2-6. The assembly is subjected to a force P that follows the normal distribution $P \sim N\left(195, 8^2\right)$ kN. If the initial gap δ is equal to 0.26 mm before the force is applied, what is the probability that the gap is filled, given E = 245 GPa?

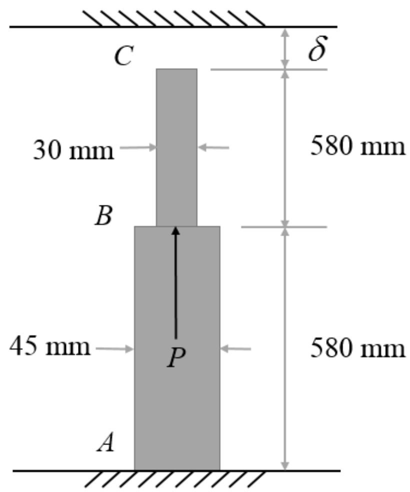

Fig. 2.6

Solution

Solve for the displacement of point C in terms of P. Do not consider if this displacement will fill the gap at this point.

$$d_C = d_B = \frac{FL}{AE} = \frac{PL_{AB}}{A_{AB}E}$$

$$\Rightarrow d_C = \frac{L_{AB}}{A_{AB}E}P$$

$$A_{AB} = \frac{\pi}{4}D_{AB}^2 = \frac{\pi}{4}(0.045)^2 = 0.00159 \text{ m}^2$$

The probability that the gap is filled is $p = \Pr\left(\delta \le d_C\right)$. Let $Y = \delta - d_C$. Then,

$$Y = \delta - d_C = \delta - \frac{L_{AB}}{A_{AB}E}P$$

Then, p is written as $p = \Pr\left(Y \le 0\right)$. Y follows a normal distribution, $Y = \left(\mu_Y, \sigma_Y^2\right)$.

$$\mu_Y = \delta - \frac{L_{AB}}{A_{AB}E}\mu_P = 2.6\times10^{-4} - \frac{0.58}{(0.00159)(245)\left(10^9\right)}(195000) = -4.026\times10^{-5} \text{ m}$$

$$= -0.04026 \text{ mm}$$

$$\sigma_Y = \sqrt{\left(\frac{L_{AB}}{A_{AB}E}\right)^2 \sigma_P^2} = \sqrt{\left(\frac{0.58}{(0.00159)(245)(10^9)}\right)^2 (8000)^2} = 1.191\times 10^{-5} \text{ m} = 0.01191 \text{ mm}$$

Therefore

$$p = \Pr(Y \le 0) = \Pr\left(\frac{Y-\mu_Y}{\sigma_Y} < \frac{-\mu_Y}{\sigma_Y}\right) = \Phi\left(-\frac{\mu_Y}{\sigma_Y}\right) = \Phi\left(-\frac{-0.04026}{0.01191}\right)$$

$$= \Phi(3.3806) = 0.99964$$ **Ans.**