1-4. The allowable average shear stress τ_b in each of the 5-mm diameter bolts follows a normal distribution, $\tau_{ab} \sim N(75, 6^2)$ MPa. Along the four shaded shear planes the allowable average shear stress also follows a normal distribution, $\tau_{ap} \sim N(0.35, 0.03^2)$ MPa. Determine the maximum axial force P that can be applied to the joint, as shown in Fig. 1.4.1, if the probabilities of failure of the bolts and shear planes of the member are less than 10^{-4}.

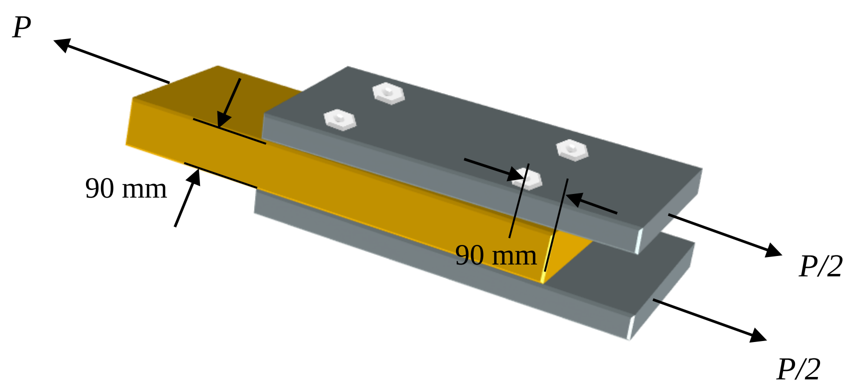

Fig. 1.4.1

Solution

Internal loading: The shear force developed on each shear plane of the bolt and the member can be determined by solving the force equation of equilibtium according to the free-body diagrams as shown in Figs. 1.4.2 and 1.4.3, respectively.

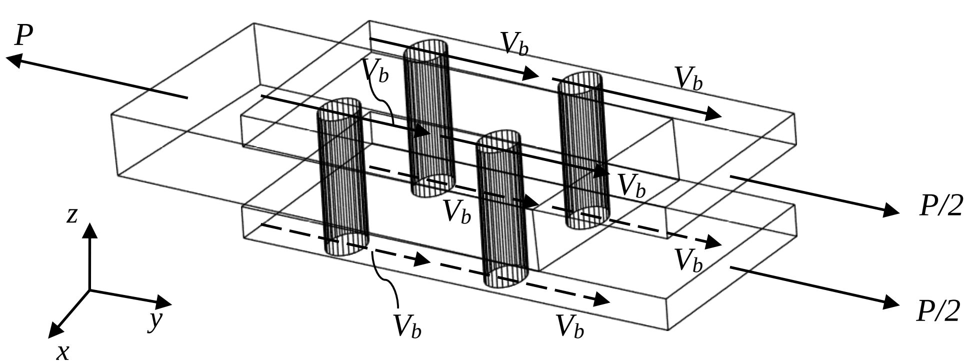

Fig. 1.4.2

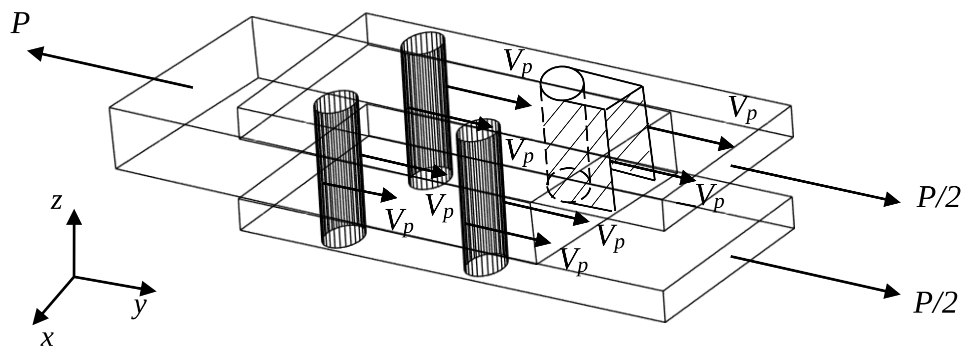

Fig. 1.4.3

$$\Sigma F_y = 0; \quad 4V_b - P = 0; \qquad V_b = P/4$$

$$\Sigma F_y = 0; \quad 4V_p - P = 0; \qquad V_p = P/4$$

Average Shear Stress: The areas of each shear plane of the bolts and the members are $A_b = \frac{\pi}{4}(0.005^2) = 19.625\times10^{-6}\ m^2$ and $A_p = 0.09^2 = 0.0081\, m^2$, respectively.

We obtain

$$\tau_b = \frac{V_b}{A_b} = \frac{P/4}{19.625\times10^{-6}} = 1.27\times10^4 P; \qquad \tau_p = \frac{V_b}{A_p} = \frac{P/4}{0.0081} = 30.864P$$

Let $Y_b = \tau_b - \tau_{ab}$, the probability of failure of each bolt can be expressed as

$$p_f = \Pr(Y_b = \tau_b - \tau_{ab} > 0) < 10^{-4}.$$

We can get μ_{Y_b} and σ_{Y_b} using

$$\mu_{Y_b} = \tau_b - \mu_{\tau_{ab}} = 1.27\times10^4 P - 75\times10^6; \qquad \sigma_{Y_b} = \sigma_{ab} = 6\times10^6$$

Then

$$p_f = \Pr(Y_b > 0) = \Pr\left(\frac{Y_b - \mu_{Y_b}}{\sigma_{Y_b}} > \frac{-\mu_{Y_b}}{\sigma_{Y_b}}\right) = 1 - \Phi\left(\frac{-\mu_{Y_b}}{\sigma_{Y_b}}\right) = 1 - \Phi\left(\frac{-(1.27\times10^4 P - 75\times10^6)}{6\times10^6}\right) < 10^{-4}$$

We can obtian that $P < 662\ \text{N}$ (controls). **Ans.**

Let $Y_p = \tau_p - \tau_{ap}$, the probability of failure of shear planes can be expressed as

$$p_f = \Pr(Y_p = \tau_p - \tau_{ap} > 0) < 10^{-4}.$$

We can get μ_{Y_p} and σ_{Y_p} using

$$\mu_{Y_p} = \tau_p - \mu_{\tau_{ap}} = 30.864P - 0.35\times10^6; \qquad \sigma_{Y_p} = \sigma_{ap} = 0.03\times10^6$$

Then

$$p_f = \Pr(Y_b > 0) = \Pr(\frac{Y_p - \mu_{Y_p}}{\sigma_{Y_p}} > \frac{-\mu_{Y_p}}{\sigma_{Y_p}}) = 1 - \Phi\left(\frac{-\mu_{Y_p}}{\sigma_{Y_p}}\right) = 1 - \Phi\left(\frac{-(30.864P - 0.35\times10^6)}{0.03\times10^6}\right) < 10^{-4}$$

We can obtian $P < 7725\mathrm{N}$.