2-1. Bar *AB* is hung by a cable at *C*. The total weight of the bar is $W \sim N(500, 45^2)$ N. A vertical force $P \sim N(700, 55^2)$ N acts on the bar at point *B*. Assuming that the allowable normal stress of the cable is $S_a \sim N(120, 10^2)$ MPa. If *W* and *P* are independent, determine the cross-cectional area of the cable to make sure that the probability of failure of the cable is less than 10^{-4}.

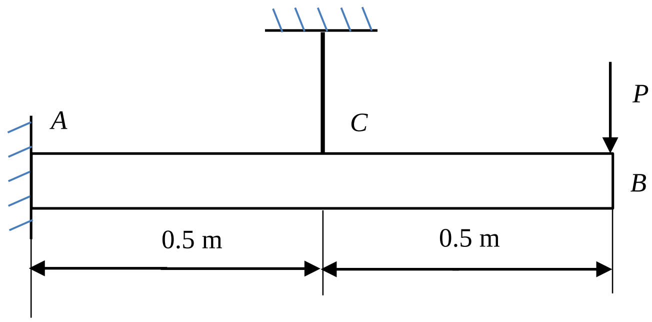

Fig. 2.1.1

Solution

From the diagram, we can find force *F* in the cable using

$\Sigma M_A = 0$, $F(0.5) - W(0.5) - P(0.5 + 0.5) = 0$, $F = W + 2P$.

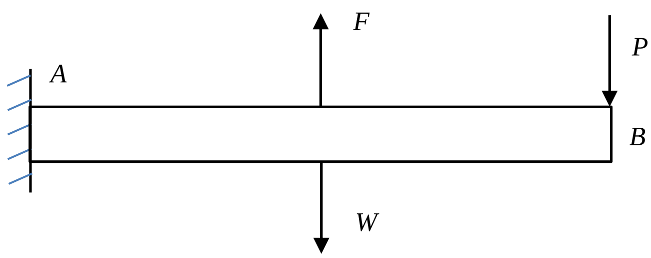

Fig. 2.1.2

Stress in the cable is

$$S = \frac{F}{A} = \frac{W + 2P}{A}$$

Set $Y = S_a - S$, then $Y \sim N(\mu_Y, \sigma_Y^2)$, where

$$\mu_Y = \mu_{S_a} - \mu_S = \mu_{S_a} - \frac{1}{A}(\mu_W + 2\mu_P) = 120\times 10^6 - \frac{1900}{A}$$

$$\sigma_Y = \sqrt{\sigma_{S_a}^2 + \frac{1}{A^2}\sigma_W^2 + \frac{1}{A^2}\sigma_P^2} = \sqrt{(10\times 10^6)^2 + \frac{1}{A^2}(45^2) + \frac{1}{A^2}(55^2)}$$

$$p_f = \Pr(Y<0) = \Pr\left(\frac{Y-\mu_Y}{\sigma_Y} < \frac{-\mu_Y}{\sigma_Y}\right) = \Phi\left(\frac{-\mu_Y}{\sigma_Y}\right) < 10^{-4} = \Phi\left(-3.719\right)$$

Thus, $\frac{-\mu_Y}{\sigma_Y} < -3.719$.

We have $A > 2.34\times 10^{-5}\,\text{m}^2 = 23.4\,\text{mm}^2$. **Ans.**