S_A \sim N(135, 15^2)$ MPa. If the mass of the chandelier follows the distribution of $m \sim N(190, 15^2)$ kg, what are the probabilities of failure of the two rods? Assume S_A and *m* are independent.



Solution

Free Body Diagram



Determine the forces in the two rods in terms of *m*.

$$\Sigma F_x = F_{BC}\cos 45^\circ - F_{AB}\sin 30^\circ = 0$$
$$\Sigma F_Y = F_{BC}\sin 45^\circ + F_{AB}\cos 30^\circ - mg = 0$$
$$\Rightarrow F_{AB} = 7.181m \quad F_{BC} = 8.795m$$

Determine the area of each rod

$$A_{AB} = \frac{\pi}{4} d_{AB}^2 = \frac{\pi}{4}(0.004)^2 = 1.26\times10^{-5} \text{ m}^2$$

$$A_{BC} = \frac{\pi}{4} d_{BC}^2 = \frac{\pi}{4}(0.006)^2 = 2.83\times10^{-5} \text{ m}^2$$

Determine the equation for normal stress in each rod

$$S_{AB} = \frac{F_{AB}}{A_{AB}} = \frac{7.181m}{A_{AB}} \qquad S_{BC} = \frac{F_{BC}}{A_{BC}} = \frac{8.795m}{A_{BC}}$$

The probability of failure in a single rod is $p_f\left(S_A < S_{rod}\right)$. Let $Y_{rod} = S_A - S_{rod}$, then

$$Y_{AB} = S_A - S_{AB} = S_A - \frac{7.181m}{A_{AB}}$$

$$Y_{BC} = S_A - S_{BC} = S_A - \frac{8.795m}{A_{BC}}$$

Then, p_f for a single rod is written as $p_{f,rod} = \left(Y_{rod} < 0\right)$. Because normal random variables S_A and m are independent, Y_{rod} also follows a normal distribution, $Y_{rod} \sim N\left(\mu_{Y,rod}, \sigma^2_{Y,rod}\right)$.

$$\mu_{Y,AB} = \mu_{S_A} - \frac{7.181\mu_m}{A_{AB}} = 135\left(10^6\right) - \frac{7.181(190)}{1.257\left(10^{-5}\right)} = 26.4 \text{ MPa}$$

$$\sigma_{Y,AB} = \sqrt{\sigma^2_{S_A} + \left(\frac{7.181}{A_{AB}}\right)^2 \sigma^2_m} = \sqrt{\left(15\left(10^6\right)\right)^2 + \left(\frac{7.181}{1.257\left(10^{-5}\right)}\right)(15)^2} = 17.3 \text{ MPa}$$

$$\mu_{Y,BC} = \mu_{S_A} - \frac{8.795\mu_m}{A_{BC}} = 135\left(10^6\right) - \frac{8.795(190)}{2.827\left(10^{-5}\right)} = 75.9 \text{ MPa}$$

$$\sigma_{Y,BC} = \sqrt{\sigma^2_{S_A} + \left(\frac{8.795}{A_{BC}}\right)^2 \sigma^2_m} = \sqrt{\left(15\left(10^6\right)\right)^2 + \left(\frac{8.795}{2.827\left(10^{-5}\right)}\right)^2 (15)^2} = 15.7 \text{ MPa}$$

Thus

$$p_{f,AB} = \Pr\left(Y_{AB} < 0\right) = \Pr\left(\frac{Y_{AB} - \mu_{Y,AB}}{\sigma_{Y,AB}} < \frac{-\mu_{Y,AB}}{\sigma_{Y,AB}}\right) = \Phi\left(-\frac{\mu_{Y,AB}}{\sigma_{Y,AB}}\right) = 1 - \Phi\left(\frac{\mu_{Y,AB}}{\sigma_{Y,AB}}\right) = 1 - \Phi\left(\frac{26.4}{17.3}\right)$$

$$= 1 - \Phi(1.53) = 1 - 0.93650 = 6.3 \times 10^{-2}$$ **Ans.**

$$p_{f,\mathrm{BC}} = \Pr\left(Y_{BC} < 0\right) = \Pr\left(\frac{Y_{BC} - \mu_{Y,\mathrm{BC}}}{\sigma_{Y,BC}} < \frac{-\mu_{Y,\mathrm{BC}}}{\sigma_{Y,\mathrm{BC}}}\right) = \Phi\left(-\frac{\mu_{Y,\mathrm{BC}}}{\sigma_{Y,\mathrm{BC}}}\right) = 1 - \Phi\left(\frac{\mu_{Y,\mathrm{BC}}}{\sigma_{Y,\mathrm{BC}}}\right) = 1 - \Phi\left(\frac{75.9}{15.7}\right)$$

$$= 1 - \Phi(4.83) = 1 - 0.99997 = 6.8 \times 10^{-7}$$ **Ans.**