1-7. A box, whose wight P follows $P \sim N(0.5, 0.05^2)\,\text{kN}$, is suspended from wires AB and BC. If the wires have a normal failure stress of $S_a \sim N(150, 15^2)\,\text{MPa}$, determine the minimum diameter of each wire to make sure that the probilities of failure of the wires are less than 10^{-4}.

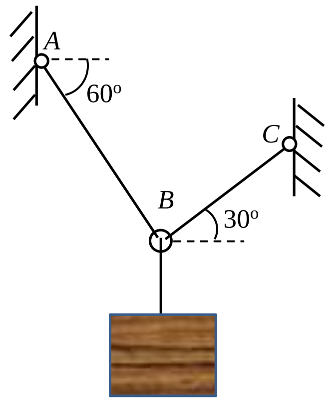

Fig. 1.7.1

Solution

Referring to the equilibrium of joint B in Fig. 1.7.2.

$\overset{+}{\rightarrow}\Sigma F_x = 0;\quad F_{BC}\cos 30^\circ - F_{AB}\cos 60^\circ = 0;\quad F_{AB} = 0.866P$;

$+\uparrow \Sigma F_y = 0;\quad F_{BC}\sin 30^\circ + F_{AB}\sin 60^\circ - P = 0;\quad F_{BC} = 0.5P$

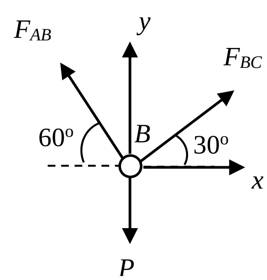

Fig. 1.7.2

Assuming that the diameter of the wires is d, and the normal stress of AB, BC are S_1, S_2 respectively. Then, the cross-section area of the wires is $A = \dfrac{\pi d^2}{4}$. We obtain

$$S_1 = \frac{F_{AB}}{A} = \frac{0.866P}{\left(\frac{1}{4}\pi d^2\right)} = \left(\frac{1.103}{d^2}\right)P, \qquad S_2 = \frac{F_{BC}}{A} = \frac{0.5P}{\left(\frac{1}{4}\pi d^2\right)} = \left(\frac{0.637}{d^2}\right)P.$$

Obviously, $S_1 > S_2$. Set $Y_1 = S_a - S_1$, then $Y_1 \sim N(\mu_{Y1},\ \sigma_{Y1}^2)$ MPa, where

$$\mu_{Y1} = \mu_{S_a} - \mu_{S1} = \mu_{S_a} - \left(\frac{1.103}{d^2}\right)\mu_P = 150\times10^6 - \left(\frac{1.103}{d^2}\right)\left(0.5\times10^3\right)$$

$$\sigma_{Y1} = \sqrt{\sigma_{S_a}^2 + \sigma_{S1}^2} = \sqrt{(15\times10^6)^2 + \left(\frac{1.103}{d^2}\times0.05\times10^3\right)^2}.$$

To make sure the probilities of failure of the wires are less than 10^{-4}, we just have to ensure that wire *AB* meets this requirement. Thus, the probability of failure of wire *AB* is

$$p_f = \Pr(Y_1 = S_a - S_1 < 0) = \Pr\left(\frac{Y_1 - \mu_{Y1}}{\sigma_{Y1}} < \frac{-\mu_{Y1}}{\sigma_{Y1}}\right) \le \Phi\left(\frac{-\mu_{Y1}}{\sigma_{Y1}}\right) = 10^{-4} = \Phi(-3.719)$$

$$\frac{-\mu_{Y1}}{\sigma_{Y1}} = -\frac{150\times10^6 - \left(\frac{1.103}{d^2}\right)\left(0.5\times10^3\right)}{\sqrt{(15\times10^6)^2 + \left(\frac{1.103}{d^2}\times0.05\times10^3\right)^2}} \le -3.719$$

$\Rightarrow d \ge 0.0025\,\text{m} = 2.5\,\text{mm}$. **Ans.**