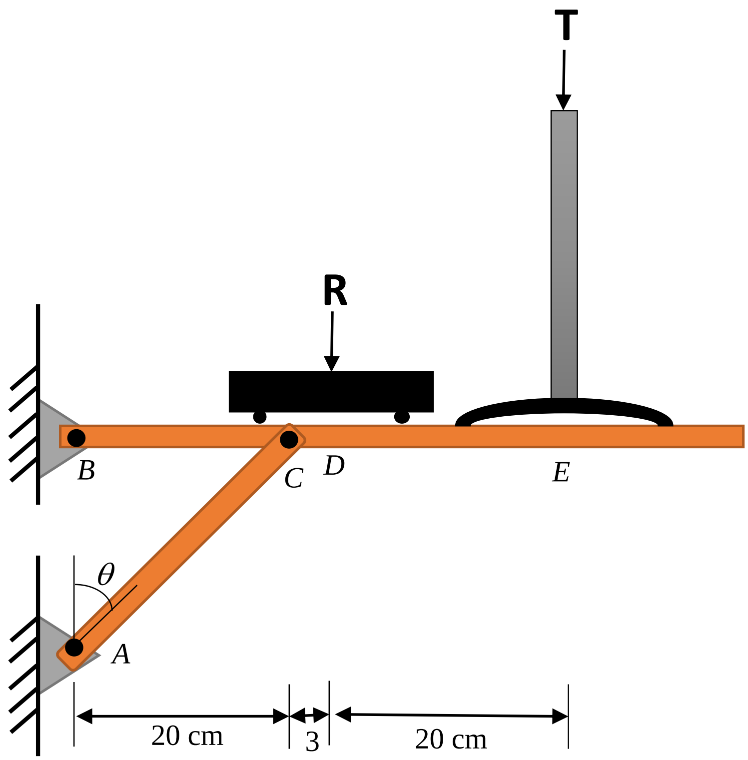
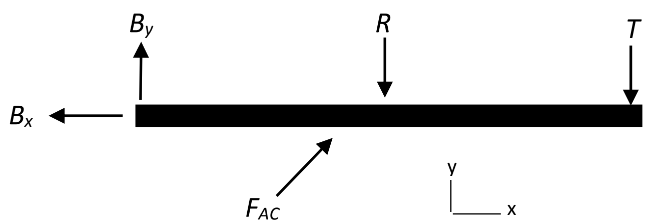
1-18. The wall mounted television stand is supported by an arm which has a cross sectional area 625 mm^2. There are two normally distributed loads applied to the stand as shown, a television $T \sim N\left(50,5^2\right)$ N and a receiver $R \sim N\left(20,3^2\right)$ N. Determine the probability of yielding failure of the arm AC if the allowable yield strength is $S_y \sim N\left(2,0.5^2\right)$ MPa. Assume P, F, and S_y are independent and $\theta = 45^o$.





Solution

Sum the forces: The force in *AC* can be found by summing the forces in the *x* and *y* directions then take the moment about point *B*.

$$+\rightarrow \sum F_x = 0; \qquad -B_x + F_{AC} \cos 45 = 0;$$

$$+\uparrow \sum F_y = 0; \qquad B_y + F_{AC}\sin 45 - R - T = 0;$$

$$+\nwarrow \sum M_B = 0; \quad F_{AC}\sin 45(0.2) - R(0.23) - T(0.43) = 0;$$

$$B_x = (1.15)R + (2.15)T; \qquad B_y = (-0.15)R + (-1.15)T; \quad F_{AC} = (1.626)R + (3.041)T$$

For the probability of failure in *AC*:

$$p_f = \Pr(S_{AC} > S_y) = \Pr(Y = S_y - S_{AC} < 0) = \Pr\left(Y = S_y - \frac{F_{AC}}{A} < 0\right)$$

$$= \Pr\left(Y = S_y - \frac{(1.626)R + (3.041)T}{A} < 0\right) \qquad \textbf{(1)}$$

Since $R \sim N(20, 3^2)$ N, $T \sim N(50, 5^2)$ N, and $S_y \sim N(2, 0.5^2)$ MPa are independent, Y also follows a normal distribution. $Y \sim N(\mu_y, \sigma_y^2)$.

$$\mu_y = \mu_{s_y} - \left(\frac{1.626}{A}\right)\mu_R + \left(\frac{3.041}{A}\right)\mu_T = 2 - \left(\frac{1.626}{625}\right)20 + \left(\frac{3.041}{625}\right)50 = 2.19125 \text{ MPa} \qquad \textbf{(2)}$$

$$\sigma_y = \sqrt{\sigma_{S_y}^2 + \left(\frac{1.626}{A}\right)^2 \mu_R{}^2 + \left(\frac{3.041}{A}\right)^2 \mu_T{}^2}$$

$$= \sqrt{0.5^2 + \left(\frac{1.626}{625}\right)^2 3^2 + \left(\frac{3.041}{625}\right)^2 5^2} = 0.50065 \text{ MPa} \qquad \textbf{(3)}$$

Equation (1) can be written as

$$p_f = \Pr(Y < 0) = \Pr\left(\frac{Y - \mu_y}{\sigma_y} < \frac{-\mu_y}{\sigma_y}\right) = \Phi\left(\frac{-\mu_y}{\sigma_y}\right) = \Phi(-4.37681) = 6.2361(10^{-5}) \qquad \textbf{Ans.}$$