1-2. The two aluminum rods support the vertical force of $P \sim N(30, 4^2)$ kN. The allowable tensile stress for the aluminum is $S_a \sim N(150, 20^2)$ MPa. Determine the diameters of the two rods to make sure that the probabilities of failure of them both are less than 10^{-4}. Assume that P and S_a are independent.

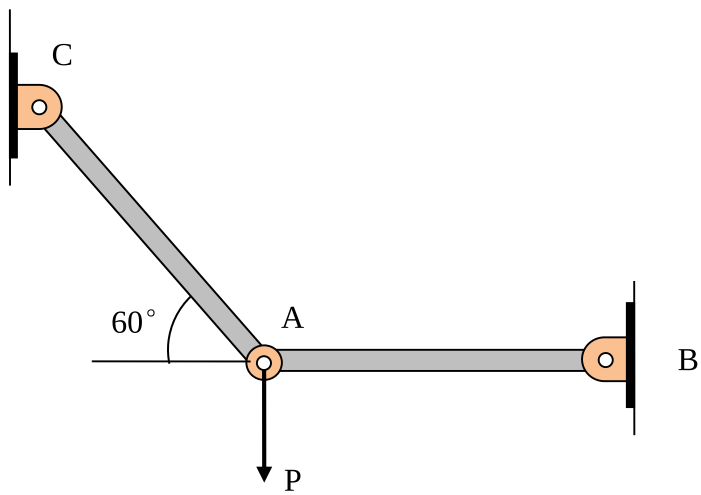

Fig. 1.2.1

Solution

Internal Loadings: The force developed in each rod can be determined by using the method of joints. Consider the equilibrium of joint A as shown in Fig. 1.2.2,

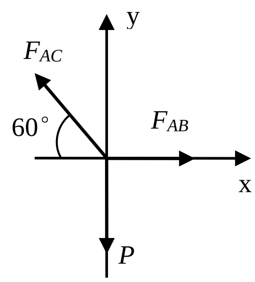

Fig. 1.2.2

$$+\uparrow \sum F_y = 0; \quad F_{AC} \cdot \sin 60^\circ - P = 0; \qquad F_{AC} = 1.155P$$

$$\underset{\rightarrow}{+} \sum F_x = 0; \quad -F_{AC} \cdot \cos 60^\circ + F_{AB} = 0; \quad F_{AB} = 0.5F_{AC} = 0.577P$$

For rod AB, the probability of failure p_f is

$$p_f = \Pr(S_{AB} > S_a) = \Pr(Y = S_a - S_{AB} < 0) = \Pr\left(Y = S_a - \frac{F_{AB}}{A} < 0\right) = \Pr\left(Y = S_a - \frac{0.577P}{A} < 0\right) \quad (1)$$

Since $P \sim N\,(30, 4^2)\,\text{kN}$, $S_a \sim N\,(150, 20^2)\,\text{MPa}$, and P and S_a are independent, Y also follows a normal distribution, $Y \sim N(\mu_Y, \sigma_Y^2)$.

$$\mu_Y = \mu_{S_a} - \left(\frac{0.577}{A}\right)\mu_P = 150 - \left(\frac{0.577}{\frac{\pi d_{AB}^2}{4}}\right)3\times10^4 \quad (2)$$

$$\sigma_Y = \sqrt{\sigma_{S_a}^2 + \left(\frac{0.577}{A}\right)^2 \sigma_P^2} = \sqrt{20^2 + \left(\frac{0.577}{\frac{\pi d_{AB}^2}{4}}\right)^2 (4)^2} \quad (3)$$

Equation (1) can be written as

$$p_f = \Pr\left(Y < 0\right) = \Pr\left(\frac{Y-\mu_Y}{\sigma_Y} < \frac{-\mu_Y}{\sigma_Y}\right) = \Phi\left(\frac{-\mu_Y}{\sigma_Y}\right) < 10^{-4} = \Phi\left(-3.719\right)$$

Therefore, $\frac{-\mu_Y}{\sigma_Y} < -3.719$

(4)

From (1), (2) and (3), we can get $d_{AB} > 8.33\,\text{mm}$.

Ans.

Similarly, For rod AC, the probability of failure p_f is

$$p_f = \Pr\,(S_{AC} > S_a) = \Pr\,(Y = S_a - S_{AC} < 0) = \Pr\left(Y = S_a - \frac{F_{AC}}{A} < 0\right) = \Pr\left(Y = S_a - \frac{1.155P}{A} < 0\right) \quad (5)$$

we can get $d_{AC} > 11.786\,\text{mm}$.

Ans.