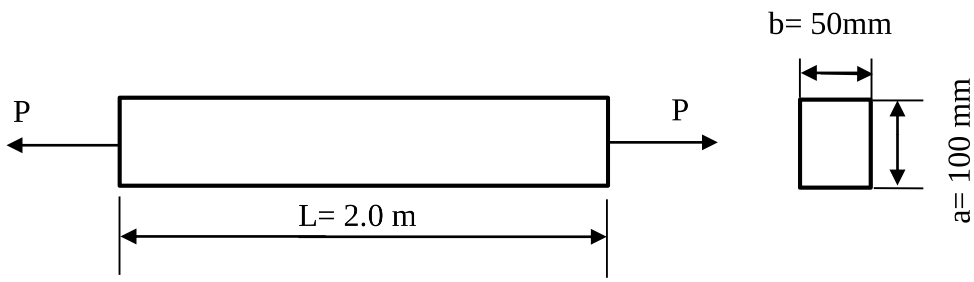
1-9. A bar is subject to an axial force P ~ $N(1000, 50^2)$ kN. If E = 120 GPa and v = 0.3, determine the distributions of the final lengths of sides L and b.



Solution

$$S = \frac{F}{A} = \frac{P}{ab}$$

$$\varepsilon_{long} = \frac{S}{E} = \frac{P}{Eab}$$

$$v = -\frac{\varepsilon_{lat}}{\varepsilon_{long}}; \varepsilon_{lat} = -v\varepsilon_{long}$$

Solving for the new length L

$$L_{new} = L + \Delta L = L + \varepsilon_{long} L$$

$$\Rightarrow L_{new} = L + \frac{PL}{Eab}$$

Thus

$$\mu_L = L + \frac{\mu_P L}{Eab} = 2.0 + \frac{(1000\times10^3)\times2.0}{(120\times10^3)\times100\times50} = 2.003 \text{ m}$$

$$\sigma_L = \sqrt{\left(\frac{L}{Eab}\right)^2 \sigma_P{}^2} = \sqrt{\left(\frac{2.0}{(120\times10^3}\right)^2 \times\left(50\times10^3\right)^2} = 1.667{\cdot}10^4 \text{ m}$$

Then

$$L \sim N\left(2.003, \left(1.667\times10^4\right)^2\right) \text{ m}$$ **Ans.**

Solving for the new width b

$$b_{new} = b + \Delta b = b + \varepsilon_{lat} b$$

$$\Rightarrow b_{new} = b - \frac{vP}{Ea}$$

Thus

$$\mu_b = b - \frac{v\mu_P}{Ea} = 50 - \frac{0.3\times(1000\times10^3)}{(120\times10^3)\times100} = 49.975 \text{ mm}$$

$$\sigma_b = \sqrt{\left(\frac{v}{Ea}\right)^2 \sigma_P{}^2} = \sqrt{\left(\frac{0.3}{(120\times10^3)\times100}\right)^2 \times \left(50\times10^3\right)^2} = 0.00125 \text{ mm}$$

Then

$$b \sim N\left(49.975, 0.00125^2\right) \text{ mm}$$ **Ans.**