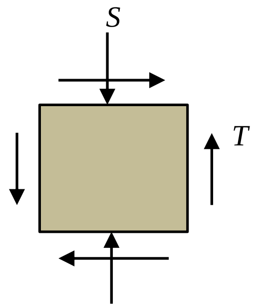
1-24. An element is subject to a normal stress $S \sim N(260, 20^2)$ psi and a shear stress $T \sim N(900, 70^2)$ psi as shown in the figure. If the element is oriented 45° clockwise from its current position, determine the distribution of the equivalent state of stress. Assume S and T are independent.



Solution:

According to the stress diagram, we have

$\sigma_x = 0$, $\sigma_y = -S$, $\tau_{xy} = T$, $\theta = -45^\circ$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta = \frac{-S}{2} + \frac{S}{2}\cos\left(-90^\circ\right) + T\sin\left(-90^\circ\right) = \frac{-S}{2} - T$$

$$\mu_{\sigma_{x'}} = \frac{-\mu_S}{2} - \mu_T = \frac{-260}{2} - 900 = -1030 \text{ psi}$$

$$\sigma_{\sigma_{x'}} = \sqrt{\left(\frac{-\sigma_S}{2}\right)^2 + \left(\sigma_T\right)^2} = \sqrt{\left(\frac{-20}{2}\right)^2 + \left(70\right)^2} = 70.7 \text{ psi}$$

Thus, $\sigma_{x'}$ follows $\sigma_{x'} \sim N(-1030, 70.7^2)$ psi .

$$\tau_{x'y'} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta = \frac{-S}{2}\sin\left(-90^\circ\right) + T\cos\left(-90^\circ\right) = \frac{S}{2}$$

$$\mu_{\tau_{x'y'}} = \frac{\mu_S}{2} = 130 \text{ psi}, \quad \sigma_{\tau_{x'y'}} = \sqrt{\left(\frac{\sigma_S}{2}\right)^2} = \frac{\sigma_S}{2} = 10 \text{ psi}$$

Thus, $\tau_{x'y'}$ follows $\tau_{x'y'} \sim N(130, 10^2)$ psi.

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta = \frac{-S}{2} - \frac{S}{2}\cos\left(-90^\circ\right) - T\sin\left(-90^\circ\right) = \frac{-S}{2} + T$$

$$\mu_{\sigma_{y'}} = \frac{-\mu_S}{2} + \mu_T = \frac{-260}{2} + 900 = -770 \text{ psi}$$

$$\sigma_{\sigma_{y'}} = \sqrt{\left(\frac{-\sigma_S}{2}\right)^2 + \left(\sigma_T\right)^2} = \sqrt{\left(\frac{-20}{2}\right)^2 + \left(70\right)^2} = 70.7 \text{ psi}$$

Thus, $\sigma_{y'}$ follows $\sigma_{y'} \sim N(-770, 70.7^2)$ psi. **Ans.**