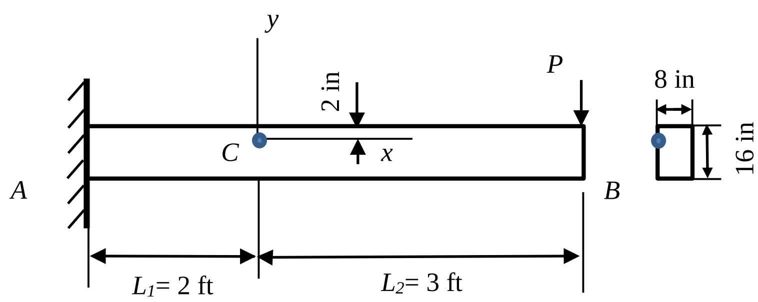
1-26. A cantilever is subject to a vertical random force $P \sim N(16, 1^2)$ kips as shown in the figure. If the allowable shear strain at point C is $\gamma_a = 9 \times 10^{-6}$, determine the probability of failure. Given that the cantilever has a Shear modulus of $G = 11 \times 10^3$ ksi. P and γ_a are independent.



Solution:

Section Properties

$$I = \frac{1}{12}(8)(16^3) = 2731\,\text{in}^4$$

$$Q_C = \overline{y}'A' = 7(8)(2) = 112\,\text{in}^3$$

Shear Stress

$$\tau_{xy} = \frac{VQ_c}{It} = \frac{PQ_c}{It} = \frac{P(112)}{(2731)(8)} = (0.005)P$$

Shear Strain

$$\gamma_{xy} = \frac{\tau_{xy}}{G} = \frac{(0.005)P}{11 \times 10^3} = \left(4.55 \times 10^{-7}\right)P$$

Since $P \sim N(16, 1^2)$ kips, we have

$$\mu_{\gamma_{xy}} = \left(4.55 \times 10^{-7}\right)\mu_P = 7.28 \times 10^{-6}$$

$$\sigma_{\gamma_{xy}} = \left(4.55 \times 10^{-7}\right)\sigma_P = 4.55 \times 10^{-7}$$

Set $Y = \gamma_a - \gamma_{xy}$, then $Y \sim N(\mu_Y, \sigma_Y^2)$, where

$$\mu_Y = \mu_{\gamma_a} - \mu_{\gamma_{xy}} = 9\times10^{-6} - 7.28\times10^{-6} = 1.72\times10^{-6}$$

$$\sigma_Y = \sqrt{\sigma_{\gamma_a}^2 + \sigma_{\gamma_{xy}}^2} = \sqrt{0 + (6.83\times10^{-7})^2} = 4.55\times10^{-7}$$

Thus, the probability of failure of the cantilever could be obtained by

$$p_f = \Pr(Y < 0) = \Pr\left(\frac{Y-\mu_Y}{\sigma_Y} < \frac{-\mu_Y}{\sigma_Y}\right) = \Phi\left(\frac{-\mu_Y}{\sigma_Y}\right) = \Phi(-3.78) = 7.84\times10^{-5}$$ **Ans.**