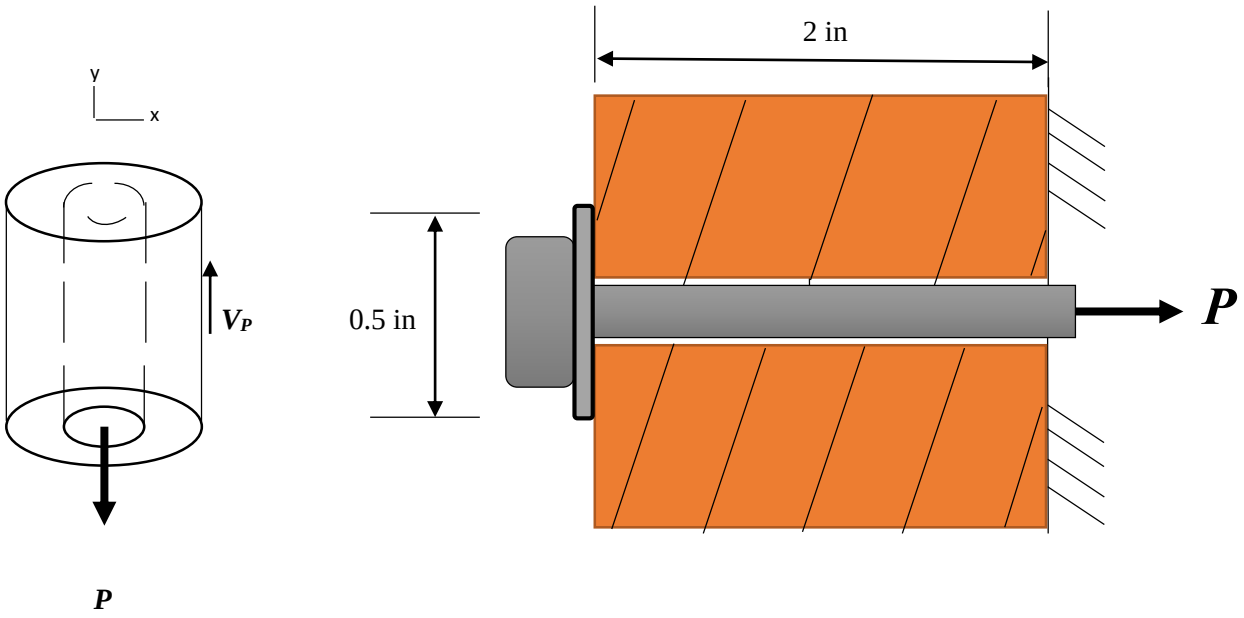
1-23. A bolt passes through a board that is 2 inches thick and the board has a normally distributed allowable shear stress $S_a \sim N\left(250, 30^2\right)$ psi. If the washer has an outer diameter of 0.5 inches and a normally distributed load is applied $P \sim N\left(400, 44^2\right)$ lb, what is the probability that the board will fail and the bolt head and washer will rip through the board? Assume S_a and P are independent variables.



Solution

Internal loading: The shear force developed in the board due to the force P on the washer is determined by summing the forces in the *y* direction.

$$+\uparrow \sum F_y = 0; \qquad -P + V_P = 0; \qquad V_P = P;$$

Average shear stress: The relevant area of the board is $A_B = 2\pi d = \pi$ in^2.
We obtain

$$\left(\tau_{avg}\right)_P = \frac{V_P}{A_P} = \left(\frac{P}{\pi}\right) = (0.31831)P;$$

Probability of failure:

$$p_f = \Pr(S_{\tau_P} > S_a) = \Pr(Y = S_a - S_{\tau_P} < 0) = \Pr\left(Y = S_a - \tau_P < 0\right)$$

$$= \Pr\left(Y = S_a - (0.31831)P < 0\right)$$

Since $P \sim N\left(400, 44^2\right)$ lb and $S_a \sim N\left(250, 30^2\right)$ psi are independent, Y also follows a normal distribution. $Y \sim N(\mu_Y, \sigma_Y^2)$.

$$\mu_Y = \mu_{S_a} - \left(0.31831\right)\mu_P = 250 - \left(0.31831\right)400 = 122.676 \text{ psi}$$

$$\sigma_Y = \sqrt{\sigma_{S_a}^2 + \left(0.31831\right)^2 \sigma_P^2} = \sqrt{30^2 + \left(0.31831\right)^2 \left(44\right)^2} = 33.1083 \text{ psi}$$

Equation (1) can be written as

$$p_f = \Pr\left(Y < S_a\right) = \Pr\left(\frac{Y - \mu_Y}{\sigma_Y} < \frac{-\mu_Y}{\sigma_Y}\right) = \Phi\left(\frac{-\mu_Y}{\sigma_Y}\right) = \Phi\left(-3.7053\right) = 1.0557\left(10^{-4}\right) \qquad \textbf{Ans.}$$