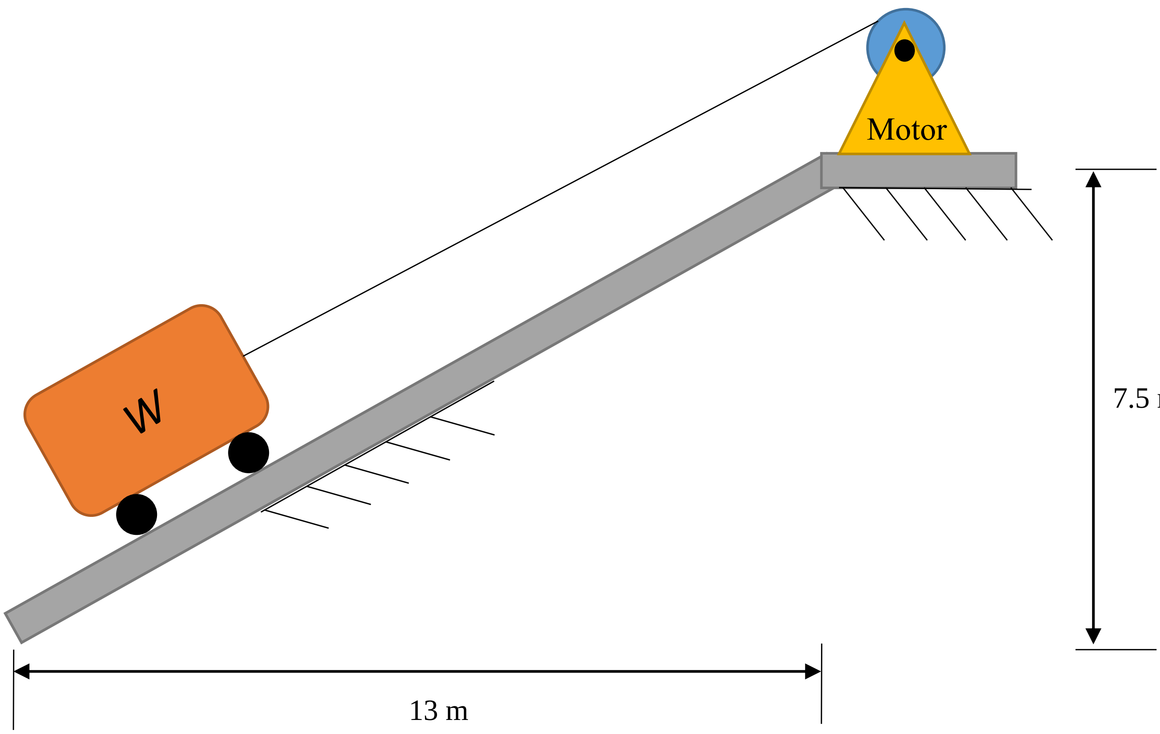
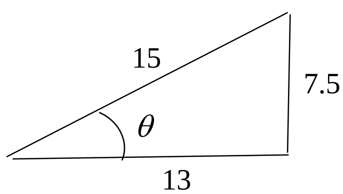
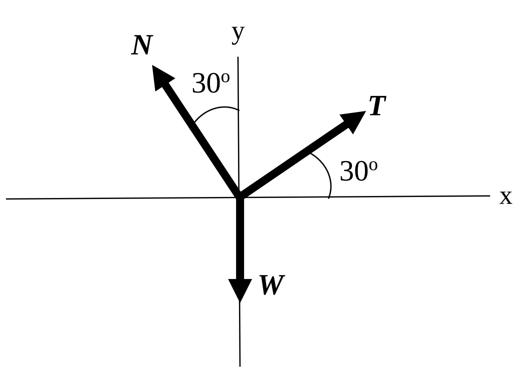
1-21. A normally distributed Weight $W \sim N(120, 15^2)$ kN is being pulled up an incline by a ramp and pulley system as shown. If the yield stress of the cable also follows a normal distribution $S_y \sim N(155, 17^2)$ MPa, and the cable has a diameter of 25 mm, what is the probability that the cable will break? Assume W and S_y are independent.



Solution





$\theta \approx 30^o$;

Sum the forces: The forces *N* and *T* can be found by summing the forces in the *x* and *y* directions.

$$+\rightarrow\sum F_x = 0; \qquad -N\cos 60 + T\cos 30 = 0;$$

$$+\uparrow\sum F_y = 0; \qquad -W + N\sin 60 + T\sin 30 = 0;$$

$$N = (0.866025)W; \qquad T = (0.5)W;$$

Since T represents the force applied to the cable, we use it to show:

$$p_f = \Pr(S_{cable} > S_y) = \Pr(Y = S_y - S_{cable} < 0) = \Pr\left(Y = S_y - \frac{T}{A} < 0\right) = \Pr\left(Y = S_y - \frac{(0.5)W}{A} < 0\right) \qquad \textbf{(1)}$$

Since $W \sim N(120,15^2)$ kN, and $S_y \sim N(155,17^2)$ MPa are independent, Y also follows a normal distribution. $Y \sim N(\mu_y, \sigma_y^2)$. Also, $A = \frac{\pi d^2}{4} = \frac{\pi(25^2)}{4} = 490.874$ mm^2.

$$\mu_y = \mu_{s_y} - \left(\frac{0.5}{A}\right)\mu_W = 155 - \left(\frac{0.5}{490.874}\right)120000 = 32.769 \text{ MPa} \qquad \textbf{(2)}$$

$$\sigma_y = \sqrt{\sigma_{S_y}^2 + \left(\frac{0.5}{A}\right)^2 \sigma_W^2} = \sqrt{17^2 + \left(\frac{0.5}{490.874}\right)^2 (15000)^2} = 22.857 \text{ MPa} \qquad \textbf{(3)}$$

Equation (1) can be written as

$$p_f = \Pr(Y < 0) = \Pr\left(\frac{Y - \mu_y}{\sigma_y} < \frac{-\mu_y}{\sigma_y}\right) = \Phi\left(\frac{-\mu_y}{\sigma_y}\right) = \Phi(-1.43365) = 0.075836 \qquad \textbf{Ans.}$$