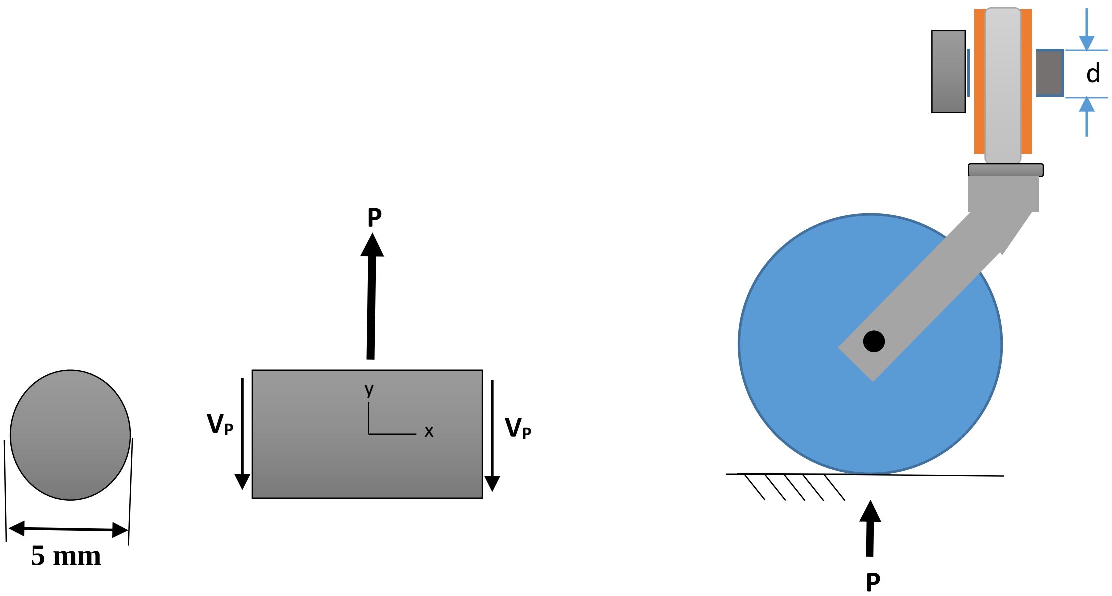
1-17. The supporting wheel on a hospital bed is connected to the leg with a 5-mm-diameter pin as shown. If the wheel is subjected to a normal force $P \sim N(2500, 200^2)$ N, determine the probability of failure of the pin which has an allowable shear stress $S_a \sim N\left(150, 20^2\right)$ MPa. Neglect friction between the inner bed leg and the wheel shaft, and assume P and S_a are independent.



Solution

Internal loading: The shear force developed on the two shear planes of the pin can be determined by the force equilibrium equation.

$$+\uparrow \sum F_y = 0; \qquad P - 2V_P = 0; \qquad V_P = P/2;$$

Average shear stress: The area of the bolt $A_P = \left(\frac{\pi}{4}\right)\left(d^2\right) = \left(\frac{\pi}{4}\right)\left(5^2\right) = 19.635\,\text{mm}^2$.

We obtain

$$\left(\tau_{avg}\right)_P = \frac{V_P}{A_P} = \left(\frac{P/2}{19.635}\right) = 0.0255P;$$

Probability of failure:

$$p_f = \Pr(S_{\tau_P} > S_a) = \Pr(Y = S_a - S_{\tau_P} < 0) = \Pr\left(Y = S_a - \tau_P < 0\right) = \Pr\left(Y = S_a - 0.0255P < 0\right) \qquad \textbf{(1)}$$

Since $P \sim N\left(2500, 200^2\right)$ N and $S_a \sim N\left(150, 20^2\right)$ MPa are independent, Y also follows a normal distribution. $Y \sim N(\mu_Y, \sigma_Y^2)$.

$$\mu_Y = \mu_{S_a} - \left(0.0255\right)\mu_P = 150 - \left(0.0255\right)2500 = 86.25 \text{ MPa} \quad \textbf{(2)}$$

$$\sigma_Y = \sqrt{\sigma_{S_a}^2 + \left(0.0255\right)^2 \sigma_P^2} = \sqrt{20^2 + \left(0.0255\right)^2 \left(200\right)^2} = 20.64 \text{ MPa} \quad \textbf{(3)}$$

Equation (1) can be written as

$$p_f = \Pr\left(Y < S_a\right) = \Pr\left(\frac{Y - \mu_Y}{\sigma_Y} < \frac{-\mu_Y}{\sigma_Y}\right) = \Phi\left(\frac{-\mu_Y}{\sigma_Y}\right) = \Phi\left(-4.18\right) = 1.4654\left(10^{-5}\right) \quad \textbf{Ans.}$$