1-6. A vertical force of $F \sim N(2, 0.25^2)\,\text{kN}$ is applied to the bell crank. Assuming that the allowable normal stress at the 10-mm diameter rod *CD* is $S_{a1} \sim N(80, 5^2)$ MPa, and the allowable shear stress at the 6-mm diameter pin *B*, subjected to double shear is $S_{a2} \sim N(60, 6^2)$ MPa. If F, S_{a1} and S_{a2} are independent, determine the probabilities of failure of rod *CD* and pin *B*.

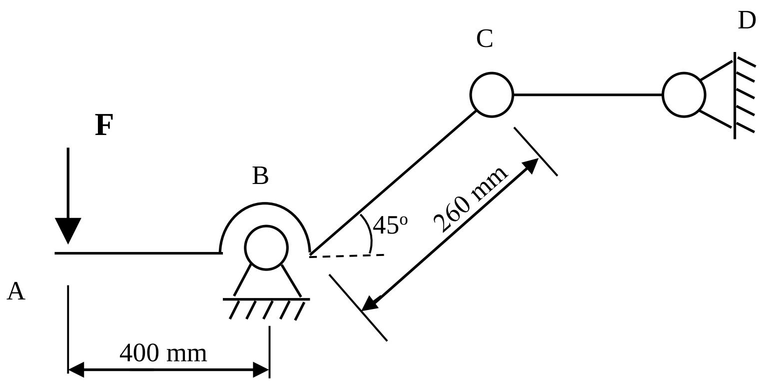

Fig. 1.6

Solution

Referring to the free-body diagram of the bell crank shown in Fig. 1.6.1.

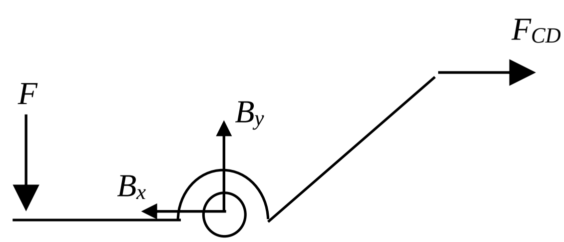

Fig. 1.6.1

$+\Sigma M_B = 0;\qquad F(0.4) - F_{CD}(0.26\sin 45^\circ) = 0;\qquad F_{CD} = 2.1757F;$

$\underset{\rightarrow}{+}\Sigma F_x = 0;\qquad -B_x + F_{CD} = 0;\qquad B_x = 2.1757F;$

$+\uparrow \Sigma F_y = 0;\qquad B_y - F = 0;\qquad B_y = F;$

Thus, the force acting on pin B is $F_B = \sqrt{B_x^2 + B_y^2} = 2.395F$.

Since pin B is in double shear, we have $V_B = \frac{F_B}{2} = 1.198F$.

The cross-section area of rod CD is $A_{CD} = \frac{\pi(0.01^2)}{4} = 78.540\times10^{-6}\,\text{m}^2$, and the area of the shear plane of pin B is $A_B = \frac{\pi(0.006^2)}{4} = 28.274\times10^{-6}\,\text{m}^2$. We obtain the normal stress S_1 at rod CD and shear stress S_2 at pin B as follows

$$S_1 = \frac{F_{CD}}{A_{CD}} = \frac{2.1757F}{78.540\times10^{-6}} = 0.0277\times10^6 F\ , \qquad S_2 = \frac{V_B}{A_B} = \frac{1.198F}{78.540\times10^{-6}} = 0.0153\times10^6 F\ .$$

Set $Y_1 = S_{a1} - S_1$, then $Y_1 \sim N(\mu_{Y1},\ \sigma_{Y1}^2)\,\text{MPa}$, where

$$\mu_{Y1} = \mu_{S_{a1}} - \mu_{S1} = \mu_{S_{a1}} - 0.0277\times10^6\,\mu_F = 80\times10^6 - 55.4\times10^6 = 24.6\,\text{MPa}$$

$$\sigma_{Y1} = \sqrt{\sigma_{S_{a1}}^2 + \sigma_{S1}^2} = \sqrt{(5\times10^6)^2 + (0.0277\times10^6\times0.25\times10^3)^2} = 8.541\text{MPa}\ .$$

Thus, the probability of failure of rod CD is

$$p_f = \Pr(Y_1 = S_{a1} - S_1 < 0) = \Pr(\frac{Y_1 - \mu_{Y1}}{\sigma_{Y1}} < \frac{-\mu_{Y1}}{\sigma_{Y1}}) = \Phi(\frac{-\mu_{Y1}}{\sigma_{Y1}}) = \Phi(-2.88) = 2\times10^{-3}\ .$$

Similarly, set $Y_2 = S_{a2} - S_2$, then $Y_2 \sim N(\mu_{Y2},\ \sigma_{Y2}^2)\,\text{MPa}$, where

$$\mu_{Y2} = \mu_{S_{a2}} - \mu_{S2} = \mu_{S_{a2}} - 0.0153\times10^6\,\mu_F = 60\times10^6 - 30.6\times10^6 = 29.4\,\text{MPa}$$

$$\sigma_{Y2} = \sqrt{\sigma_{S_{a2}}^2 + \sigma_{S2}^2} = \sqrt{(6\times10^6)^2 + (0.0153\times10^6\times0.25\times10^3)^2} = 7.116\text{MPa}\ .$$

Thus, the probability of pin B is

$$p_f = \Pr(Y_2 = S_{a2} - S_2 < 0) = \Pr(\frac{Y_2 - \mu_{Y2}}{\sigma_{Y2}} < \frac{-\mu_{Y2}}{\sigma_{Y2}}) = \Phi(\frac{-\mu_{Y2}}{\sigma_{Y2}}) = \Phi(-4.132) = 1.8\times10^{-5}\ .$$

Ans.