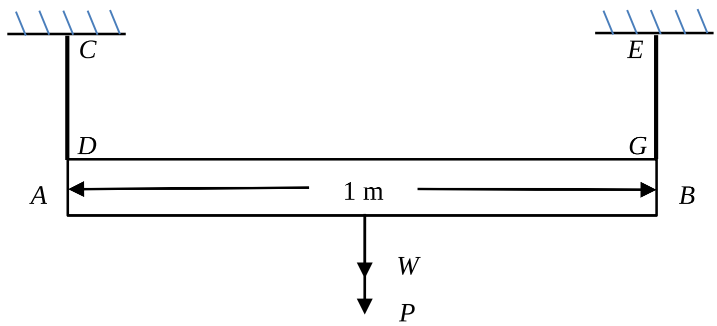
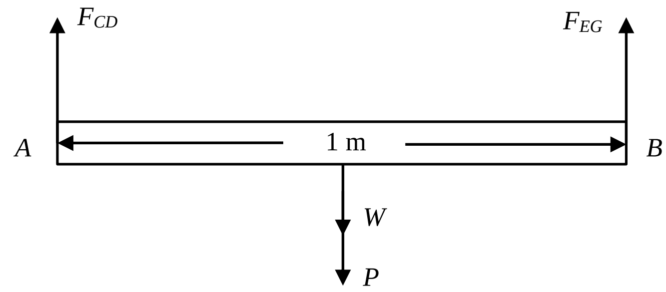
2-7. Bar *AB* is hung by two cables *CD* and *EG*. The total weight of the bar is $W \sim N(800,\ 10^2)$ N. A vertical force $P \sim N(900,\ 12^2)$ N acts on the bar at the middle point. Assume that the allowable normal stress of the cables is $S_a \sim N(110, 10^2)$ MPa. If W, P, and S_a are independent, determine the cross-cectional area of the cables to make sure that the probability of failure of each cable is less than 10^{-4}.



Solution:

The internal forces developed in cables *CD* and *EG* are shown in the free-body diagram below.



$$\circlearrowleft +\Sigma M_A = 0, \quad F_{EG}(1) - P(0.5) - W(0.5) = 0, \quad F_{EG} = 0.5(W+P)$$

$$\circlearrowleft +\Sigma M_B = 0, \quad -F_{CD}(1) + P(0.5) + W(0.5) = 0, \quad F_{CD} = 0.5(W+P)$$

The stress in cables *CD* and *EG* is

$$S = \frac{F_{CD}}{A} = \frac{F_{EG}}{A} = \frac{0.5(W+P)}{A}$$

Set $Y = S_a - S$, and then $Y \sim N(\mu_Y, \sigma_Y^2)$, where

$$\mu_Y = \mu_{S_a} - \mu_S = \mu_{S_a} - \frac{0.5(\mu_W + \mu_P)}{A} = 110\times10^6 - \frac{850}{A}$$

$$\sigma_Y = \sqrt{\sigma_{S_a}^2 + \left(\frac{0.5}{A}\right)^2 \sigma_W^2 + \left(\frac{0.5}{A}\right)^2 \sigma_P^2} = \sqrt{(10^7)^2 + \left(\frac{0.5}{A}\right)^2 \left(10^2\right) + \left(\frac{0.5}{A}\right)^2 \left(12^2\right)}$$

$$p_f = \Pr(Y < 0) = \Pr\left(\frac{Y-\mu_Y}{\sigma_Y} < \frac{-\mu_Y}{\sigma_Y}\right) = \Phi\left(\frac{-\mu_Y}{\sigma_Y}\right) < 10^{-4} = \Phi\left(-3.719\right)$$

Thus, $\frac{-\mu_Y}{\sigma_Y} < -3.719$.

We have $A > 1.17 \times 10^{-5} \text{m}^2 = 11.7 \text{mm}^2$. **Ans.**