

2-3. The force acting on a round bar is $F \sim N(80, 10^2)$ lb. The bar has a diameter $d = 0.12$ in and a length $L = 10$ ft. The modulus of elasticity is $E = 10,600$ ksi. If the maximum permissible elongation is $\delta_a = 0.11$ in, determine the probability of failure.

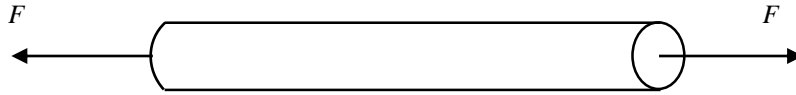


Fig. 2.3

Solution

The elongation of the wire under force F is

$$\delta = \frac{FL}{EA} = \frac{10(12)}{(10600 \times 10^3) \left(\frac{0.12^2 \pi}{4} \right)} F = 0.001F$$

Thus $\delta \sim N(\mu_\delta, \sigma_\delta)$, where

$$\mu_\delta = 0.001\mu_F = 0.001 \times 80 = 0.08$$

$$\sigma_\delta = 0.001\sigma_F = 0.001 \times 10 = 0.01.$$

Then

$$p_f = \Pr(\delta > \delta_a) = 1 - \Pr(\delta \leq \delta_a) = 1 - \Pr\left(\frac{\delta - \mu_\delta}{\sigma_\delta} \leq \frac{0.11 - 0.08}{0.01}\right) = 1 - \Phi(3) = 0.0013 \quad \text{Ans.}$$