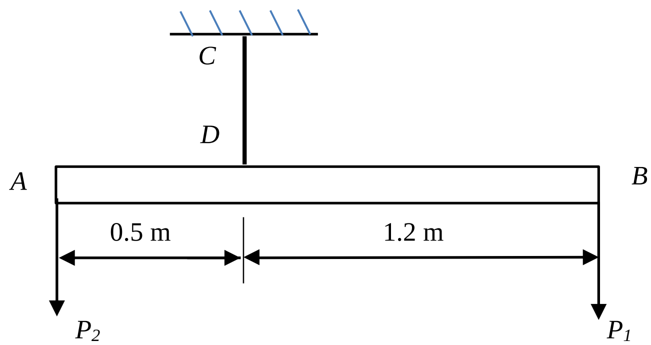
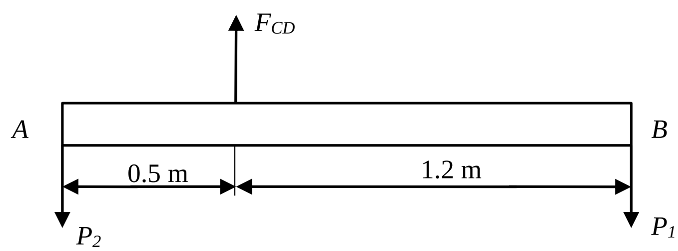
2-9. Bar *AB*, which is hung by a cable *CD*, is used to support a load. The weight of the bar is negligible. Force $P_1 \sim N(700, 55^2)$ N acts on the bar at point *B* to support the load P_2 that acts at point *A*. Assuming that the allowable normal stress of the cable is $S_a \sim N(150, 12^2)$ MPa, and that P_1, P_2, and S_a are independent, determine the diameter of the cable to make sure that the probability of failure is less than 10^{-4}.



Solution:

The internal force developed in cable *CD* is shown below.



$$\circlearrowleft +\Sigma M_A = 0, \quad F_{CD}(0.5) - P_1(1.7) = 0, \quad F_{CD} = 3.4P_1$$

The stress in cable *CD* is

$$S_{CD} = \frac{F_{CD}}{A} = \frac{3.4P_1}{A}$$

Set $Y = S_a - S_{CD}$, and then $Y \sim N(\mu_Y, \sigma_Y^2)$, where

$$\mu_Y = \mu_{S_a} - \mu_{S_{CD}} = \mu_{S_a} - \frac{3.4\mu_{P_1}}{A} = 150\times10^6 - \frac{2380}{A}$$

$$\sigma_Y = \sqrt{\sigma_{S_a}^2 + \left(\frac{3.4}{A}\right)^2 \sigma_P^2} = \sqrt{(12\times10^6)^2 + \left(\frac{3.4}{A}\right)^2 \left(55^2\right)}$$

$$p_f = \Pr(Y < 0) = \Pr\left(\frac{Y-\mu_Y}{\sigma_Y} < \frac{-\mu_Y}{\sigma_Y}\right) = \Phi\left(\frac{-\mu_Y}{\sigma_Y}\right) < 10^{-4} = \Phi\left(-3.719\right)$$

Then, $\frac{-\mu_Y}{\sigma_Y} < -3.719$. We have $A = \frac{\pi d_{CD}^2}{4} > 2.45\times10^{-5}\,\text{m}^2 = 24.5\,\text{mm}^2$.

Thus, $d_{CD} > 5.59\,\text{mm}$. **Ans.**