2-4. The rod is subject to forces P and Q, which follow the distribution $P \sim N\left(7, 0.5^2\right)$ kN and $Q \sim N\left(16, 1^2\right)$ kN. Determine the distributions of the displacements of point B and point A given $E_{BC} = 190$ GPa, $E_{AB} = 74$ GPa, $d_{AB} = 11$ mm, and $d_{BC} = 15$ mm. Assume P and Q are independent.

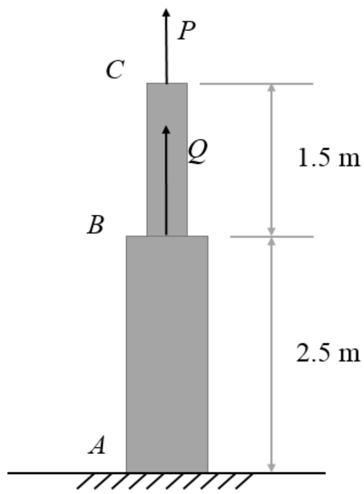

Fig. 2.4

Solution

Solve for the areas of each section.

$$A_{BC} = \frac{\pi}{4} d_{BC}^2 = \frac{\pi}{4}(0.015)^2 = 1.767 \times 10^{-4} \text{ m}^2$$

$$A_{AB} = \frac{\pi}{4} d_{AB}^2 = \frac{\pi}{4}(0.011)^2 = 9.503 \times 10^{-5} \text{ m}^2$$

Find the displacement of point B in terms of Q.

$$\delta_B = \frac{FL}{AE} = \frac{QL_{BC}}{A_{BC}E_{BC}}$$

Using the equation above, solve for μ_B and σ_B.

$$\mu_B = \frac{\mu_Q L_{BC}}{A_{BC}E_{BC}} = \frac{(16)\left(10^3\right)(2.5)}{\left(1.767 \times 10^{-4}\right)(190)\left(10^9\right)} = 0.0012 \text{ m} = 1.2 \text{ mm}$$

$$\sigma_B = \sqrt{\left(\frac{L_{BC}}{A_{BC}E_{BC}}\right)^2 \sigma_Q^2} = \sqrt{\left(\frac{2.5}{\left(1.767\times10^{-4}\right)(190)\left(10^9\right)}\right)^2 \left((1)\left(10^3\right)\right)^2} = 7.45\times10^{-5}\text{ m} = 0.0745\text{ mm}$$

Thus

$$\delta_B \sim N\left(1.2, 0.0745^2\right)\text{ mm} \qquad \textbf{Ans.}$$

Find the displacement of point A in terms of P and Q.

$$\delta_A = \Sigma\frac{FL}{AE} = \frac{QL_{BC}}{A_{BC}E_{BC}} + \frac{PL_{AB}}{A_{AB}E_{AB}}$$

Using the equation above, solve for μ_A and σ_A.

$$\mu_A = \frac{\mu_Q L_{BC}}{A_{BC}E_{BC}} + \frac{\mu_P L_{AB}}{A_{AB}E_{AB}} = \frac{(16)\left(10^3\right)(2.5)}{\left(1.767\times10^{-4}\right)(190)\left(10^9\right)} + \frac{(7)\left(10^3\right)(1.5)}{\left(9.503\times10^{-5}\right)(74)\left(10^9\right)}$$

$$= 0.0027\text{ m} = 2.7\text{ mm}$$

$$\sigma_A = \sqrt{\left(\frac{L_{BC}}{A_{BC}E_{BC}}\right)^2 \sigma_Q^2 + \left(\frac{L_{AB}}{A_{AB}E_{AB}}\right)^2 \sigma_P^2}$$

$$= \sqrt{\left(\frac{2.5}{\left(1.767\times10^{-4}\right)(190)\left(10^9\right)}\right)^2 (1000)^2 + \left(\frac{1.5}{\left(9.503\times10^{-5}\right)(74)\left(10^9\right)}\right)^2 (500)^2}$$

$$= 1.30\times10^{-4}\text{ m} = 0.13\text{ mm}$$

Thus

$$\delta_A \sim N\left(2.7, 0.13^2\right)\text{ mm} \qquad \textbf{Ans.}$$