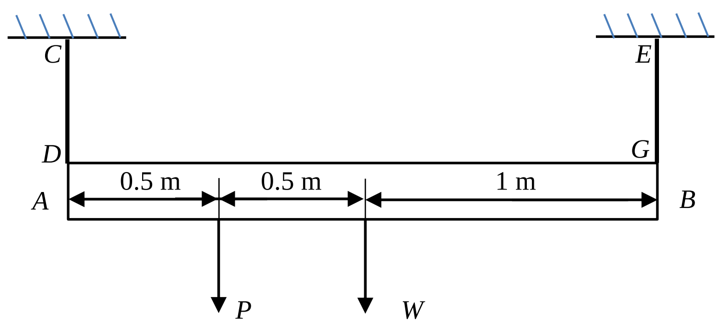
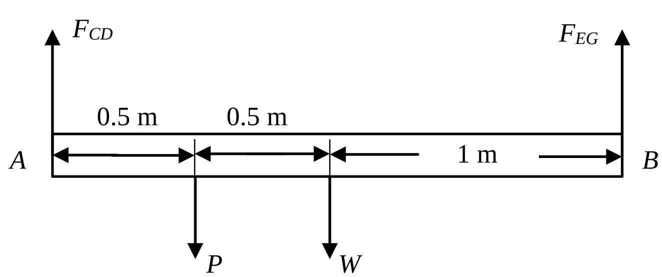
2-8. Bar *AB* is hung by two cables *CD* and *EG*. The total weight of the bar is $W \sim N(1200,\ 15^2)$ N. A vertical force $P \sim N(800,\ 12^2)$ N acts on the bar as shown in the figure. Assume that the allowable normal stress of the cables is $S_a \sim N(110, 10^2)$ MPa. If W, P, and S_a are independent, determine the diameters of the cables to make sure that the probability of failure of each cable is less than 10^{-4}.



Solution:

The internal forces developed in cables *CD* and *EG* are shown below.



$$\circlearrowleft +\Sigma M_A = 0, \quad F_{EG}(2) - P(0.5) - W(1) = 0, \quad F_{EG} = \frac{W}{2} + \frac{P}{4}$$

$$\circlearrowleft +\Sigma M_B = 0, \quad -F_{CD}(2) + P(1.5) + W(1) = 0, \quad F_{CD} = \frac{W}{2} + \frac{3P}{4}$$

The stress in cable *CD* is

$$S_{CD} = \frac{F_{CD}}{A} = \frac{W}{2A} + \frac{P}{4A}$$

Set $Y_1 = S_a - S_{CD}$, and then $Y_1 \sim N(\mu_{Y_1}, \sigma_{Y_1}^2)$, where

$$\mu_{Y_1} = \mu_{S_a} - \mu_{S_{CD}} = \mu_{S_a} - \frac{\mu_W}{2A} - \frac{\mu_P}{4A} = 110\times10^6 - \frac{800}{A}$$

$$\sigma_Y = \sqrt{\sigma_{S_a}^2 + \left(\frac{1}{2A}\right)^2 \sigma_W^2 + \left(\frac{1}{4A}\right)^2 \sigma_P^2} = \sqrt{(10\times10^6)^2 + \left(\frac{0.5}{A}\right)^2 \left(15^2\right) + \left(\frac{0.5}{A}\right)^2 \left(12^2\right)}$$

$$p_f = \Pr(Y<0) = \Pr\left(\frac{Y-\mu_Y}{\sigma_Y} < \frac{-\mu_Y}{\sigma_Y}\right) = \Phi\left(\frac{-\mu_Y}{\sigma_Y}\right) < 10^{-4} = \Phi\left(-3.719\right)$$

Then, $\frac{-\mu_Y}{\sigma_Y} < -3.719$. We have $A = \frac{\pi d_{CD}^2}{4} > 1.1\times10^{-5}\,\text{m}^2 = 11\,\text{mm}^2$.

Thus, $d_{CD} > 3.74\,\text{mm}$. **Ans.**

The stress in cable *EG* is

$$S_{EG} = \frac{F_{EG}}{A} = \frac{W}{2A} + \frac{3P}{4A}$$

Set $Y_2 = S_a - S_{EG}$, and then $Y_2 \sim N(\mu_{Y_2}, \sigma_{Y_2}^2)$, where

$$\mu_{Y_2} = \mu_{S_a} - \mu_{S_{EG}} = \mu_{S_a} - \frac{\mu_W}{2A} - \frac{3\mu_P}{4A} = 110\times10^6 - \frac{1200}{A}$$

$$\sigma_Y = \sqrt{\sigma_{S_a}^2 + \left(\frac{1}{2A}\right)^2 \sigma_W^2 + \left(\frac{3}{4A}\right)^2 \sigma_P^2} = \sqrt{(10\times10^6)^2 + \left(\frac{0.5}{A}\right)^2 \left(15^2\right) + \left(\frac{0.75}{A}\right)^2 \left(12^2\right)}$$

$$p_f = \Pr(Y<0) = \Pr\left(\frac{Y-\mu_Y}{\sigma_Y} < \frac{-\mu_Y}{\sigma_Y}\right) = \Phi\left(\frac{-\mu_Y}{\sigma_Y}\right) < 10^{-4} = \Phi\left(-3.719\right)$$

Then, $\frac{-\mu_Y}{\sigma_Y} < -3.719$. We have $A = \frac{\pi d_{EG}^2}{4} > 1.65\times10^{-5}\,\text{m}^2 = 16.5\,\text{mm}^2$.

Thus, $d_{EG} > 4.58\,\text{mm}$. **Ans.**