1-16. A rod has a length of L = 250 mm and a diameter of d = 17 mm. And axial load $P \sim N(315, 10^2)$ N is applied to the rod. Determine the distributions of the change in length and the change in diameter given E = 2.65 GPa and v = 0.3.

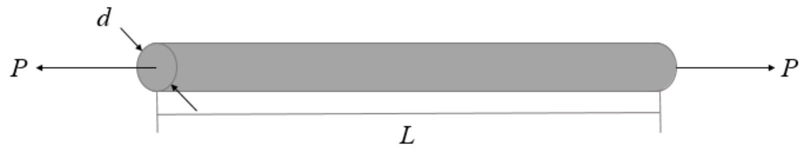

Fig. 1.16

Solution

Find the area of the rod

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4} (0.017)^2 = 2.27 \times 10^{-4} \text{ m}^2$$

Find an equation for the change in length

$$S = \frac{P}{A}$$

$$\varepsilon_{long} = \frac{S}{E} = \frac{P}{AE}$$

$$\varepsilon_{long} = \frac{\Delta L}{L} = \frac{P}{AE}$$

$$\Rightarrow \Delta L = \frac{PL}{AE}$$

$$\mu_{\Delta L} = \frac{\mu_P L}{AE} = \frac{(315)(0.25)}{(2.27 \times 10^{-4})(2.65)} = 0.131 \text{ mm}$$

$$\sigma_{\Delta L} = \sqrt{\left(\frac{L}{AE}\right)^2 \sigma_P^2} = \sqrt{\left(\frac{0.25}{(2.27 \times 10^{-4})(2.65)(10^9)}\right)(10)^2} = 0.00416 \text{ mm}$$

Then

$$\Delta L \sim N\left(0.131, (0.000416)^2\right) \text{ mm}$$ **Ans.**

Find an equation for the change in the diameter

$$\varepsilon_{lat} = -v\varepsilon_{long} = -\frac{vP}{AE}$$

$$\varepsilon_{lat} = \frac{\Delta d}{d} = -\frac{vP}{AE}$$

$$\Rightarrow \Delta d = -\frac{vPd}{AE}$$

$$\mu_{\Delta d} = -\frac{v\mu_P d}{AE} = -\frac{(0.3)(315)(0.017)}{(2.27\times 10^{-4})(2.65)(10^9)} = -0.00267 \text{ mm}$$

$$\sigma_{\Delta d} = \sqrt{\left(\frac{vd}{AE}\right)^2 \sigma_P^2} = \sqrt{\left(\frac{(0.3)(0.017)}{(2.27\times 10^{-4})(2.65)(10^9)}\right)^2 (10)^2} = 8.48\times 10^{-5} \text{ mm}$$

Then

$$\Delta d \sim N\left(-0.00267, \left(8.48\times 10^{-5}\right)^2\right) \text{ mm}$$ **Ans.**