1-8. The cover of a pressurized circular cylinder is fastened by bolts. The pressure is given by $P \sim N(350, 30^2)$ psi, and the allowable tensile stress in a bolt is $S_a \sim N(15000, 2000^2)$ psi. P and S_a are independent. The inner diameter of the cylinder is $D = 12.0$ in, and the diameter of the bolt is $d_B = 0.8$ in. To maintain the reliability of the bolts at least at 99.99%, how many bolts should be used?

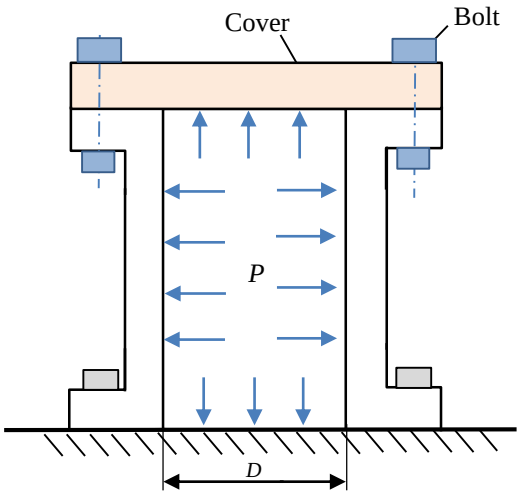

Fig. 1.8

Solution

The force acting on the cover plate is

$$F = P\left(\frac{\pi D^2}{4}\right) = \left(\frac{3.14\times 12^2}{4}\right)P = 113.04P .$$

Assumed that n bolts are needed to maintain the reliability of the bolts at at least 99.99%.

The tensile stress in all these bolts are $S = \dfrac{F}{A} = \dfrac{P\left(\dfrac{\pi D^2}{4}\right)}{n\left(\dfrac{\pi d_B{}^2}{4}\right)} = \dfrac{225P}{n}$.

It is known that $\mu_P = 350$, $\sigma_P = 30$, $\mu_{S_a} = 15000$, and $\sigma_{S_a} = 2000$. Since P and S_a are independent, if we set $Y = S_a - S$, then $Y \sim N(\mu_Y, \sigma_Y^2)$, and

$$\mu_Y = \mu_{S_a} - \mu_S = \mu_{S_a} - \left(\frac{225}{n}\right)\mu_P = 15000 - \left(\frac{225}{n}\right)350,$$

$$\sigma_Y = \sqrt{\sigma_{S_a}^2 + \sigma_S^2} = \sqrt{\sigma_{S_a}^2 + \left(\frac{225}{n}\right)^2 \sigma_P^2} = \sqrt{2000^2 + \left(\frac{225}{n}\right)^2 (30)^2}\ .$$

To maintain the reliability of the bolts at least at 99.99%, we have to make sure that the probolity of failure, p_f, of these bolts should be less than or equal to 0.01%; that is, to make sure that $p_f = \Pr(Y = S_a - S \le 0) \le 0.01\%$. Thus, we have

$$p_f = \Pr(Y \le 0) = \Pr\left(\frac{Y - \mu_Y}{\sigma_Y} \le \frac{-\mu_Y}{\sigma_Y}\right) = \Phi\left(\frac{-\mu_Y}{\sigma_Y}\right) \le 0.01\% = \Phi(-3.719),$$

$$\frac{-\mu_Y}{\sigma_Y} \le -3.719 \Rightarrow \frac{15000 - \left(\frac{225}{n}\right)350}{\sqrt{2000^2 + \left(\frac{225}{n}\right)^2 (30)^2}} \ge 3.719 \Rightarrow n \ge 10.416\ .$$

At least $n = 11$ bolts are needed to make sure that the reliability of these bolts is no less than 99.99%.