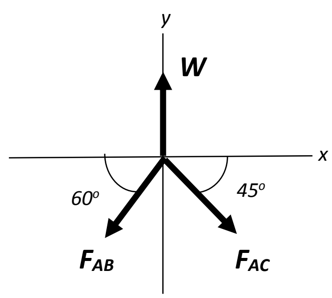
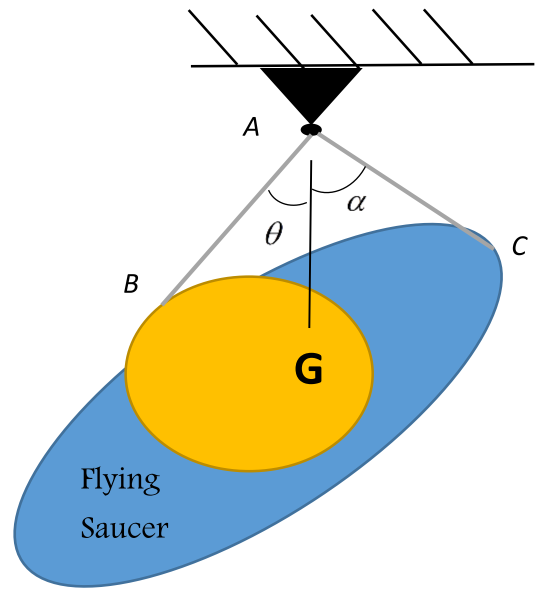
1-19. The Flying Saucer café sign has a center of mass at G which has a normally distributed load $W \sim N(120, 20^2)$ N. It is suspended from cables AB and AC as shown. If the allowable yield stress in both cables is $S_y \sim N(20, 2.5^2)$ MPa, what is the required minimum diameter of the cables so that the probability of failure is less than 10^{-3}? Assume W and S_y are independent. $\theta = 30^o, \alpha = 45^o$.





Solution

Sum the forces: The forces in AB and AC can be found by summing the forces in the x and y directions.

$$+\rightarrow \sum F_x = 0; \qquad -F_{AB}\cos 60 + F_{AC}\cos 45 = 0;$$

$$+\uparrow \sum F_y = 0; \qquad W - F_{AB}\sin 60 - F_{AC}\sin 45 = 0;$$

$$F_{AB} = (0.7321)W; \qquad F_{AC} = (0.51764)W;$$

We choose the probability of failure of AB due to the fact that it carries more of the weight:

$$p_f = \Pr(S_{AB} > S_y) = \Pr(Y = S_y - S_{AB} < 0) = \Pr\left(Y = S_y - \frac{F_{AB}}{A} < 0\right)$$

$$= \Pr\left(Y = S_y - \frac{(0.7321)W}{A} < 0\right) \qquad \textbf{(1)}$$

Since $W \sim N(120, 20^2)$ N, and $S_y \sim N(20, 2.5^2)$ MPa are independent, Y also follows a normal distribution. $Y \sim N(\mu_y, \sigma_y^2)$.

$$\mu_y = \mu_{s_y} - \left(\frac{0.7321}{A}\right)\mu_W = 20 - \left(\frac{0.7321}{A}\right)120 = 20 - \frac{87.852}{\left(\frac{\pi d^2}{4}\right)} \text{ MPa} \tag{2}$$

$$\sigma_y = \sqrt{\sigma_{S_y}^2 + \left(\frac{0.7321}{A}\right)^2 \sigma_W^2} = \sqrt{2.5^2 + \left(\frac{0.7321}{A}\right)^2 (20)^2} = \sqrt{6.25 + \left(\frac{214.388}{\left(\frac{\pi^2 d^4}{16}\right)}\right)} \text{ MPa} \tag{3}$$

Equation (1) can be written as

$$p_f = \Pr(Y < 0) = \Pr\left(\frac{Y - \mu_y}{\sigma_y} < \frac{-\mu_y}{\sigma_y}\right) = \Phi\left(\frac{-\mu_y}{\sigma_y}\right) = \Phi(-3.09) = 10^{-3}$$

From (1), (2), (3), and some algebra we obtain

$$\frac{-\left(20 - \frac{87.852}{\left(\frac{\pi d^2}{4}\right)}\right)}{\sqrt{6.25 + \left(\frac{214.388}{\left(\frac{\pi^2 d^4}{16}\right)}\right)}} > -3.09 \Rightarrow d > 2.61156 \text{ mm}^2 \qquad \textbf{Ans.}$$