2-5. Six reinforcing rods, each with a diamter of $d_{rod} = 19$ mm, are embedded in a concrete piece with a diameter of $D = 275$ mm. If the piece is subjected to an axial force of $P \sim N(815, 11^2)$ kN, determine the normal stress in the concrete and in the rods. The constants $E_{rod} = 210$ GPa and $E_{con} = 25$ GPa are given.

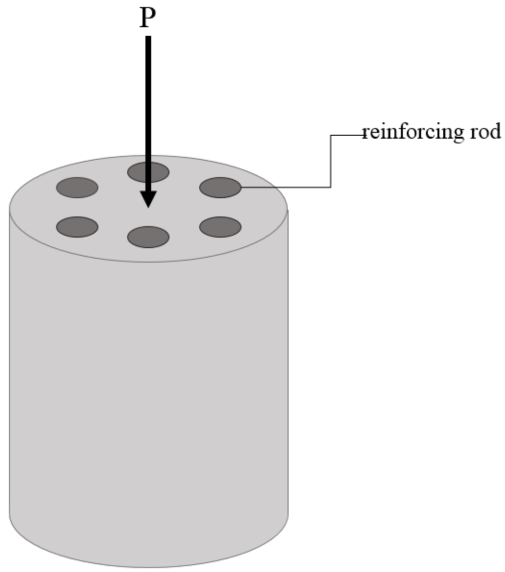

Fig. 2.5

Solution

Find the area of the rod and of the concrete.

$$A_{rod} = 6\frac{\pi}{4}d_{rod}^2 = 6\frac{\pi}{4}(0.019)^2 = 0.00170 \text{ m}^2$$

$$A_{con} = \frac{\pi}{4}D^2 - 6A_{rod} = \frac{\pi}{4}(0.275)^2 - (0.017) = 0.0577 \text{ m}^2$$

Develop the equilibrium equation for the piece.

$$+\uparrow \Sigma F_Y = F_{con} + F_{rod} - P = 0$$

Next, develop a compatability equation.

$$\delta_{rod} = \delta_c$$

$$\frac{F_{rod}L}{A_{rods}E_{rod}} = \frac{F_{con}L}{A_{con}E_{con}}$$

$$\Rightarrow F_{rod} = F_{con}\frac{A_{rod}}{A_{con}}\frac{E_{rod}}{E_{con}}$$

$$\Rightarrow F_{con} = F_{rod} \frac{A_{con}}{A_{rod}} \frac{E_{con}}{E_{rod}}$$

To solve for the normal stress in the rods, combine the equilibruim equation and the compatability equation to derive an equation for F_{rod} in terms of *P*.

$$F_{rod} \frac{A_{con}}{A_{rod}} \frac{E_{con}}{E_{rod}} + F_{rod} = P$$

$$F_{rod} \left(1 + \frac{A_{con}}{A_{rod}} \frac{E_{con}}{E_{rod}}\right) = P$$

$$\Rightarrow F_{rod} = P \left(1 + \frac{A_{con}}{A_{rod}} \frac{E_{con}}{E_{rod}}\right)^{-1}$$

Solve for the normal stress in the rods.

$$S_{rod} = \frac{F_{rod}}{A_{rod}}$$

$$S_{rod} = P \left(1 + \frac{A_{con}}{A_{rod}} \frac{E_{con}}{E_{rod}}\right)^{-1} \left(\frac{1}{A_{rod}}\right)$$

$$\mu_{rod} = \mu_P \left(1 + \frac{A_{con}}{A_{rod}} \frac{E_{con}}{E_{rod}}\right)^{-1} \left(\frac{1}{A_{rod}}\right) = (815000)\left(1 + \left(\frac{0.0577}{0.00170}\right)\left(\frac{25}{210}\right)\right)^{-1} \left(\frac{1}{0.00171}\right)$$

= 95.1 MPa

$$\sigma_{rod} = \sqrt{\left(\left(1 + \frac{A_{con}}{A_{rod}} \frac{E_{con}}{E_{rod}}\right)^{-1} \left(\frac{1}{A_{rod}}\right)\right)^2 \sigma_P^2}$$

$$= \sqrt{\left(\left(1 + \left(\frac{0.0577}{0.00170}\right)\left(\frac{25}{210}\right)\right)^{-1} \left(\frac{1}{0.00170}\right)\right)^2 (11000)^2}$$

= 0.00128 MPa

Thus,

$$S_{rod} \sim N\left(95.1, 0.00128^2\right) \text{ MPa}$$ **Ans.**

To solve for the normal stress in the concrete, combine the equilibrium equation and the compatability equation to derive an equation for F_{con} in terms of *P*.

$$F_{con} + F_{con} \frac{A_{rod}}{A_{con}} \frac{E_{rod}}{E_{con}} = P$$

$$F_{con} \left(1 + \frac{A_{rod}}{A_{con}} \frac{E_{rod}}{E_{con}}\right) = P$$

$$F_{con} = P\left(1+\frac{A_{rod}}{A_{con}}\frac{E_{rod}}{E_{con}}\right)^{-1}$$

Solve for the normal stress in the concrete.

$$S_{con} = \frac{F_{con}}{A_{con}}$$

$$S_{con} = P\left(1+\frac{A_{rod}}{A_{con}}\frac{E_{rod}}{E_{rod}}\right)^{-1}\left(\frac{1}{A_{con}}\right)$$

$$\mu_{con} = \mu_P\left(1+\frac{A_{rod}}{A_{con}}\frac{E_{rod}}{E_{con}}\right)^{-1}\left(\frac{1}{A_{con}}\right) = (815000)\left(1+\frac{0.00170}{0.0577}\frac{210}{25}\right)^{-1}\left(\frac{1}{0.0577}\right)$$

= 11.3 MPa

$$\sigma_{con} = \sqrt{\left(\left(1+\frac{A_{rod}}{A_{con}}\frac{E_{rod}}{E_{con}}\right)^{-1}\left(\frac{1}{A_{con}}\right)\right)^2\sigma_P^2}$$

$$= \sqrt{\left(\left(1+\frac{0.00170}{0.0577}\frac{210}{25}\right)^{-1}\left(\frac{1}{0.0577}\right)\right)^2(11000)^2}$$

= 0.000153 MPa

Thus

$$S_{con} \sim N\left(11.3, 0.000153^2\right) \text{ MPa}$$ **Ans.**