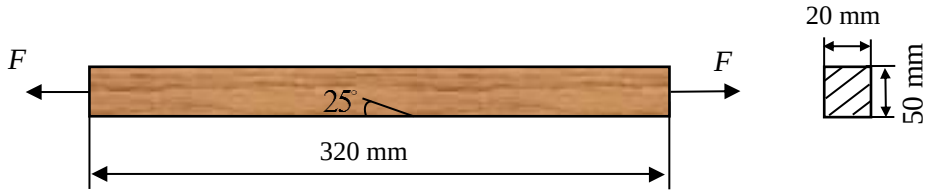
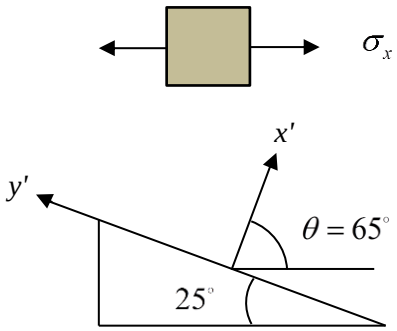
1-25. A wooden board is subject to a load $F \sim N(200, 12^2)$ N. The grains of the wood have an angle of 25° as shown in the figure. Determine the distributions of the normal and shear stress that act perpendicular and parallel to the grains.



Solution:



$$\sigma_x = \frac{F}{A} = \frac{F}{0.02(0.05)} = 1\times 10^3 F\,,\ \ \sigma_y = 0\,,\ \ \tau_{xy} = 0\,,\ \ \theta = 65^\circ$$

In the new coordinates system, we have

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta = \frac{\sigma_x}{2} + \frac{\sigma_x}{2}\cos\left(130^\circ\right) + \tau_{xy}\sin\left(130^\circ\right) = 0.179\sigma_x = 179F$$

$$\mu_{\sigma_{x'}} = 179\mu_F = 179(200) = 35.8\,\text{kPa}$$

$$\sigma_{\sigma_{x'}} = 179\sigma_F = 179(12) = 2.15\,\text{kPa}$$

Thus, $\sigma_{x'}$ follows $\sigma_{x'} \sim N(35.8, 2.15^2)$ kPa.

$$\tau_{x'y'} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta = \frac{-\sigma_x}{2}\sin\left(130\right) + \tau_{xy}\cos\left(130^\circ\right) = -0.383\sigma_x = -383F$$

$$\mu_{\sigma_{x'}} = -383\mu_F = -383(200) = -76.6\,\text{kPa}$$

$\sigma_{\sigma_{x'}} = -383\sigma_F = -383(12) = -4.6\,\text{kPa}$

Thus, $\tau_{x'y'}$ follows $\tau_{x'y'} \sim N(-76.6, -4.6^2)\,\text{kPa}$. **Ans.**