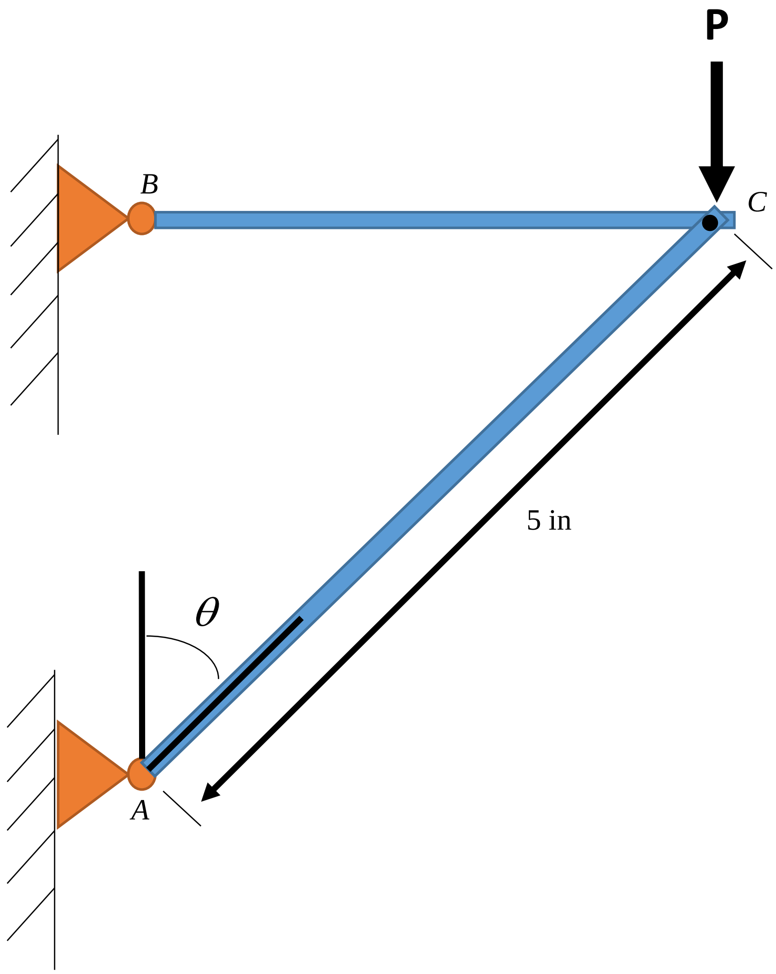
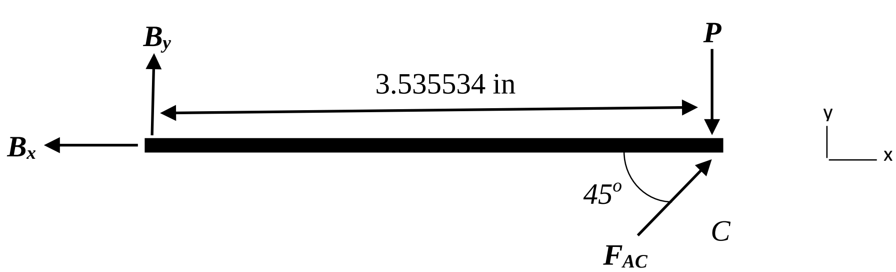
1-20. The system supports a normally distributed load $P \sim N\left(6000, 750^2\right)$ lb. Determine the minimum diameter of bar BC if the allowable yield strength is $S_y \sim N\left(2000, 250^2\right)$ psi. Let the maximum probability of failure for of bar BC be 10^{-4}. Assume P and S_y are independent variables. $\theta = 45^o$.





Solution

Sum the forces: The force in AC can be found by taking the moment about B or by first taking the moment about C then sum the forces in the y direction.

$$+\nwarrow \sum M_B = 0; \qquad F_{AC}\sin 45(3.535534) - P(3.535534) = 0;$$

Alternative solution:

$$+\nwarrow \sum M_C = 0; \qquad -B_y(3.535534) = 0; \qquad B_y = 0;$$

$$+\uparrow \sum F_y = 0; \qquad F_{AC}\sin 45 - P = 0;$$

$$F_{AC} = \frac{P}{\sin 45};$$

For the probability of failure in AC:

$$p_f = \Pr(S_{AC} > S_n) = \Pr(Y = S_n - S_{AC} < 0) = \Pr\left(Y = S_n - \frac{F_{AC}}{A} < 0\right)$$

$$= \Pr\left(Y = S_y - \frac{\left(\frac{P}{\sin 45}\right)}{A} < 0\right) \qquad \textbf{(1)}$$

Since $P \sim N(6000, 750^2)$ lb, and $S_y \sim N(2000, 250^2)$ psi are independent, Y also follows a normal distribution. $Y \sim N(\mu_y, \sigma_y^2)$.

$$\mu_y = \mu_{S_y} - \left(\frac{1}{A\sin 45}\right)\mu_P = 2000 - \left(\frac{1}{A\sin 45}\right)6000 = 2000 - \frac{8485.2814}{A} \text{ psi} \qquad \textbf{(2)}$$

$$\sigma_y = \sqrt{\sigma_{S_y}^2 + \left(\frac{1}{A\sin 45}\right)^2 \sigma_P^2} = \sqrt{250^2 + \left(\frac{1}{A\sin 45}\right)^2 (750)^2} = \sqrt{62500 + \left(\frac{1125000}{A^2}\right)} \text{ psi} \qquad \textbf{(3)}$$

Equation (1) can be written as

$$p_f = \Pr(Y < 0) = \Pr\left(\frac{Y - \mu_y}{\sigma_y} < \frac{-\mu_y}{\sigma_y}\right) = \Phi\left(\frac{-\mu_y}{\sigma_y}\right) = \Phi(-3.719) = 10^{-4}$$

From (1), (2), (3), and some algebra we obtain

$$\frac{-\left(2000-\frac{8485.2814}{A}\right)}{\sqrt{62500+\left(\frac{1125000}{A^2}\right)}}=-3.719\Rightarrow A=5.2844=\frac{\pi d^2}{4}\Rightarrow d=2.594\ \text{in}$$ **Ans.**