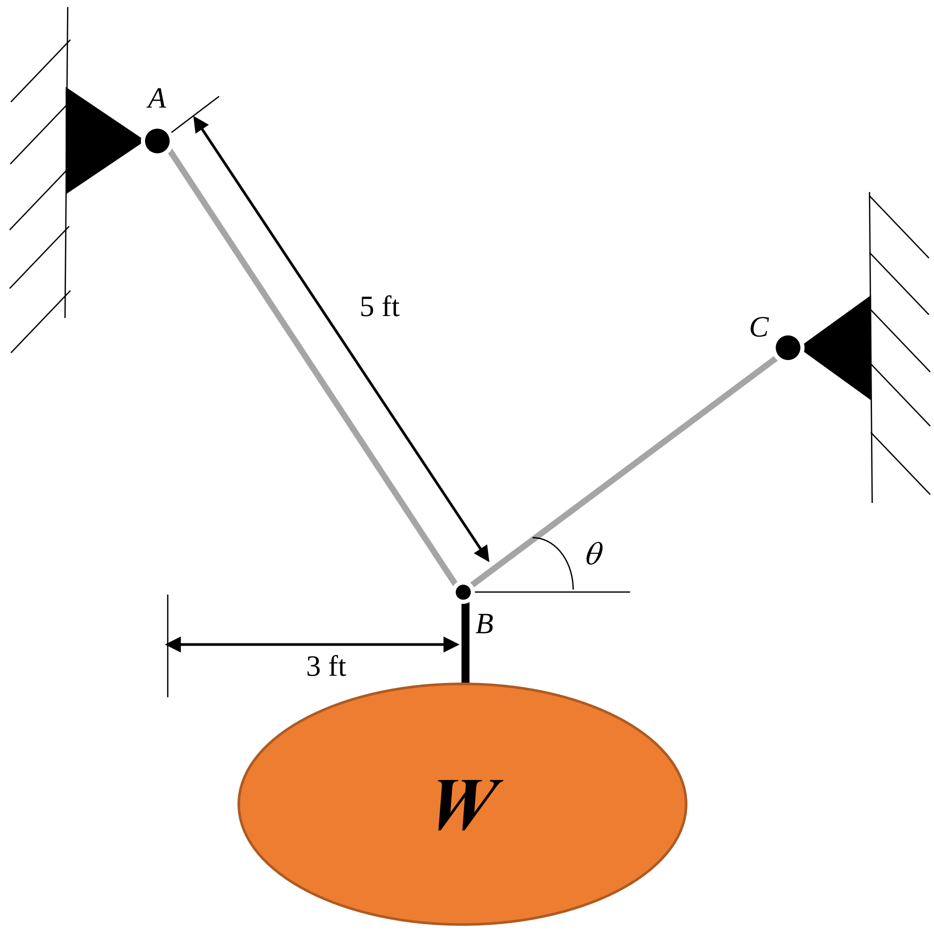
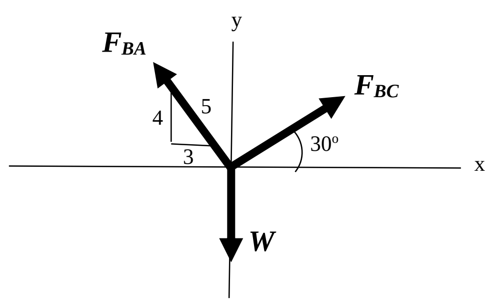
1-22. A normally distributed weight $W \sim N\left(350, 40^2\right)$ lb is supported by two cables as shown. The yield stress of cables is $S_y \sim N\left(2800, 300^2\right)$ psi and they have a diameter of 0.5 inches. What is the probability that the weight will fall? Assume W and S_y are independent variables and $\theta = 30^o$.



Solution:



Sum the forces: The forces F_{BA} and F_{BC} can be found by summing the forces in the x and y directions.

$$+\rightarrow \sum F_x = 0; \qquad -F_{BA}\left(\frac{3}{5}\right) + F_{BC}\cos 30 = 0;$$

$$+\uparrow \sum F_y = 0; \qquad F_{BA}\left(\frac{4}{5}\right) + F_{BC}\sin 30 - W = 0;$$

$F_{BA} = (0.872288)W; \quad F_{BC} = (0.604339)W;$

Since *W* produces a larger force in the cable *AB*, we use it to show:

$$p_f = \Pr(S_{cable} > S_y) = \Pr(Y = S_y - S_{cable} < 0) = \Pr\left(Y = S_y - \frac{F_{BA}}{A} < 0\right) = \Pr\left(Y = S_y - \frac{(0.872288)W}{A} < 0\right)$$

Since $W \sim N\left(350, 40^2\right)$ lb, and $S_y \sim N\left(2800, 300^2\right)$ psi are independent, Y also follows a normal distribution. $Y \sim N(\mu_y, \sigma_y^2)$. Also, $A = \frac{\pi d^2}{4} = \frac{\pi\left(0.5^2\right)}{4} = 0.19635$ in^2.

$$\mu_y = \mu_{s_y} - \left(\frac{0.872288}{A}\right)\mu_W = 2800 - \left(\frac{0.872288}{0.19635}\right)350 = 1245.1158 \text{ psi}$$

$$\sigma_y = \sqrt{\sigma_{S_y}^2 + \left(\frac{0.872288}{A}\right)^2 \sigma_W^2} = \sqrt{300^2 + \left(\frac{0.872288}{0.19635}\right)^2 (40)^2} = 348.68 \text{ psi}$$

Equation (1) can be written as

$$p_f = \Pr\left(Y < 0\right) = \Pr\left(\frac{Y - \mu_y}{\sigma_y} < \frac{-\mu_y}{\sigma_y}\right) = \Phi\left(\frac{-\mu_y}{\sigma_y}\right) = \Phi\left(-3.57094\right) = 1.7785\left(10^{-4}\right) \qquad \textbf{Ans.}$$