2-2. Truss *ABC* shown is subjected to two independent random forces P_1 and P_2, that follow a distribution of $N(400, 35^2)\,\text{kN}$. The span is $L = 4\,\text{m}$. Each member has the cross-sectional area $A = 4000\,\text{mm}^2$ and modulus of elasticity $E = 200\,\text{GPa}$. Determine the probability that the displacement of joint *B* is greater than 1.8 mm.

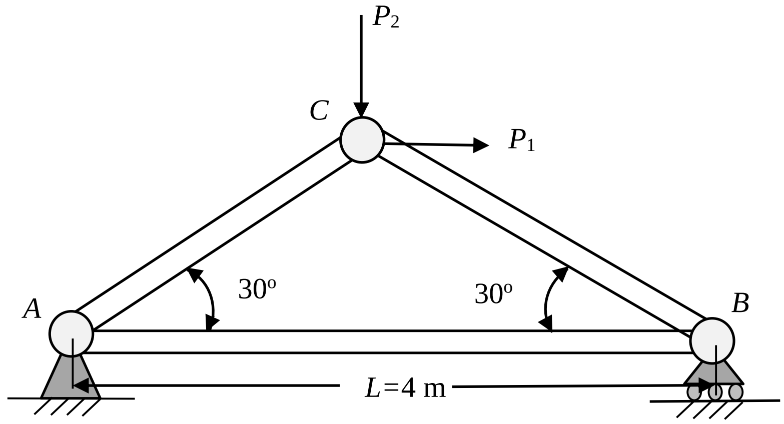

Fig. 2.2.1

Solution

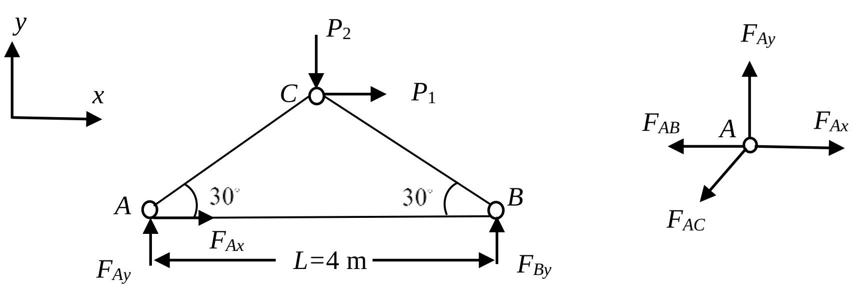

Fig. 2.2.2

(a) Find the displacement of joint *B* in the *x* direction

$$\Sigma M_A = 0, \qquad F_{By}L - P_1\left(\frac{L}{2}\right)\tan(30^\circ) - P_2\left(\frac{L}{2}\right) = 0, \qquad F_{By} = \left(\frac{1}{2}\right)\left(P_1\tan(30^\circ) + P_2\right)$$

$$\Sigma F_x = 0\ , \qquad F_{Ax} + P_1 = 0, \qquad F_{Ax} = -P_1$$

$\Sigma F_y = 0$, $\quad F_{Ay} - P_2 + F_{By} = 0$, $\quad F_{Ay} = P_2 - \left(\frac{1}{2}\right)\left(P_1 \tan(30^\circ) + P_2\right)$

Methods of joints:

$F_{ACy} = F_{Ay}$, $\quad F_{AC} = \dfrac{F_{Ay}}{\sin 30^\circ}$, $\quad F_{ACx} = F_{Ay} \cot 30^\circ = 0.866P_2 - 0.5P_1$

Since $F_{Ax} = F_{AB} + F_{ACx}$. We have $F_{AB} = F_{Ax} - F_{ACx} = P_1 - (0.866P_2 - 0.5P_1) = 1.5P_1 - 0.866P_2$.

The elongation of *AB* is equal to δ_B (the displacement of joint *B*) in the *x* direction; then

$$\delta_B = \frac{F_{AB}L}{EA} = \frac{L}{EA}\left(1.5P_1 - 0.866P_2\right) = \frac{4\left(1.5P_1 - 0.866P_2\right)}{2\times10^{11}\times4\times10^{-3}} = 0.75\times10^{-8}P_1 - 0.433\times10^{-8}P_2$$

Since $\delta_B \sim N(\mu_{\delta_B}, \sigma_{\delta_B}^2)$, $\mu_{P1} = \mu_{P2} = \mu_P = 400\times10^3$, $\sigma_{P1} = \sigma_{P2} = \sigma_P = 35\times10^3$, we have

$$\mu_{\delta_B} = 0.75\times10^{-8}\mu_{P1} - 0.433\times10^{-8}\mu_{P2} = 0.317\times10^{-8}\mu_P = 1.268\times10^{-3}$$

$$\sigma_{\delta_B} = \sqrt{(0.75\times10^{-8})^2\sigma_{P1}^2 - (0.433\times10^{-8})^2\sigma_{P_2}^2} = 0.6124\times10^{-8}\sigma_P = 2.1434\times10^{-4}$$

Thus, $\Pr(\delta_B > 1.8\times10^{-3}) = 1 - \Pr(\delta_B \le 1.8\times10^{-3}) = 1 - \Pr\left(\dfrac{\delta_B - \mu_P}{\sigma_P} \le \dfrac{1.8\times10^{-3} - 1.268\times10^{-3}}{2.1434\times10^{-4}}\right)$

$$= 1 - \Phi\left(2.482\right) = 1 - 0.9935 = 0.0065$$ **Ans.**