1-5. The shaft is subjected to an axial force of $F \sim N(500,\ 80^2)\,\text{kN}$. If the allowable bearing stress of the collar A is $S_a \sim N\,(150,\ 20^2)\ \text{MPa}$. Calculate the probability of failure of the collar.

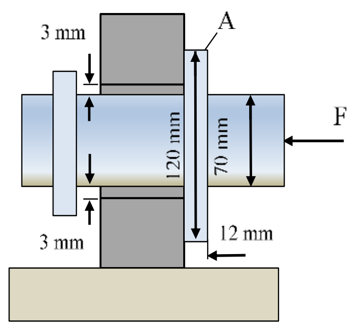

Fig. 1.5

Solution

The bearing area on the collar is $A_b = \dfrac{\pi(0.12^2 - 0.076^2)}{4} = 6.77\times10^{-3}\,\text{m}^2$. The bearing stress of the collar can be expressed as $S = \dfrac{F}{A_b}$. Set $Y = S_a - S$, then $Y \sim N(\mu_Y,\ \sigma_Y^2)\ \text{MPa}$, where

$$\mu_Y = \mu_{S_a} - \mu_S = \mu_{S_a} - \frac{\mu_F}{A_b} = 150\times10^6 - \frac{500\times10^3}{6.77\times10^{-3}} = 76.145\ \text{MPa}\ ,$$

$$\sigma_Y = \sqrt{\sigma_{S_a}^2 + \sigma_S^2} = \sqrt{(20\times10^6)^2 + \frac{(80\times10^3)^2}{(6.77\times10^{-3})^2}} = 23.23\ \text{MPa}\ .$$

Thus, the probability of failure of the collar is

$$p_f = \Pr\,(Y = S_a - S < 0) = \Pr\,(\frac{Y-\mu_Y}{\sigma_Y} < \frac{-\mu_Y}{\sigma_Y}) = \Phi(\frac{-\mu_Y}{\sigma_Y}) = \Phi(-3.27) = 5.23\times10^{-4}$$ **Ans.**